1.6 Classify Polygons

Before You classified angles.

Now You will classify polygons.

Why? So you can find lengths in a floor plan, as in Ex. 32.

Key Vocabulary
- **polygon**
  side, vertex
- **convex**
- **concave**
- ***n*-gon**
- **equilateral**
- **equiangular**
- **regular**

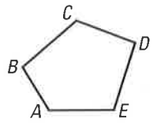
KEY CONCEPT *For Your Notebook*

Identifying Polygons

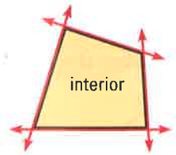
In geometry, a figure that lies in a plane is called a *plane figure*. A **polygon** is a closed plane figure with the following properties.

1. It is formed by three or more line segments called **sides**.
2. Each side intersects exactly two sides, one at each endpoint, so that no two sides with a common endpoint are collinear.

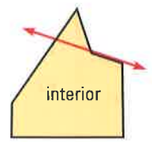
Each endpoint of a side is a **vertex** of the polygon. The plural of vertex is *vertices*. A polygon can be named by listing the vertices in consecutive order. For example, *ABCDE* and *CDEAB* are both correct names for the polygon at the right.



A polygon is **convex** if no line that contains a side of the polygon contains a point in the interior of the polygon. A polygon that is not convex is called *nonconvex* or **concave**.



convex polygon

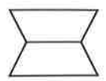


concave polygon

EXAMPLE 1 Identify polygons

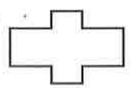
READ VOCABULARY
A *plane figure* is two-dimensional. Later, you will study three-dimensional *space figures* such as prisms and cylinders.

Tell whether the figure is a polygon and whether it is *convex* or *concave*.

a. 

b. 

c. 

d. 

Solution

a. Some segments intersect more than two segments, so it is not a polygon.

b. The figure is a convex polygon.

c. Part of the figure is not a segment, so it is not a polygon.

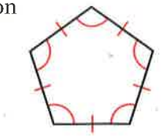
d. The figure is a concave polygon.

CLASSIFYING POLYGONS A polygon is named by the number of its sides.

Number of sides	Type of polygon	Number of sides	Type of polygon
3	Triangle	8	Octagon
4	Quadrilateral	9	Nonagon
5	Pentagon	10	Decagon
6	Hexagon	12	Dodecagon
7	Heptagon	*n*	*n*-gon

The term ***n*-gon**, where n is the number of a polygon's sides, can also be used to name a polygon. For example, a polygon with 14 sides is a 14-gon.

In an **equilateral** polygon, all sides are congruent. In an **equiangular** polygon, all angles in the interior of the polygon are congruent. A **regular** polygon is a convex polygon that is both equilateral and equiangular.

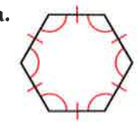


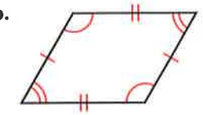
regular pentagon

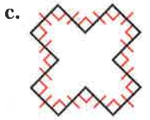
EXAMPLE 2 Classify polygons

READ DIAGRAMS
Double marks are used in part (b) of Example 2 to show that more than one pair of sides are congruent and more than one pair of angles are congruent.

Classify the polygon by the number of sides. Tell whether the polygon is equilateral, equiangular, or regular. Explain your reasoning.

a. 

b. 

c. 

Solution

a. The polygon has 6 sides. It is equilateral and equiangular, so it is a regular hexagon.

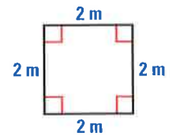
b. The polygon has 4 sides, so it is a quadrilateral. It is not equilateral or equiangular, so it is not regular.

c. The polygon has 12 sides, so it is a dodecagon. The sides are congruent, so it is equilateral. The polygon is not convex, so it is not regular.

Animated Geometry at classzone.com

GUIDED PRACTICE for Examples 1 and 2

1. Sketch an example of a convex heptagon and an example of a concave heptagon.
2. Classify the polygon shown at the right by the number of sides. *Explain* how you know that the sides of the polygon are congruent and that the angles of the polygon are congruent.



EXAMPLE 3 Find side lengths

READ VOCABULARY
Hexagonal means "shaped like a hexagon."

xy ALGEBRA **A table is shaped like a regular hexagon. The expressions shown represent side lengths of the hexagonal table. Find the length of a side.**

Solution

First, write and solve an equation to find the value of x. Use the fact that the sides of a regular hexagon are congruent.

