

1.1 Identify Points, Lines, and Planes



Before

You studied basic concepts of geometry.

Now

You will name and sketch geometric figures.

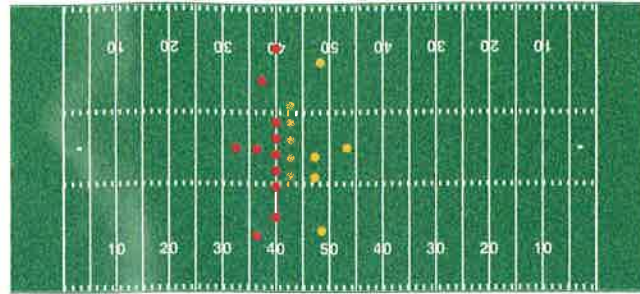
Why

So you can use geometry terms in the real world, as in Ex. 13.

Key Vocabulary

- undefined terms
point, line, plane
- collinear points
- coplanar points
- defined terms
line segment
- endpoints
- ray
- opposite rays
- intersection

In the diagram of a football field, the positions of players are represented by *points*. The yard lines suggest *lines*, and the flat surface of the playing field can be thought of as a *plane*.



In geometry, the words *point*, *line*, and *plane* are **undefined terms**. These words do not have formal definitions, but there is agreement about what they mean.

KEY CONCEPT

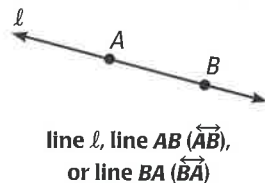
For Your Notebook

Undefined Terms

Point A **point** has no dimension. It is represented by a dot.

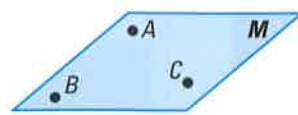


Line A **line** has one dimension. It is represented by a line with two arrowheads, but it extends without end.



Through any two points, there is exactly one line. You can use any two points on a line to name it.

Plane A **plane** has two dimensions. It is represented by a shape that looks like a floor or a wall, but it extends without end.



Through any three points not on the same line, there is exactly one plane. You can use three points that are not all on the same line to name a plane.

plane *M* or plane *ABC*

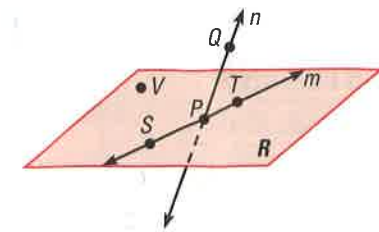
Collinear points are points that lie on the same line. **Coplanar points** are points that lie in the same plane.

EXAMPLE 1 Name points, lines, and planes

VISUAL REASONING

There is a line through points *S* and *Q* that is not shown in the diagram. Try to imagine what plane *SPQ* would look like if it were shown.

- Give two other names for \vec{PQ} and for plane *R*.
- Name three points that are collinear. Name four points that are coplanar.



Solution

- Other names for \vec{PQ} are \vec{QP} and line *n*. Other names for plane *R* are plane *SVT* and plane *PTV*.
- Points *S*, *P*, and *T* lie on the same line, so they are collinear. Points *S*, *P*, *T*, and *V* lie in the same plane, so they are coplanar.

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GUIDED PRACTICE for Example 1

- Use the diagram in Example 1. Give two other names for \vec{ST} . Name a point that is *not* coplanar with points *Q*, *S*, and *T*.

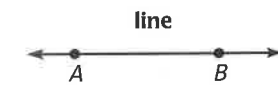
DEFINED TERMS In geometry, terms that can be described using known words such as *point* or *line* are called **defined terms**.

KEY CONCEPT

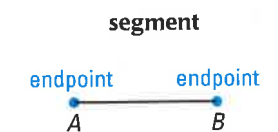
For Your Notebook

Defined Terms: Segments and Rays

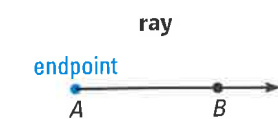
Line *AB* (written as \vec{AB}) and points *A* and *B* are used here to define the terms below.



Segment The **line segment** *AB*, or **segment** *AB*, (written as \overline{AB}) consists of the **endpoints** *A* and *B* and all points on \vec{AB} that are between *A* and *B*. Note that \overline{AB} can also be named \overline{BA} .



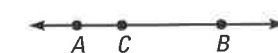
Ray The **ray** *AB* (written as \vec{AB}) consists of the endpoint *A* and all points on \vec{AB} that lie on the same side of *A* as *B*.



Note that \vec{AB} and \vec{BA} are different rays.



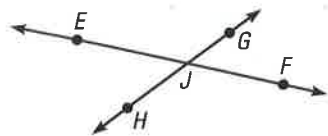
If point *C* lies on \vec{AB} between *A* and *B*, then \vec{CA} and \vec{CB} are **opposite rays**.



Segments and rays are collinear if they lie on the same line. So, opposite rays are collinear. Lines, segments, and rays are coplanar if they lie in the same plane.

EXAMPLE 2 Name segments, rays, and opposite rays

- Give another name for \overline{GH} .
- Name all rays with endpoint J . Which of these rays are opposite rays?



Solution

- Another name for \overline{GH} is \overline{HG} .
- The rays with endpoint J are \overrightarrow{JE} , \overrightarrow{JG} , \overrightarrow{JF} , and \overrightarrow{JH} . The pairs of opposite rays with endpoint J are \overrightarrow{JE} and \overrightarrow{JF} , and \overrightarrow{JG} and \overrightarrow{JH} .

AVOID ERRORS

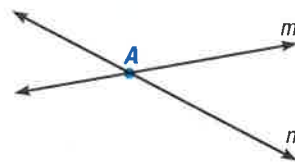
In Example 2, \overrightarrow{JG} and \overrightarrow{JF} have a common endpoint, but are not collinear. So they are not opposite rays.

GUIDED PRACTICE for Example 2

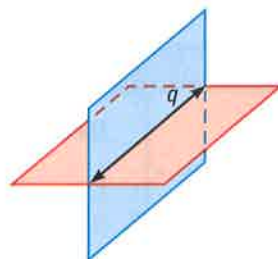
Use the diagram in Example 2.

- Give another name for \overline{EF} .
- Are \overrightarrow{HJ} and \overrightarrow{JH} the same ray? Are \overrightarrow{HJ} and \overrightarrow{HG} the same ray? Explain.

INTERSECTIONS Two or more geometric figures *intersect* if they have one or more points in common. The **intersection** of the figures is the set of points the figures have in common. Some examples of intersections are shown below.



The intersection of two different lines is a point.

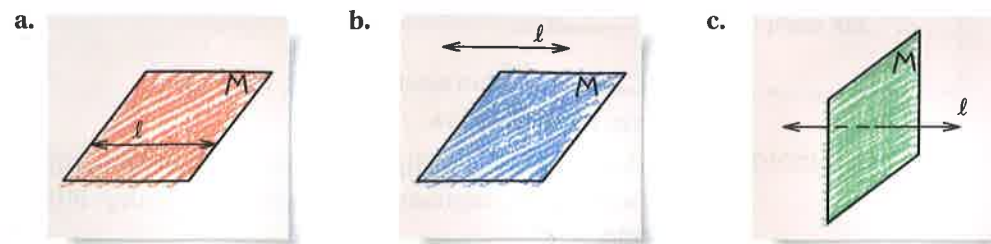


The intersection of two different planes is a line.

EXAMPLE 3 Sketch intersections of lines and planes

- Sketch a plane and a line that is in the plane.
- Sketch a plane and a line that does not intersect the plane.
- Sketch a plane and a line that intersects the plane at a point.

Solution

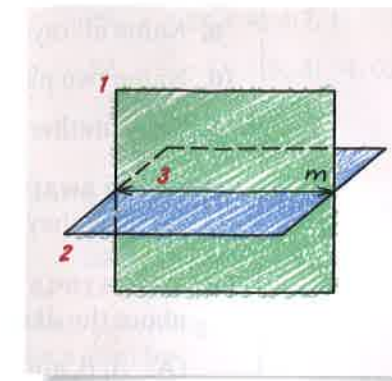


EXAMPLE 4 Sketch intersections of planes

Sketch two planes that intersect in a line.

Solution

- STEP 1** Draw a vertical plane. Shade the plane.
- STEP 2** Draw a second plane that is horizontal. Shade this plane a different color. Use dashed lines to show where one plane is hidden.
- STEP 3** Draw the line of intersection.

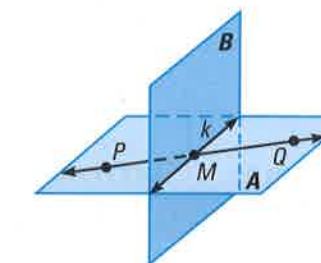


GUIDED PRACTICE for Examples 3 and 4

- Sketch two different lines that intersect a plane at the same point.

Use the diagram at the right.

- Name the intersection of \overrightarrow{PQ} and line k .
- Name the intersection of plane A and plane B .
- Name the intersection of line k and plane A .