$3x + 6 = 4x - 2$ **Write equation.**

$6 = x - 2$ **Subtract 3x from each side.**

$8 = x$ **Add 2 to each side.**

Then find a side length. Evaluate one of the expressions when $x = 8$.

$3x + 6 = 3(8) + 6 = 30$

▸ The length of a side of the table is 30 inches.

GUIDED PRACTICE for Example 3

3. The expressions $8y°$ and $(9y - 15)°$ represent the measures of two of the angles in the table in Example 3. Find the measure of an angle.

1.6 EXERCISES

HOMEWORK KEY
○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 13, 19, and 33
★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 7, 37, 39, and 40

SKILL PRACTICE

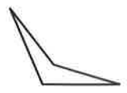
1. VOCABULARY *Explain* what is meant by the term n-gon.

2. ★ WRITING Imagine that you can tie a string tightly around a polygon. If the polygon is convex, will the length of the string be equal to the distance around the polygon? What if the polygon is concave? *Explain.*

EXAMPLE 1 on p. 42 for Exs. 3–7

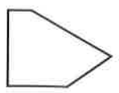
IDENTIFYING POLYGONS **Tell whether the figure is a polygon. If it is not, *explain* why. If it is a polygon, tell whether it is *convex* or *concave*.**

3.



4.

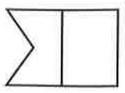
5.



6.

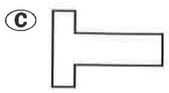
7. ★ MULTIPLE CHOICE Which of the figures is a concave polygon?

A

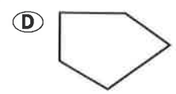


B

C



D

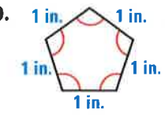


EXAMPLE 2 on p. 43 for Exs. 8–14

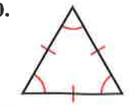
CLASSIFYING **Classify the polygon by the number of sides. Tell whether the polygon is equilateral, equiangular, or regular. *Explain* your reasoning.**

8.

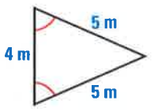
9.



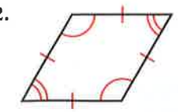
10.



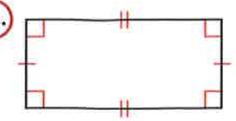
11.



12.

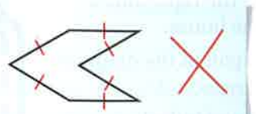


13.

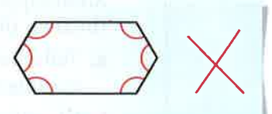


14. ERROR ANALYSIS Two students were asked to draw a regular hexagon, as shown below. *Describe* the error made by each student.

Student A



Student B



EXAMPLE 3 on p. 44 for Exs. 15–17

15. xy ALGEBRA The lengths (in inches) of two sides of a regular pentagon are represented by the expressions $5x - 27$ and $2x - 6$. Find the length of a side of the pentagon.

16. xy ALGEBRA The expressions $(9x + 5)°$ and $(11x - 25)°$ represent the measures of two angles of a regular nonagon. Find the measure of an angle of the nonagon.

17. xy ALGEBRA The expressions $3x - 9$ and $23 - 5x$ represent the lengths (in feet) of two sides of an equilateral triangle. Find the length of a side.

USING PROPERTIES **Tell whether the statement is *always*, *sometimes*, or *never* true.**

18. A triangle is convex.

19. A decagon is regular.

20. A regular polygon is equiangular.

21. A circle is a polygon.

22. A polygon is a plane figure.

23. A concave polygon is regular.

DRAWING **Draw a figure that fits the description.**

24. A triangle that is not regular

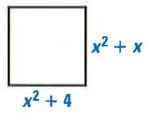
25. A concave quadrilateral

26. A pentagon that is equilateral but not equiangular

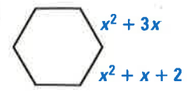
27. An octagon that is equiangular but not equilateral

xy ALGEBRA **Each figure is a regular polygon. Expressions are given for two side lengths. Find the value of x.**

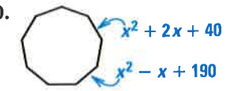
28.



29.



30.



Investigating Geometry **ACTIVITY** *Use before Lesson 1.7*

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Keystrokes

1.7 Investigate Perimeter and Area

MATERIALS • graph paper • graphing calculator

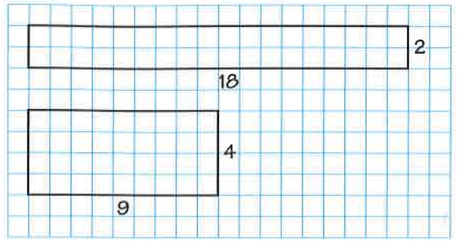
QUESTION **How can you use a graphing calculator to find the smallest possible perimeter for a rectangle with a given area?**

You can use the formulas below to find the perimeter P and the area A of a rectangle with length ℓ and width w.