1.1 EXERCISES

HOMEWORK KEY

- = WORKED-OUT SOLUTIONS on p. WS1 for Exs. 15, 19, and 43
- ★ = STANDARDIZED TEST PRACTICE Exs. 2, 7, 13, 16, and 43

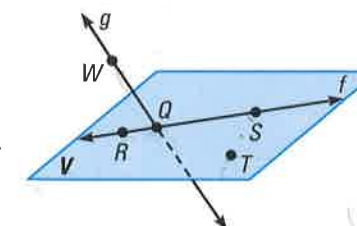
SKILL PRACTICE

- VOCABULARY** Write in words what each of the following symbols means.
 - Q
 - \overline{MN}
 - \overrightarrow{ST}
 - \overleftrightarrow{FG}

- ★ WRITING** Compare collinear points and coplanar points. Are collinear points also coplanar? Are coplanar points also collinear? Explain.

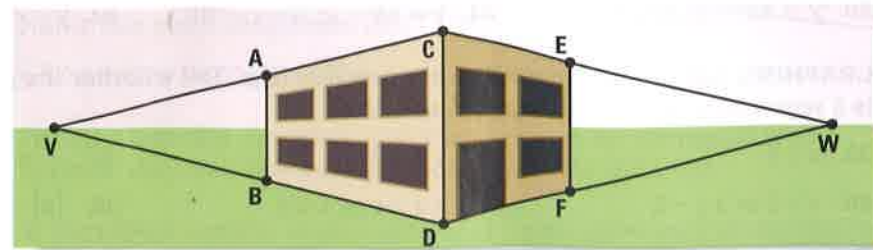
NAMING POINTS, LINES, AND PLANES In Exercises 3–7, use the diagram.

- Give two other names for \overleftrightarrow{WQ} .
- Give another name for plane V .
- Name three points that are collinear. Then name a fourth point that is *not* collinear with these three points.
- Name a point that is *not* coplanar with R , S , and T .
- ★ WRITING** Is point W coplanar with points Q and R ? Explain.



EXAMPLE 1
on p. 3
for Exs. 3–7

45. **MULTI-STEP PROBLEM** In a *perspective drawing*, lines that do not intersect in real life are represented by lines that appear to intersect at a point far away on the horizon. This point is called a *vanishing point*. The diagram shows a drawing of a house with two vanishing points.



- Trace the black line segments in the drawing. Using lightly dashed lines, join points A and B to the vanishing point W . Join points E and F to the vanishing point V .
 - Label the intersection of \overleftrightarrow{EV} and \overleftrightarrow{AW} as G . Label the intersection of \overleftrightarrow{FV} and \overleftrightarrow{BW} as H .
 - Using heavy dashed lines, draw the hidden edges of the house: \overline{AG} , \overline{EG} , \overline{BH} , \overline{FH} , and \overline{GH} .
46. **CHALLENGE** Each street in a particular town intersects every existing street exactly one time. Only two streets pass through each intersection.



2 streets



3 streets



4 streets

- A traffic light is needed at each intersection. How many traffic lights are needed if there are 5 streets in the town? 6 streets?
- Describe a pattern you can use to find the number of additional traffic lights that are needed each time a street is added to the town.

MIXED REVIEW

Find the difference. (p. 869)

- | | | |
|---------------|-----------------|-------------------|
| 47. $-15 - 9$ | 48. $6 - 10$ | 49. $-25 - (-12)$ |
| 50. $13 - 20$ | 51. $16 - (-4)$ | 52. $-5 - 15$ |

Evaluate the expression. (p. 870)

- | | | |
|------------------------|--------------------|-------------------------|
| 53. $5 \cdot -2 + 1 $ | 54. $ -8 + 7 - 6$ | 55. $-7 \cdot 8 - 10 $ |
|------------------------|--------------------|-------------------------|

Plot the point in a coordinate plane. (p. 878)

- | | | |
|---------------|----------------|-----------------|
| 56. $A(2, 4)$ | 57. $B(-3, 6)$ | 58. $E(6, 7.5)$ |
|---------------|----------------|-----------------|

PREVIEW
Prepare for
Lesson 1.2
in Exs. 53–58.

1.2 Use Segments and Congruence



Before

You learned about points, lines, and planes.

Now

You will use segment postulates to identify congruent segments.

Why?

So you can calculate flight distances, as in Ex. 33.

Key Vocabulary

- postulate, axiom
- coordinate
- distance
- between
- congruent segments

In Geometry, a rule that is accepted without proof is called a **postulate** or **axiom**. A rule that can be proved is called a *theorem*, as you will see later. Postulate 1 shows how to find the distance between two points on a line.

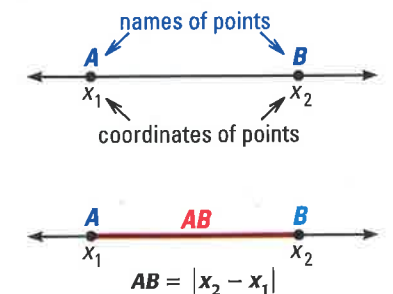
POSTULATE

For Your Notebook

POSTULATE 1 Ruler Postulate

The points on a line can be matched one to one with the real numbers. The real number that corresponds to a point is the **coordinate** of the point.

The **distance** between points A and B , written as AB , is the absolute value of the difference of the coordinates of A and B .



In the diagrams above, the small numbers in the coordinates x_1 and x_2 are called *subscripts*. The coordinates are read as “ x sub one” and “ x sub two.”

The distance between points A and B , or AB , is also called the *length* of \overline{AB} .

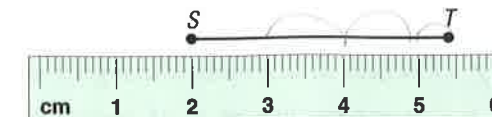
EXAMPLE 1 Apply the Ruler Postulate

Measure the length of \overline{ST} to the nearest tenth of a centimeter.



Solution

Align one mark of a metric ruler with S . Then estimate the coordinate of T . For example, if you align S with 2, T appears to align with 5.4.



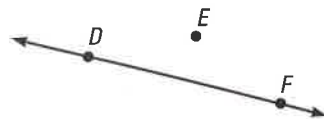
$$ST = |5.4 - 2| = 3.4 \quad \text{Use Ruler Postulate.}$$

► The length of \overline{ST} is about 3.4 centimeters.

ADDING SEGMENT LENGTHS When three points are collinear, you can say that one point is **between** the other two.



Point B is between points A and C .



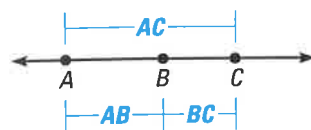
Point E is not between points D and F .

POSTULATE

For Your Notebook

POSTULATE 2 Segment Addition Postulate

If B is between A and C , then $AB + BC = AC$.
If $AB + BC = AC$, then B is between A and C .



EXAMPLE 2 Apply the Segment Addition Postulate

MAPS The cities shown on the map lie approximately in a straight line. Use the given distances to find the distance from Lubbock, Texas, to St. Louis, Missouri.



Solution

Because Tulsa, Oklahoma, lies between Lubbock and St. Louis, you can apply the Segment Addition Postulate.

$$LS = LT + TS = 380 + 360 = 740$$

▶ The distance from Lubbock to St. Louis is about 740 miles.

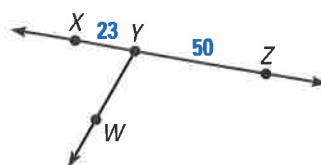
GUIDED PRACTICE for Examples 1 and 2

Use a ruler to measure the length of the segment to the nearest $\frac{1}{8}$ inch.

-
-

In Exercises 3 and 4, use the diagram shown.

- Use the Segment Addition Postulate to find XZ .
- In the diagram, $WY = 30$. Can you use the Segment Addition Postulate to find the distance between points W and Z ? Explain your reasoning.



EXAMPLE 3 Find a length

Use the diagram to find GH .

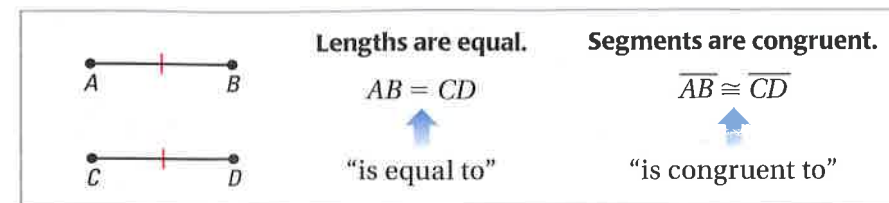


Solution

Use the Segment Addition Postulate to write an equation. Then solve the equation to find GH .

$$\begin{aligned} FH &= FG + GH && \text{Segment Addition Postulate} \\ 36 &= 21 + GH && \text{Substitute 36 for FH and 21 for FG.} \\ 15 &= GH && \text{Subtract 21 from each side.} \end{aligned}$$

CONGRUENT SEGMENTS Line segments that have the same length are called **congruent segments**. In the diagram below, you can say “the length of \overline{AB} is equal to the length of \overline{CD} ,” or you can say “ \overline{AB} is congruent to \overline{CD} .” The symbol \cong means “is congruent to.”



READ DIAGRAMS

In the diagram, the red tick marks indicate that $\overline{AB} \cong \overline{CD}$.