$P = 2\ell + 2w$ $\qquad$ $A = \ell w$

EXPLORE **Find perimeters of rectangles with fixed areas**

STEP 1 ***Draw rectangles*** Draw different rectangles, each with an area of 36 square units. Use lengths of 2, 4, 6, 8, 10, 12, 14, 16, and 18 units.



STEP 2 ***Enter data*** Use the STATISTICS menu on a graphing calculator. Enter the rectangle lengths in List 1. Use the keystrokes below to calculate and enter the rectangle widths and perimeters in Lists 2 and 3.

Keystrokes for entering widths in List 2:

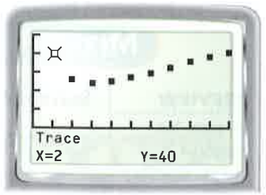
36 [÷] [2nd] [L1] [ENTER]

Keystrokes for entering perimeters in List 3:

2 [×] [2nd] [L1] [+] 2 [×] [2nd] [L2] [ENTER]

STEP 3 ***Make a scatter plot*** Make a scatter plot using the lengths from List 1 as the x-values and the perimeters from List 3 as the y-values. Choose an appropriate viewing window. Then use the *trace* feature to see the coordinates of each point.

How does the graph show which of your rectangles from Step 1 has the smallest perimeter?



DRAW CONCLUSIONS **Use your observations to complete these exercises**

1. Repeat the steps above for rectangles with areas of 64 square units.
2. Based on the Explore and your results from Exercise 1, what do you notice about the shape of the rectangle with the smallest perimeter?

1.7 Find Perimeter, Circumference, and Area

Before You classified polygons.
Now You will find dimensions of polygons.
Why? So you can use measures in science, as in Ex. 46.

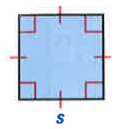
Key Vocabulary
- **perimeter,** *p. 923*
- **circumference,** *p. 923*
- **area,** *p. 923*
- **diameter,** *p. 923*
- **radius,** *p. 923*

Recall that *perimeter* is the distance around a figure, *circumference* is the distance around a circle, and *area* is the amount of surface covered by a figure. Perimeter and circumference are measured in units of length, such as meters (m) and feet (ft). Area is measured in square units, such as square meters (m^2) and square feet (ft^2).

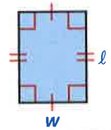
KEY CONCEPT *For Your Notebook*

Formulas for Perimeter P, Area A, and Circumference C

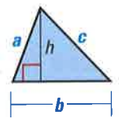
Square
side length s
$P = 4s$
$A = s^2$



Rectangle
length ℓ and width w
$P = 2\ell + 2w$
$A = \ell w$



Triangle
side lengths a, b, and c, base b, and height h
$P = a + b + c$
$A = \frac{1}{2}bh$

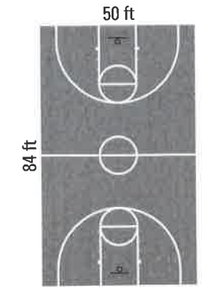


Circle
diameter d and radius r
$C = \pi d = 2\pi r$
$A = \pi r^2$

Pi (π) is the ratio of a circle's circumference to its diameter.

EXAMPLE 1 **Find the perimeter and area of a rectangle**

BASKETBALL **Find the perimeter and area of the rectangular basketball court shown.**



Perimeter	Area
$P = 2\ell + 2w$	$A = \ell w$
$= 2(84) + 2(50)$	$= 84(50)$
$= 268$	$= 4200$

▶ The perimeter is 268 feet and the area is 4200 square feet.

EXAMPLE 2 Find the circumference and area of a circle

TEAM PATCH You are ordering circular cloth patches for your soccer team's uniforms. Find the approximate circumference and area of the patch shown.

APPROXIMATE π
The approximations 3.14 and $\frac{22}{7}$ are commonly used as approximations for the irrational number π. Unless told otherwise, use 3.14 for π.

Solution

First find the radius. The diameter is 9 centimeters, so the radius is $\frac{1}{2}(9) = 4.5$ centimeters.

Then find the circumference and area. Use 3.14 to approximate the value of π.