EXAMPLE 4 Compare segments for congruence

Plot $J(-3, 4)$, $K(2, 4)$, $L(1, 3)$, and $M(1, -2)$ in a coordinate plane. Then determine whether \overline{JK} and \overline{LM} are congruent.

Solution

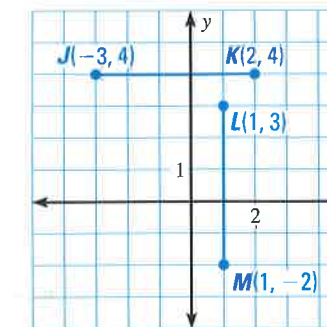
To find the length of a horizontal segment, find the absolute value of the difference of the x -coordinates of the endpoints.

$$JK = |2 - (-3)| = 5 \quad \text{Use Ruler Postulate.}$$

To find the length of a vertical segment, find the absolute value of the difference of the y -coordinates of the endpoints.

$$LM = |-2 - 3| = 5 \quad \text{Use Ruler Postulate.}$$

▶ \overline{JK} and \overline{LM} have the same length. So, $\overline{JK} \cong \overline{LM}$.

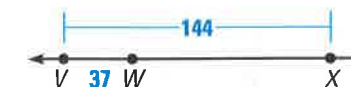


REVIEW USING A COORDINATE PLANE

For help with using a coordinate plane, see p. 878.

GUIDED PRACTICE for Examples 3 and 4

- Use the diagram at the right to find WX .
- Plot the points $A(-2, 4)$, $B(3, 4)$, $C(0, 2)$, and $D(0, -2)$ in a coordinate plane. Then determine whether \overline{AB} and \overline{CD} are congruent.



1.2 EXERCISES

HOMEWORK KEY

- = WORKED-OUT SOLUTIONS on p. WS1 for Exs. 13, 17, and 33
- ★ = STANDARDIZED TEST PRACTICE Exs. 2, 20, 27, and 34

SKILL PRACTICE

In Exercises 1 and 2, use the diagram at the right.

- VOCABULARY** Explain what \overline{MN} means and what MN means.
- ★ WRITING** Explain how you can find PN if you know PQ and QN . How can you find PN if you know MP and MN ?



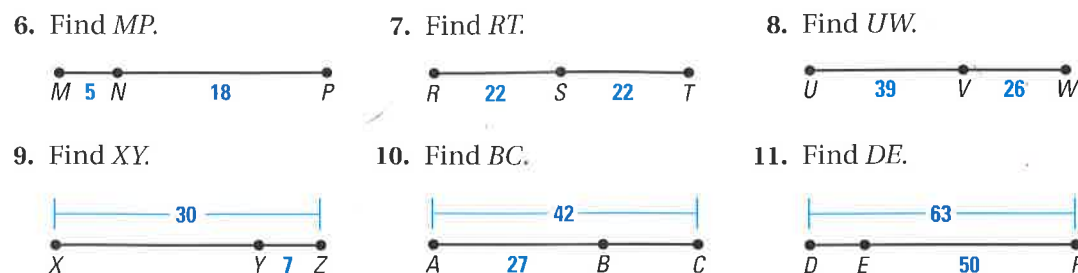
EXAMPLE 1
on p. 9
for Exs. 3–5

MEASUREMENT Measure the length of the segment to the nearest tenth of a centimeter.

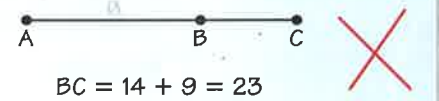


EXAMPLES 2 and 3
on pp. 10–11
for Exs. 6–12

SEGMENT ADDITION POSTULATE Find the indicated length.



- ERROR ANALYSIS** In the figure at the right, $AC = 14$ and $AB = 9$. Describe and correct the error made in finding BC .



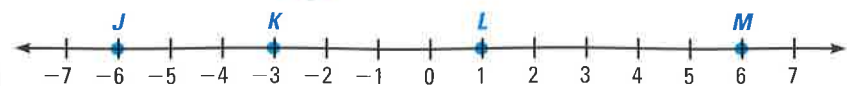
EXAMPLE 4
on p. 11
for Exs. 13–12

CONGRUENCE In Exercises 13–15, plot the given points in a coordinate plane. Then determine whether the line segments named are congruent.

- $A(0, 1), B(4, 1), C(1, 2), D(1, 6)$; \overline{AB} and \overline{CD}
- $J(-6, -8), K(-6, 2), L(-2, -4), M(-6, -4)$; \overline{JK} and \overline{LM}
- $R(-200, 300), S(200, 300), T(300, -200), U(300, 100)$; \overline{RS} and \overline{TU}

xy ALGEBRA Use the number line to find the indicated distance.

- JK
- JL
- JM
- KM



- ★ SHORT RESPONSE** Use the diagram. Is it possible to use the Segment Addition Postulate to show that $FB > CB$ or that $AC > DB$? Explain.



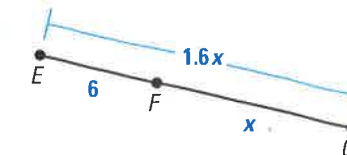
FINDING LENGTHS In the diagram, points $V, W, X, Y,$ and Z are collinear, $VZ = 52, XZ = 20,$ and $WX = XY = YZ$. Find the indicated length.

- WX
- VW
- WY
- VX
- WZ
- VY



- ★ MULTIPLE CHOICE** Use the diagram. What is the length of \overline{EG} ?

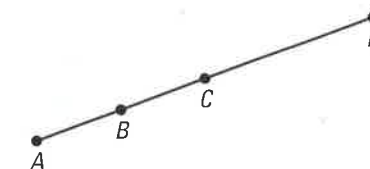
- (A) 1
- (B) 4.4
- (C) 10
- (D) 16



xy ALGEBRA Point S is between R and T on \overline{RT} . Use the given information to write an equation in terms of x . Solve the equation. Then find RS and ST .

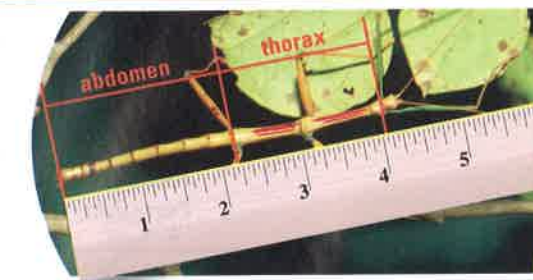
- $RS = 2x + 10$
 $ST = x - 4$
 $RT = 21$
- $RS = 3x - 16$
 $ST = 4x - 8$
 $RT = 60$
- $RS = 2x - 8$
 $ST = 3x - 10$
 $RT = 17$

- CHALLENGE** In the diagram, $\overline{AB} \cong \overline{BC}, \overline{AC} \cong \overline{CD}$, and $AD = 12$. Find the lengths of all the segments in the diagram. Suppose you choose one of the segments at random. What is the probability that the measure of the segment is greater than 3? Explain.



PROBLEM SOLVING

- SCIENCE** The photograph shows an insect called a walkingstick. Use the ruler to estimate the length of the abdomen and the length of the thorax to the nearest $\frac{1}{4}$ inch. About how much longer is the walkingstick's abdomen than its thorax?



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EXAMPLE 2
on p. 10
for Ex. 33

- MODEL AIRPLANE** In 2003, a remote-controlled model airplane became the first ever to fly nonstop across the Atlantic Ocean. The map shows the airplane's position at three different points during its flight.



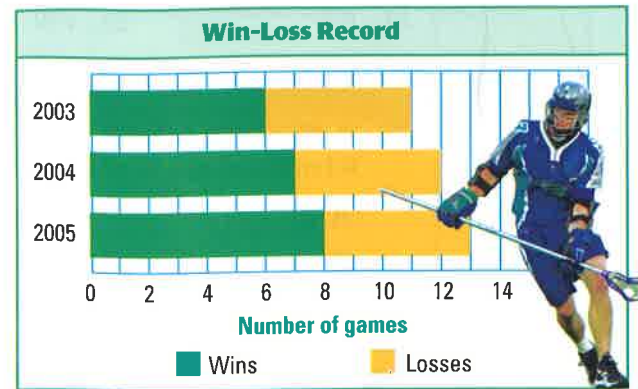
- A Leave Cape Spear, Newfoundland
- B Approximate position after about 1 day
- C Land at Mannin Bay, Ireland, after nearly 38 hours

- Find the total distance the model airplane flew.
- The model airplane's flight lasted nearly 38 hours. Estimate the airplane's average speed in miles per hour.

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34. **★ SHORT RESPONSE** The bar graph shows the win-loss record for a lacrosse team over a period of three years.

- Use the scale to find the length of the yellow bar for each year. What does the length represent?
- For each year, find the percent of games lost by the team.
- Explain how you are applying the Segment Addition Postulate when you find information from a stacked bar graph like the one shown.



35. **MULTI-STEP PROBLEM** A climber uses a rope to descend a vertical cliff. Let A represent the point where the rope is secured at the top of the cliff, let B represent the climber's position, and let C represent the point where the rope is secured at the bottom of the cliff.
- Model** Draw and label a line segment that represents the situation.
 - Calculate** If AC is 52 feet and AB is 31 feet, how much farther must the climber descend to reach the bottom of the cliff?

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36. **CHALLENGE** Four cities lie along a straight highway in this order: City A, City B, City C, and City D. The distance from City A to City B is 5 times the distance from City B to City C. The distance from City A to City D is 2 times the distance from City A to City B. Copy and complete the mileage chart.

	City A	City B	City C	City D
City A		?	?	?
City B	?		?	?
City C	?	?		10 mi
City D	?	?	?	

MIXED REVIEW

PREVIEW

Prepare for Lesson 1.3 in Exs. 37–42.

Simplify the expression. Write your answer in simplest radical form. (p. 874)

37. $\sqrt{45 + 99}$

38. $\sqrt{14 + 36}$

39. $\sqrt{42 + (-2)^2}$

Solve the equation. (p. 875)

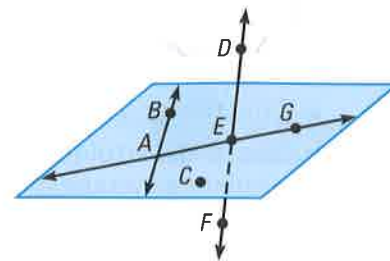
40. $4m + 5 = 7 + 6m$

41. $13 - 4h = 3h - 8$

42. $17 + 3x = 18x - 28$

Use the diagram to decide whether the statement is true or false. (p. 2)

- Points A , C , E , and G are coplanar.
- \vec{DF} and \vec{AG} intersect at point E .
- \vec{AE} and \vec{EG} are opposite rays.



1.3 Use Midpoint and Distance Formulas



Before

You found lengths of segments.

Now

You will find lengths of segments in the coordinate plane.

Why?

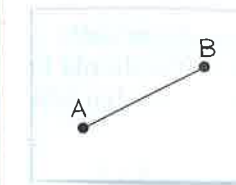
So you can find an unknown length, as in Example 1.

Key Vocabulary

- midpoint
- segment bisector

ACTIVITY FOLD A SEGMENT BISECTOR

STEP 1



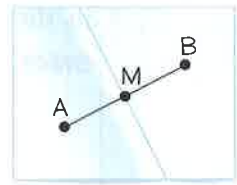
Draw \overline{AB} on a piece of paper.

STEP 2



Fold the paper so that B is on top of A .

STEP 3

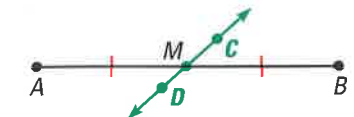


Label point M . Compare AM , MB , and AB .

MIDPOINTS AND BISECTORS The **midpoint** of a segment is the point that divides the segment into two congruent segments. A **segment bisector** is a point, ray, line, line segment, or plane that intersects the segment at its midpoint. A midpoint or a segment bisector *bisects* a segment.



M is the midpoint of \overline{AB} .
So, $\overline{AM} \cong \overline{MB}$ and $AM = MB$.



\vec{CD} is a segment bisector of \overline{AB} .
So, $\overline{AM} \cong \overline{MB}$ and $AM = MB$.

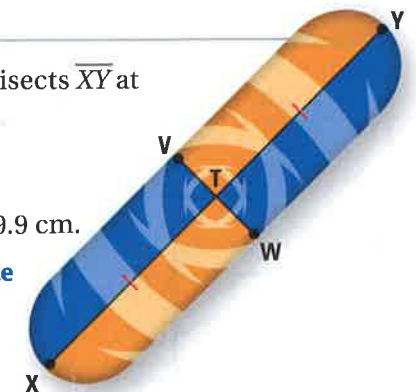
EXAMPLE 1 Find segment lengths

SKATEBOARD In the skateboard design, \overline{VW} bisects \overline{XY} at point T , and $XT = 39.9$ cm. Find XY .

Solution

Point T is the midpoint of \overline{XY} . So, $XT = TY = 39.9$ cm.

$$\begin{aligned} XY &= XT + TY && \text{Segment Addition Postulate} \\ &= 39.9 + 39.9 && \text{Substitute.} \\ &= 79.8 \text{ cm} && \text{Add.} \end{aligned}$$



EXAMPLE 2 Use algebra with segment lengths

xy **ALGEBRA** Point M is the midpoint of \overline{VW} . Find the length of \overline{VM} .



Solution

STEP 1 Write and solve an equation. Use the fact that $VM = MW$.

$$\begin{aligned} VM &= MW && \text{Write equation.} \\ 4x - 1 &= 3x + 3 && \text{Substitute.} \\ x - 1 &= 3 && \text{Subtract } 3x \text{ from each side.} \\ x &= 4 && \text{Add 1 to each side.} \end{aligned}$$

STEP 2 Evaluate the expression for VM when $x = 4$.

$$VM = 4x - 1 = 4(4) - 1 = 15$$

► So, the length of \overline{VM} is 15.

CHECK Because $VM = MW$, the length of \overline{MW} should be 15. If you evaluate the expression for MW , you should find that $MW = 15$.

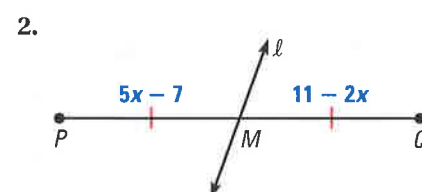
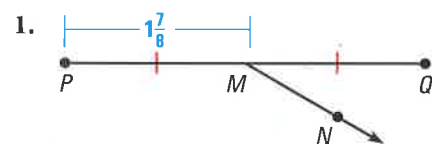
$$MW = 3x + 3 = 3(4) + 3 = 15 \checkmark$$

REVIEW ALGEBRA

For help with solving equations, see p. 875.

GUIDED PRACTICE for Examples 1 and 2

In Exercises 1 and 2, identify the segment bisector of \overline{PQ} . Then find PQ .



READ DIRECTIONS

Always read direction lines carefully. Notice that this direction line has two parts.

COORDINATE PLANE You can use the coordinates of the endpoints of a segment to find the coordinates of the midpoint.

KEY CONCEPT

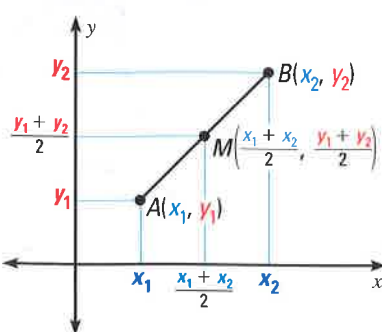
For Your Notebook

The Midpoint Formula

The coordinates of the midpoint of a segment are the averages of the x -coordinates and of the y -coordinates of the endpoints.

If $A(x_1, y_1)$ and $B(x_2, y_2)$ are points in a coordinate plane, then the midpoint M of \overline{AB} has coordinates

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$



CLEAR FRACTIONS

Multiply each side of the equation by the denominator to clear the fraction.

EXAMPLE 3 Use the Midpoint Formula

- a. **FIND MIDPOINT** The endpoints of \overline{RS} are $R(1, -3)$ and $S(4, 2)$. Find the coordinates of the midpoint M .
- b. **FIND ENDPOINT** The midpoint of \overline{JK} is $M(2, 1)$. One endpoint is $J(1, 4)$. Find the coordinates of endpoint K .

Solution

a. **FIND MIDPOINT** Use the Midpoint Formula.

$$M\left(\frac{1+4}{2}, \frac{-3+2}{2}\right) = M\left(\frac{5}{2}, -\frac{1}{2}\right)$$

► The coordinates of the midpoint M are $\left(\frac{5}{2}, -\frac{1}{2}\right)$.

b. **FIND ENDPOINT** Let (x, y) be the coordinates of endpoint K . Use the Midpoint Formula.

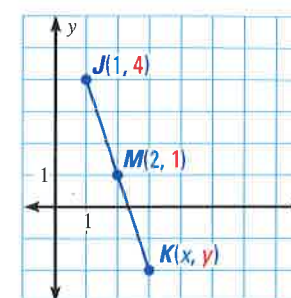
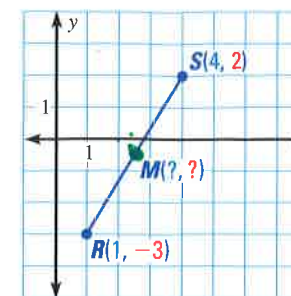
STEP 1 Find x .

$$\begin{aligned} \frac{1+x}{2} &= 2 \\ 1+x &= 4 \\ x &= 3 \end{aligned}$$

STEP 2 Find y .

$$\begin{aligned} \frac{4+y}{2} &= 1 \\ 4+y &= 2 \\ y &= -2 \end{aligned}$$

► The coordinates of endpoint K are $(3, -2)$.



GUIDED PRACTICE for Example 3

- 3. The endpoints of \overline{AB} are $A(1, 2)$ and $B(7, 8)$. Find the coordinates of the midpoint M .
- 4. The midpoint of \overline{VW} is $M(-1, -2)$. One endpoint is $W(4, 4)$. Find the coordinates of endpoint V .

DISTANCE FORMULA The Distance Formula is a formula for computing the distance between two points in a coordinate plane.