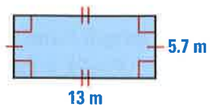
$C = 2\pi r \approx 2(3.14)(4.5) = 28.26$

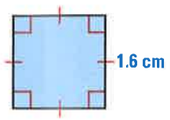
$A = \pi r^2 \approx 3.14(4.5)^2 = 63.585$

▶ The circumference is about 28.3 cm. The area is about 63.6 cm^2.

GUIDED PRACTICE for Examples 1 and 2

Find the area and perimeter (or circumference) of the figure. If necessary, round to the nearest tenth.

1. 

2. 

3. 

EXAMPLE 3 Standardized Test Practice

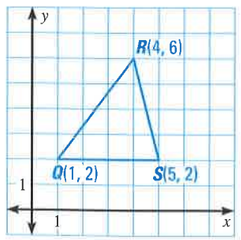
Triangle QRS has vertices $Q(1, 2)$, $R(4, 6)$, and $S(5, 2)$. What is the approximate perimeter of triangle QRS?

Ⓐ 8 units Ⓑ 8.3 units Ⓒ 13.1 units Ⓓ 25.4 units

AVOID ERRORS
Write down your calculations to make sure you do not make a mistake substituting values in the Distance Formula.

Solution

First draw triangle QRS in a coordinate plane. Find the side lengths. Use the Distance Formula to find QR and RS.



$QS = |5 - 1| = 4$ units

$QR = \sqrt{(4-1)^2 + (6-2)^2} = \sqrt{25} = 5$ units

$RS = \sqrt{(5-4)^2 + (2-6)^2} = \sqrt{17} \approx 4.1$ units

Then find the perimeter.

$P = QS + QR + RS \approx 4 + 5 + 4.1 = 13.1$ units

▶ The correct answer is C. Ⓐ Ⓑ Ⓒ Ⓓ

EXAMPLE 4 Solve a multi-step problem

SKATING RINK An ice-resurfacing machine is used to smooth the surface of the ice at a skating rink. The machine can resurface about 270 square yards of ice in one minute.

About how many minutes does it take the machine to resurface a rectangular skating rink that is 200 feet long and 90 feet wide?

ANOTHER WAY
For an alternative method for solving the problem in Example 4, turn to page 57 for the **Problem Solving Workshop**.

Solution

The machine can resurface the ice at a rate of 270 square yards per minute. So, the amount of time it takes to resurface the skating rink depends on its area.

STEP 1 **Find** the area of the rectangular skating rink.

Area $= \ell w = 200(90) = 18{,}000$ ft^2

The resurfacing rate is in square yards per minute. Rewrite the area of the rink in square yards. There are 3 feet in 1 yard, and $3^2 = 9$ square feet in 1 square yard.

$18{,}000\ ft^2 \cdot \frac{1\ yd^2}{9\ ft^2} = 2000\ yd^2$ **Use unit analysis.**

STEP 2 **Write** a verbal model to represent the situation. Then write and solve an equation based on the verbal model.

Let t represent the total time (in minutes) needed to resurface the skating rink.

Area of rink (yd^2)	=	Resurfacing rate (yd^2 per min)	×	Total time (min)

$2000 = 270 \cdot t$ **Substitute.**

$7.4 \approx t$ **Divide each side by 270.**

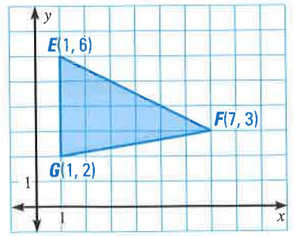
▶ It takes the ice-resurfacing machine about 7 minutes to resurface the skating rink.

GUIDED PRACTICE for Examples 3 and 4

4. Describe how to find the height from F to $\overline{EG}$ in the triangle at the right.

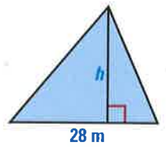
5. Find the perimeter and the area of the triangle shown at the right.

6. **WHAT IF?** In Example 4, suppose the skating rink is twice as long and twice as wide. Will it take an ice-resurfacing machine twice as long to resurface the skating rink? *Explain* your reasoning.



EXAMPLE 5 Find unknown length

The base of a triangle is 28 meters. Its area is 308 square meters. Find the height of the triangle.



Solution

$A = \frac{1}{2}bh$ Write formula for the area of a triangle.

$308 = \frac{1}{2}(28)h$ Substitute 308 for A and 28 for b.

$22 = h$ Solve for h.

▸ The height is 22 meters.