KEY CONCEPT

For Your Notebook

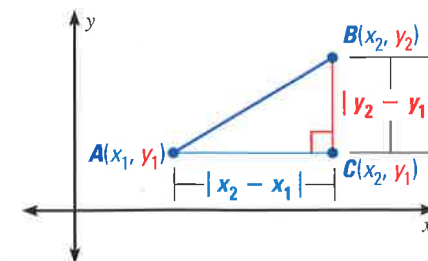
The Distance Formula

If $A(x_1, y_1)$ and $B(x_2, y_2)$ are points in a coordinate plane, then the distance between A and B is

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

READ DIAGRAMS

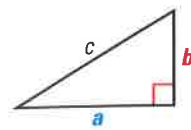
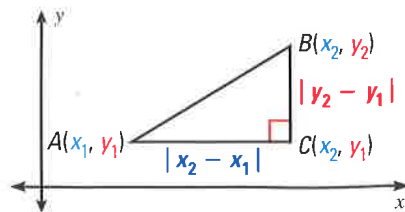
The red mark at one corner of the triangle shown indicates a right triangle.



The Distance Formula is based on the *Pythagorean Theorem*, which you will see again when you work with right triangles in Chapter 7.

Distance Formula
 $(AB)^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$

Pythagorean Theorem
 $c^2 = a^2 + b^2$



EXAMPLE 4 Standardized Test Practice

ELIMINATE CHOICES

Drawing a diagram can help you eliminate choices. You can see that choice A is not large enough to be RS.

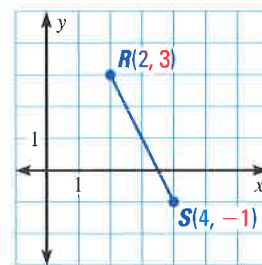
What is the approximate length of \overline{RS} with endpoints $R(2, 3)$ and $S(4, -1)$?

- (A) 1.4 units (B) 4.0 units (C) 4.5 units (D) 6 units

Solution

Use the Distance Formula. You may find it helpful to draw a diagram.

$$\begin{aligned} RS &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} && \text{Distance Formula} \\ &= \sqrt{[(4 - 2)]^2 + [(-1) - 3]^2} && \text{Substitute.} \\ &= \sqrt{(2)^2 + (-4)^2} && \text{Subtract.} \\ &= \sqrt{4 + 16} && \text{Evaluate powers.} \\ &= \sqrt{20} && \text{Add.} \\ &\approx 4.47 && \text{Use a calculator to approximate the square root.} \end{aligned}$$



READ SYMBOLS

The symbol \approx means "is approximately equal to."

▶ The correct answer is C. (A) (B) (C) (D)

GUIDED PRACTICE for Example 4

- In Example 4, does it matter which ordered pair you choose to substitute for (x_1, y_1) and which ordered pair you choose to substitute for (x_2, y_2) ? Explain.
- What is the approximate length of \overline{AB} , with endpoints $A(-3, 2)$ and $B(1, -4)$?
 (A) 6.1 units (B) 7.2 units (C) 8.5 units (D) 10.0 units

1.3 EXERCISES

HOMEWORK KEY

- O = WORKED-OUT SOLUTIONS on p. WS1 for Exs. 15, 35, and 49
- ★ = STANDARDIZED TEST PRACTICE Exs. 2, 23, 34, 41, 42, and 53

SKILL PRACTICE

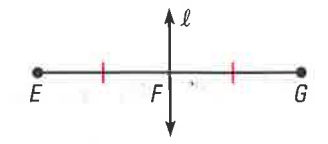
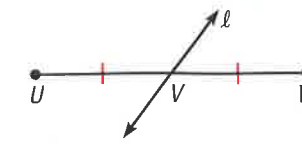
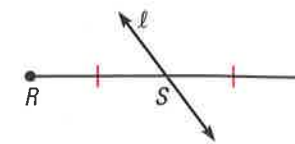
- VOCABULARY** Copy and complete: To find the length of \overline{AB} , with endpoints $A(-7, 5)$ and $B(4, -6)$, you can use the $\underline{\hspace{1cm}}$.
- ★ **WRITING** Explain what it means to bisect a segment. Why is it impossible to bisect a line?

EXAMPLE 1

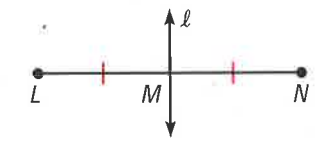
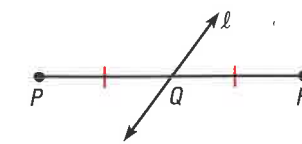
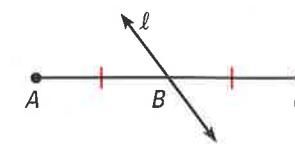
on p. 15 for Exs. 3–10

FINDING LENGTHS Line l bisects the segment. Find the indicated length.

- Find RT if $RS = 5\frac{1}{8}$ in.
- Find UW if $VW = \frac{5}{8}$ in.
- Find EG if $EF = 13$ cm.



- Find BC if $AC = 19$ cm.
- Find QR if $PR = 9\frac{1}{2}$ in.
- Find LM if $LN = 137$ mm.



- SEGMENT BISECTOR** Line RS bisects \overline{PQ} at point R . Find RQ if $PQ = 4\frac{3}{4}$ inches.
- SEGMENT BISECTOR** Point T bisects \overline{UV} . Find UV if $UT = 2\frac{7}{8}$ inches.

EXAMPLE 2

on p. 16 for Exs. 11–16

ALGEBRA In each diagram, M is the midpoint of the segment. Find the indicated length.

- Find AM .
- Find EM .
- Find JM .
- Find PR .
- Find SU .
- Find XZ .

EXAMPLE 3

on p. 17 for Exs. 17–30

FINDING MIDPOINTS Find the coordinates of the midpoint of the segment with the given endpoints.

- $C(3, 5)$ and $D(7, 5)$
- $E(0, 4)$ and $F(4, 3)$
- $G(-4, 4)$ and $H(6, 4)$
- $J(-7, -5)$ and $K(-3, 7)$
- $P(-8, -7)$ and $Q(11, 5)$
- $S(-3, 3)$ and $T(-8, 6)$
- ★ **WRITING** Develop a formula for finding the midpoint of a segment with endpoints $A(0, 0)$ and $B(m, n)$. Explain your thinking.

1.4 Measure and Classify Angles



Before

You named and measured line segments.

Now

You will name, measure, and classify angles.

Why?

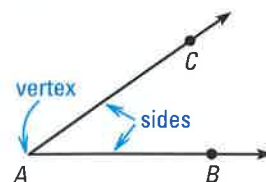
So you can identify congruent angles, as in Example 4.

Key Vocabulary

- angle
 - acute, right, obtuse, straight
- sides, vertex of an angle
- measure of an angle
- congruent angles
- angle bisector

An **angle** consists of two different rays with the same endpoint. The rays are the **sides** of the angle. The endpoint is the **vertex** of the angle.

The angle with sides \overrightarrow{AB} and \overrightarrow{AC} can be named $\angle BAC$, $\angle CAB$, or $\angle A$. Point A is the vertex of the angle.



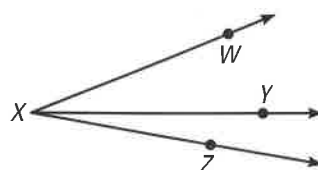
EXAMPLE 1 Name angles

Name the three angles in the diagram.

$\angle WXY$, or $\angle YXW$

$\angle YXZ$, or $\angle ZXY$

$\angle WXZ$, or $\angle ZXW$



You should not name any of these angles $\angle X$ because all three angles have X as their vertex.

MEASURING ANGLES A protractor can be used to approximate the *measure* of an angle. An angle is measured in units called *degrees* ($^\circ$). For instance, the measure of $\angle WXZ$ in Example 1 above is 32° . You can write this statement in two ways.

Words The measure of $\angle WXZ$ is 32° .

Symbols $m\angle WXZ = 32^\circ$

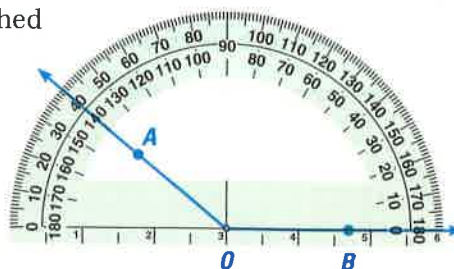
POSTULATE

For Your Notebook

POSTULATE 3 Protractor Postulate

Consider \overrightarrow{OB} and a point A on one side of \overrightarrow{OB} . The rays of the form \overrightarrow{OA} can be matched one to one with the real numbers from 0 to 180.

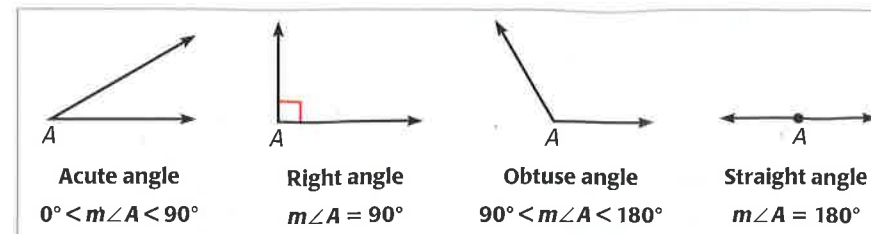
The **measure** of $\angle AOB$ is equal to the absolute value of the difference between the real numbers for \overrightarrow{OA} and \overrightarrow{OB} .