✓ GUIDED PRACTICE for Example 5

7. The area of a triangle is 64 square meters, and its height is 16 meters. Find the length of its base.

1.7 EXERCISES

HOMEWORK KEY

- ○ = **WORKED-OUT SOLUTIONS** on p. WS2 for Exs. 7, 21, and 41
- ★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 19, 26, 38, and 45
- ◆ = **MULTIPLE REPRESENTATIONS** Ex. 44

SKILL PRACTICE

1. **VOCABULARY** How are the diameter and radius of a circle related?

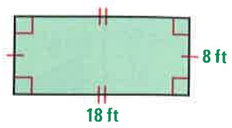
2. ★ **WRITING** *Describe* a real-world situation in which you would need to find a perimeter, and a situation in which you would need to find an area. What measurement units would you use in each situation?

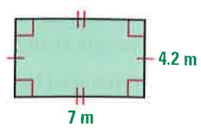
EXAMPLE 1 on p. 49 for Exs. 3–10

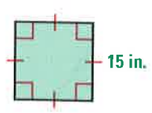
3. **ERROR ANALYSIS** *Describe* and correct the error made in finding the area of a triangle with a height of 9 feet and a base of 52 feet.

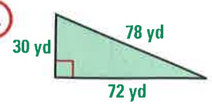
$A = 52(9) = 468 \text{ ft}^2$ ✗

PERIMETER AND AREA **Find the perimeter and area of the shaded figure.**

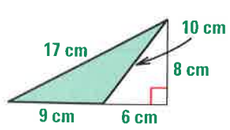
4. 

5. 

6. 

7. 

8. 

9. 

Animated Geometry at classzone.com

10. **DRAWING A DIAGRAM** The base of a triangle is 32 feet. Its height is $16\frac{1}{2}$ feet. Sketch the triangle and find its area.

EXAMPLE 2 on p. 50 for Exs. 11–15

CIRCUMFERENCE AND AREA **Use the given diameter d or radius r to find the circumference and area of the circle. Round to the nearest tenth.**

11. $d = 27$ cm 

12. $d = 5$ in. 

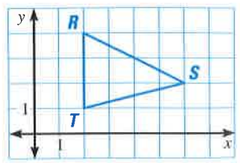
13. $r = 12.1$ cm 

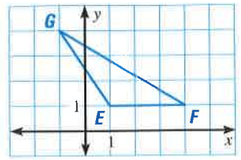
14. $r = 3.9$ cm 

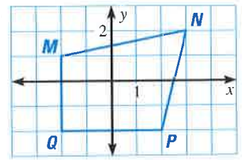
15. **DRAWING A DIAGRAM** The diameter of a circle is 18.9 centimeters. Sketch the circle and find its circumference and area. Round your answers to the nearest tenth.

EXAMPLE 3 on p. 50 for Exs. 16–19

DISTANCE FORMULA **Find the perimeter of the figure. Round to the nearest tenth of a unit.**

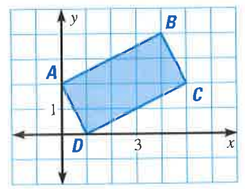
16. 

17. 

18. 

19. ★ **MULTIPLE CHOICE** What is the approximate area (in square units) of the rectangle shown at the right?

Ⓐ 6.7 Ⓑ 8.0 Ⓒ 9.0 Ⓓ 10.0



EXAMPLE 4 on p. 51 for Exs. 20–26

CONVERTING UNITS **Copy and complete the statement.**

20. $187 \text{ cm}^2 = \_?\_ \text{ m}^2$

21. $13 \text{ ft}^2 = \_?\_ \text{ yd}^2$

22. $18 \text{ in.}^2 = \_?\_ \text{ ft}^2$

23. $8 \text{ km}^2 = \_?\_ \text{ m}^2$

24. $12 \text{ yd}^2 = \_?\_ \text{ ft}^2$

25. $24 \text{ ft}^2 = \_?\_ \text{ in.}^2$

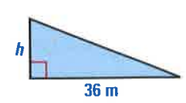
26. ★ **MULTIPLE CHOICE** A triangle has an area of 2.25 square feet. What is the area of the triangle in square inches?

Ⓐ 27 in.^2 Ⓑ 54 in.^2 Ⓒ 144 in.^2 Ⓓ 324 in.^2

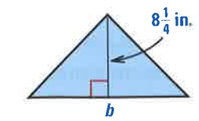
EXAMPLE 5 on p. 52 for Exs. 27–30

UNKNOWN MEASURES **Use the information about the figure to find the indicated measure.**

27. Area = 261 m^2 Find the height h.



28. Area = 66 in.^2 Find the base b.



29. Perimeter = 25 in. Find the width w.