CLASSIFYING ANGLES Angles can be classified as **acute**, **right**, **obtuse**, and **straight**, as shown below.

READ DIAGRAMS

A red square inside an angle indicates that the angle is a right angle.



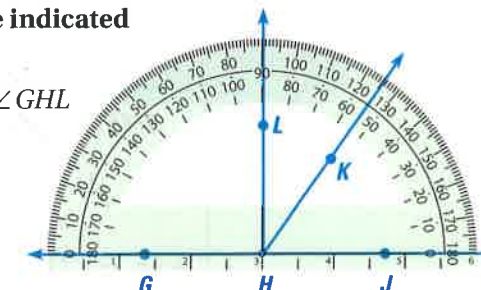
EXAMPLE 2 Measure and classify angles

Use the diagram to find the measure of the indicated angle. Then classify the angle.

- a. $\angle KHJ$ b. $\angle GHK$ c. $\angle GHJ$ d. $\angle GHL$

Solution

A protractor has an inner and an outer scale. When you measure an angle, check to see which scale to use.

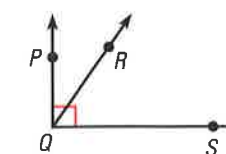


- a. \overrightarrow{HJ} is lined up with the 0° on the inner scale of the protractor. \overrightarrow{HK} passes through 55° on the inner scale. So, $m\angle KHJ = 55^\circ$. It is an acute angle.
- b. \overrightarrow{HG} is lined up with the 0° on the outer scale, and \overrightarrow{HK} passes through 125° on the outer scale. So, $m\angle GHK = 125^\circ$. It is an obtuse angle.
- c. $m\angle GHJ = 180^\circ$. It is a straight angle.
- d. $m\angle GHL = 90^\circ$. It is a right angle.

Animated Geometry at classzone.com

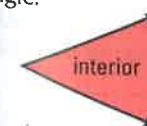
GUIDED PRACTICE for Examples 1 and 2

1. Name all the angles in the diagram at the right. Which angle is a right angle?
2. Draw a pair of opposite rays. What type of angle do the rays form?



READ DIAGRAMS

A point is in the **interior** of an angle if it is between points that lie on each side of the angle.



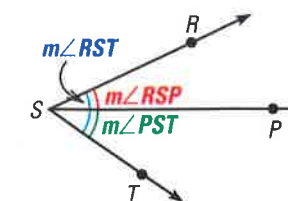
POSTULATE

For Your Notebook

POSTULATE 4 Angle Addition Postulate

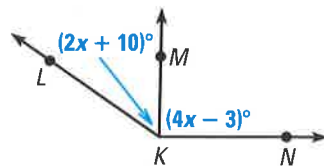
Words If P is in the interior of $\angle RST$, then the measure of $\angle RST$ is equal to the sum of the measures of $\angle RSP$ and $\angle PST$.

Symbols If P is in the interior of $\angle RST$, then $m\angle RST = m\angle RSP + m\angle PST$.



EXAMPLE 3 Find angle measures

xy ALGEBRA Given that $m\angle LKN = 145^\circ$, find $m\angle LKM$ and $m\angle MKN$.



Solution

STEP 1 Write and solve an equation to find the value of x .

$$\begin{aligned}
 m\angle LKN &= m\angle LKM + m\angle MKN && \text{Angle Addition Postulate} \\
 145^\circ &= (2x + 10)^\circ + (4x - 3)^\circ && \text{Substitute angle measures.} \\
 145 &= 6x + 7 && \text{Combine like terms.} \\
 138 &= 6x && \text{Subtract 7 from each side.} \\
 23 &= x && \text{Divide each side by 6.}
 \end{aligned}$$

STEP 2 Evaluate the given expressions when $x = 23$.

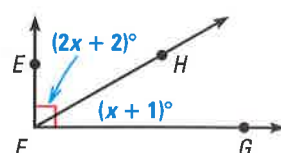
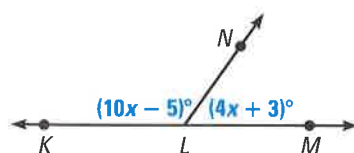
$$\begin{aligned}
 m\angle LKM &= (2x + 10)^\circ = (2 \cdot 23 + 10)^\circ = 56^\circ \\
 m\angle MKN &= (4x - 3)^\circ = (4 \cdot 23 - 3)^\circ = 89^\circ
 \end{aligned}$$

► So, $m\angle LKM = 56^\circ$ and $m\angle MKN = 89^\circ$.

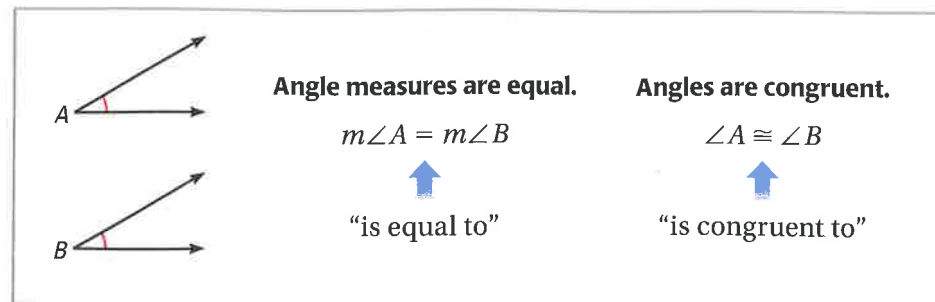
GUIDED PRACTICE for Example 3

Find the indicated angle measures.

3. Given that $\angle KLM$ is a straight angle, find $m\angle KLN$ and $m\angle NLM$.
 4. Given that $\angle EFG$ is a right angle, find $m\angle EFH$ and $m\angle HFG$.



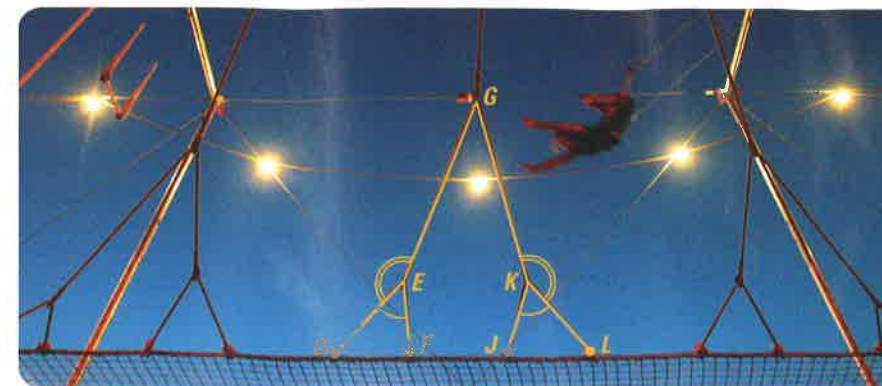
CONGRUENT ANGLES Two angles are **congruent angles** if they have the same measure. In the diagram below, you can say that “the measure of angle A is equal to the measure of angle B,” or you can say “angle A is congruent to angle B.”



READ DIAGRAMS
 Matching arcs are used to show that angles are congruent. If more than one pair of angles are congruent, double arcs are used, as in Example 4 on page 27.

EXAMPLE 4 Identify congruent angles

TRAPEZE The photograph shows some of the angles formed by the ropes in a trapeze apparatus. Identify the congruent angles. If $m\angle DEG = 157^\circ$, what is $m\angle GKL$?



Solution

There are two pairs of congruent angles:

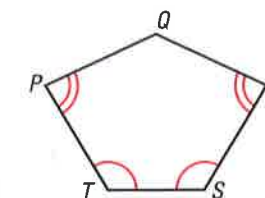
$$\angle DEF \cong \angle JKL \text{ and } \angle DEG \cong \angle GKL.$$

Because $\angle DEG \cong \angle GKL$, $m\angle DEG = m\angle GKL$. So, $m\angle GKL = 157^\circ$.

GUIDED PRACTICE for Example 4

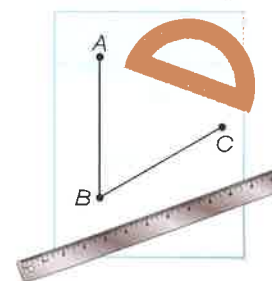
Use the diagram shown at the right.

5. Identify all pairs of congruent angles in the diagram.
 6. In the diagram, $m\angle PQR = 130^\circ$, $m\angle QRS = 84^\circ$, and $m\angle TSR = 121^\circ$. Find the other angle measures in the diagram.



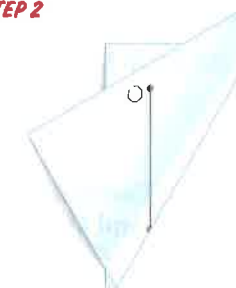
ACTIVITY FOLD AN ANGLE BISECTOR

STEP 1



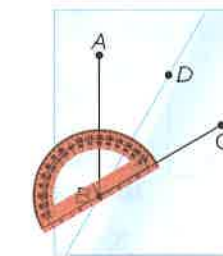
Use a straightedge to draw and label an acute angle, $\angle ABC$.

STEP 2



Fold the paper so that \vec{BC} is on top of \vec{BA} .

STEP 3

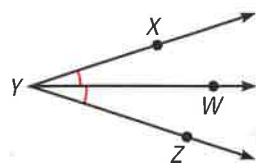


Draw a point D on the fold inside $\angle ABC$. Then measure $\angle ABD$, $\angle DBC$, and $\angle ABC$. What do you observe?

An **angle bisector** is a ray that divides an angle into two angles that are congruent. In the activity on page 27, \overrightarrow{BD} bisects $\angle ABC$. So, $\angle ABD \cong \angle DBC$ and $m\angle ABD = m\angle DBC$.

EXAMPLE 5 Double an angle measure

In the diagram at the right, \overrightarrow{YW} bisects $\angle XYZ$, and $m\angle XYW = 18^\circ$. Find $m\angle XYZ$.



Solution

By the Angle Addition Postulate, $m\angle XYZ = m\angle XYW + m\angle WYZ$. Because \overrightarrow{YW} bisects $\angle XYZ$, you know that $\angle XYW \cong \angle WYZ$.

So, $m\angle XYW = m\angle WYZ$, and you can write

$$m\angle XYZ = m\angle XYW + m\angle WYZ = 18^\circ + 18^\circ = 36^\circ.$$

GUIDED PRACTICE for Example 5

7. Angle MNP is a straight angle, and \overrightarrow{NQ} bisects $\angle MNP$. Draw $\angle MNP$ and \overrightarrow{NQ} . Use arcs to mark the congruent angles in your diagram, and give the angle measures of these congruent angles.

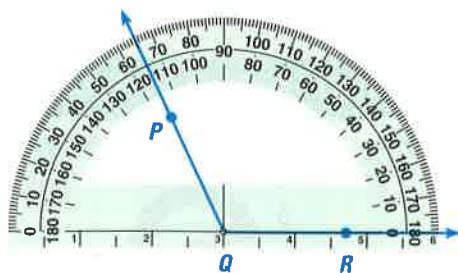
1.4 EXERCISES

HOMEWORK KEY

- = WORKED-OUT SOLUTIONS on p. WS1 for Exs. 15, 23, and 53
- ★ = STANDARDIZED TEST PRACTICE Exs. 2, 21, 27, 43, and 62

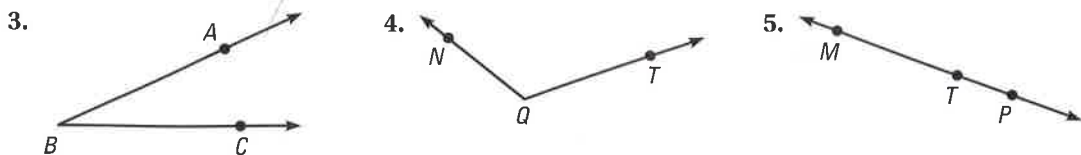
SKILL PRACTICE

- VOCABULARY** Sketch an example of each of the following types of angles: acute, obtuse, right, and straight.
- ★ WRITING** Explain how to find the measure of $\angle PQR$, shown at the right.

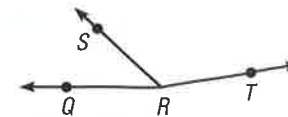


EXAMPLE 1 on p. 24 for Exs. 3–6

NAMING ANGLES AND ANGLE PARTS In Exercises 3–5, write three names for the angle shown. Then name the vertex and sides of the angle.



6. **NAMING ANGLES** Name three different angles in the diagram at the right.



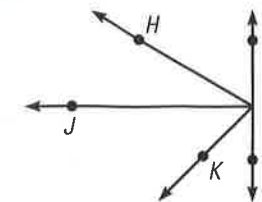
EXAMPLE 2 on p. 25 for Exs. 7–21

CLASSIFYING ANGLES Classify the angle with the given measure as *acute*, *obtuse*, *right*, or *straight*.

7. $m\angle W = 180^\circ$ 8. $m\angle X = 30^\circ$ 9. $m\angle Y = 90^\circ$ 10. $m\angle Z = 95^\circ$

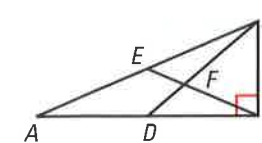
MEASURING ANGLES Trace the diagram and extend the rays. Use a protractor to find the measure of the given angle. Then classify the angle as *acute*, *obtuse*, *right*, or *straight*.

11. $\angle JFL$ 12. $\angle GFH$
13. $\angle GFK$ 14. $\angle GFL$



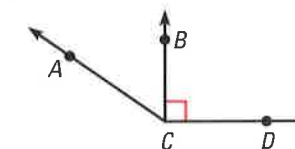
NAMING AND CLASSIFYING Give another name for the angle in the diagram below. Tell whether the angle appears to be *acute*, *obtuse*, *right*, or *straight*.

15. $\angle ACB$ 16. $\angle ABC$
17. $\angle BFD$ 18. $\angle AEC$
19. $\angle BDC$ 20. $\angle BEC$



21. **★ MULTIPLE CHOICE** Which is a correct name for the obtuse angle in the diagram?

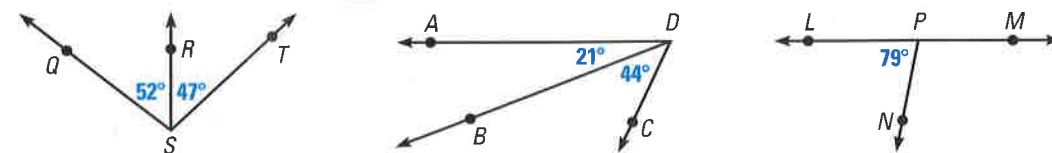
- (A) $\angle ACB$ (B) $\angle ACD$
(C) $\angle BCD$ (D) $\angle C$



EXAMPLE 3 on p. 26 for Exs. 22–27

ANGLE ADDITION POSTULATE Find the indicated angle measure.

22. $m\angle QST = \underline{\quad?}$ 23. $m\angle ADC = \underline{\quad?}$ 24. $m\angle NPM = \underline{\quad?}$



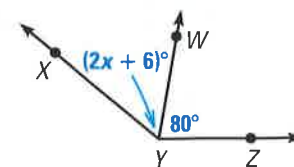
xy ALGEBRA Use the given information to find the indicated angle measure.

25. Given $m\angle WXZ = 80^\circ$, find $m\angle YXZ$. 26. Given $m\angle FJH = 168^\circ$, find $m\angle FJG$.



27. **★ MULTIPLE CHOICE** In the diagram, the measure of $\angle XYZ$ is 140° . What is the value of x ?

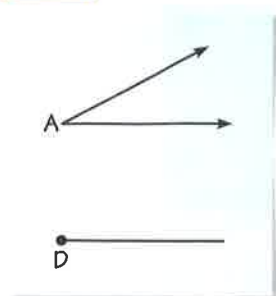
- (A) 27 (B) 33
(C) 67 (D) 73



EXPLORE 3 Copy an angle

Use the following steps to construct an angle that is congruent to $\angle A$. In this construction, the *radius* of an arc is the distance from the point where the compass point rests (the *center* of the arc) to a point on the arc drawn by the compass.

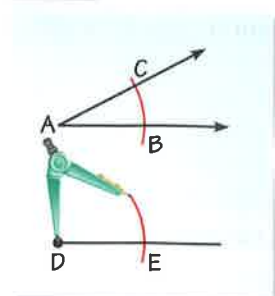
STEP 1



Draw a segment

Draw a segment. Label a point D on the segment.

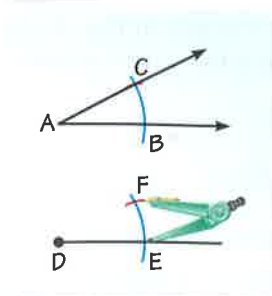
STEP 2



Draw arcs

Draw an arc with center A . Using the same radius, draw an arc with center D .

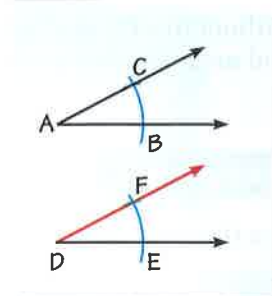
STEP 3



Draw arcs

Label B , C , and E . Draw an arc with radius BC and center E . Label the intersection F .

STEP 4



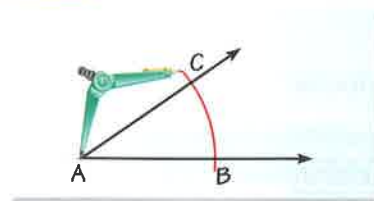
Draw a ray

Draw \overrightarrow{DF} . $\angle EDF \cong \angle BAC$.

EXPLORE 4 Bisect an angle

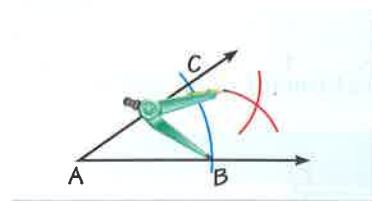
Use the following steps to construct an angle bisector of $\angle A$.

STEP 1



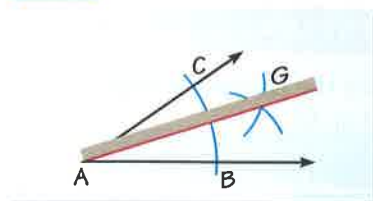
Draw an arc Place the compass at A . Draw an arc that intersects both sides of the angle. Label the intersections C and B .

STEP 2



Draw arcs Place the compass at C . Draw an arc. Then place the compass point at B . Using the same radius, draw another arc.

STEP 3



Draw a ray Label the intersection G . Use a straightedge to draw a ray through A and G . \overrightarrow{AG} bisects $\angle A$.

DRAW CONCLUSIONS Use your observations to complete these exercises

- Describe how you could use a compass and a straightedge to draw a segment that is twice as long as a given segment.
- Draw an obtuse angle. Copy the angle using a compass and a straightedge. Then bisect the angle using a compass and straightedge.

1.5 Describe Angle Pair Relationships



Before

You used angle postulates to measure and classify angles.

Now

You will use special angle relationships to find angle measures.

Why?

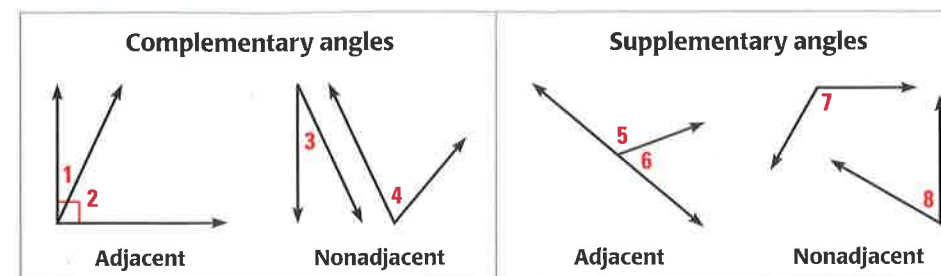
So you can find measures in a building, as in Ex. 53.

Key Vocabulary

- complementary angles
- supplementary angles
- adjacent angles
- linear pair
- vertical angles

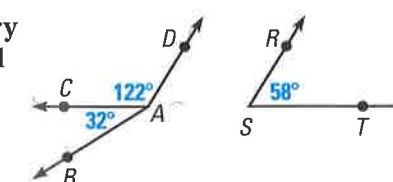
Two angles are **complementary angles** if the sum of their measures is 90° . Each angle is the *complement* of the other. Two angles are **supplementary angles** if the sum of their measures is 180° . Each angle is the *supplement* of the other.

Complementary angles and supplementary angles can be *adjacent angles* or *nonadjacent angles*. **Adjacent angles** are two angles that share a common vertex and side, but have no common interior points.



EXAMPLE 1 Identify complements and supplements

In the figure, name a pair of complementary angles, a pair of supplementary angles, and a pair of adjacent angles.



Solution

Because $32^\circ + 58^\circ = 90^\circ$, $\angle BAC$ and $\angle RST$ are complementary angles.

Because $122^\circ + 58^\circ = 180^\circ$, $\angle CAD$ and $\angle RST$ are supplementary angles.

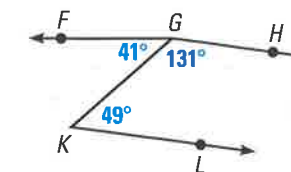
Because $\angle BAC$ and $\angle CAD$ share a common vertex and side, they are adjacent.

AVOID ERRORS

In Example 1, $\angle DAC$ and $\angle DAB$ share a common vertex. But they share common interior points, so they are *not* adjacent angles.

GUIDED PRACTICE for Example 1

- In the figure, name a pair of complementary angles, a pair of supplementary angles, and a pair of adjacent angles.
- Are $\angle KGH$ and $\angle LKG$ adjacent angles? Are $\angle FGK$ and $\angle FGH$ adjacent angles? Explain.



EXAMPLE 2 Find measures of a complement and a supplement

READ DIAGRAMS

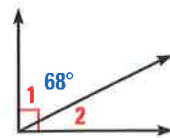
Angles are sometimes named with numbers. An angle measure in a diagram has a degree symbol. An angle name does not.

- Given that $\angle 1$ is a complement of $\angle 2$ and $m\angle 1 = 68^\circ$, find $m\angle 2$.
- Given that $\angle 3$ is a supplement of $\angle 4$ and $m\angle 4 = 56^\circ$, find $m\angle 3$.

Solution

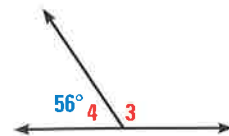
- You can draw a diagram with complementary adjacent angles to illustrate the relationship.

$$m\angle 2 = 90^\circ - m\angle 1 = 90^\circ - 68^\circ = 22^\circ$$



- You can draw a diagram with supplementary adjacent angles to illustrate the relationship.

$$m\angle 3 = 180^\circ - m\angle 4 = 180^\circ - 56^\circ = 124^\circ$$

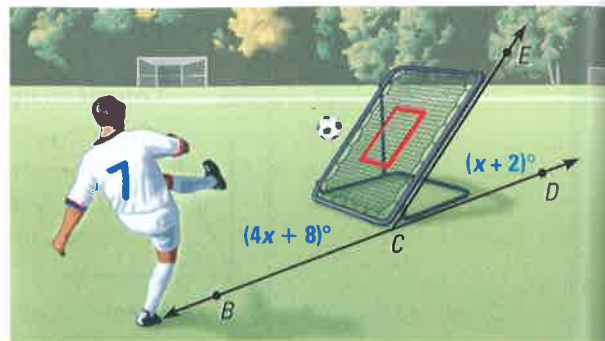


EXAMPLE 3 Find angle measures

READ DIAGRAMS

In a diagram, you can assume that a line that looks straight is straight. In Example 3, B , C , and D lie on \overleftrightarrow{BD} . So, $\angle BCD$ is a straight angle.

SPORTS When viewed from the side, the frame of a ball-return net forms a pair of supplementary angles with the ground. Find $m\angle BCE$ and $m\angle ECD$.



Solution

STEP 1 Use the fact that the sum of the measures of supplementary angles is 180° .

$$m\angle BCE + m\angle ECD = 180^\circ \quad \text{Write equation.}$$

$$(4x + 8)^\circ + (x + 2)^\circ = 180^\circ \quad \text{Substitute.}$$

$$5x + 10 = 180 \quad \text{Combine like terms.}$$

$$5x = 170 \quad \text{Subtract 10 from each side.}$$

$$x = 34 \quad \text{Divide each side by 5.}$$

STEP 2 Evaluate the original expressions when $x = 34$.

$$m\angle BCE = (4x + 8)^\circ = (4 \cdot 34 + 8)^\circ = 144^\circ$$

$$m\angle ECD = (x + 2)^\circ = (34 + 2)^\circ = 36^\circ$$

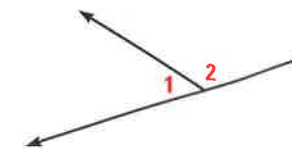
▶ The angle measures are 144° and 36° .

GUIDED PRACTICE for Examples 2 and 3

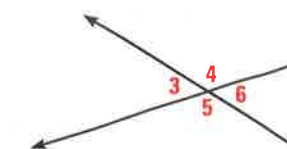
- Given that $\angle 1$ is a complement of $\angle 2$ and $m\angle 2 = 8^\circ$, find $m\angle 1$.
- Given that $\angle 3$ is a supplement of $\angle 4$ and $m\angle 3 = 117^\circ$, find $m\angle 4$.
- $\angle LMN$ and $\angle PQR$ are complementary angles. Find the measures of the angles if $m\angle LMN = (4x - 2)^\circ$ and $m\angle PQR = (9x + 1)^\circ$.

ANGLE PAIRS Two adjacent angles are a **linear pair** if their noncommon sides are opposite rays. The angles in a linear pair are supplementary angles.

Two angles are **vertical angles** if their sides form two pairs of opposite rays.



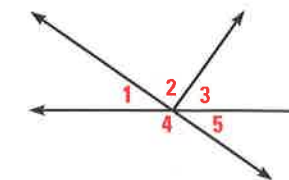
$\angle 1$ and $\angle 2$ are a linear pair.



$\angle 3$ and $\angle 6$ are vertical angles.
 $\angle 4$ and $\angle 5$ are vertical angles.

EXAMPLE 4 Identify angle pairs

Identify all of the linear pairs and all of the vertical angles in the figure at the right.



Solution

To find vertical angles, look for angles formed by intersecting lines.

▶ $\angle 1$ and $\angle 5$ are vertical angles.

To find linear pairs, look for adjacent angles whose noncommon sides are opposite rays.

▶ $\angle 1$ and $\angle 4$ are a linear pair. $\angle 4$ and $\angle 5$ are also a linear pair.

AVOID ERRORS

In the diagram, one side of $\angle 1$ and one side of $\angle 3$ are opposite rays. But the angles are not a linear pair because they are not adjacent.

EXAMPLE 5 Find angle measures in a linear pair

xy ALGEBRA Two angles form a linear pair. The measure of one angle is 5 times the measure of the other. Find the measure of each angle.

Solution

Let x° be the measure of one angle. The measure of the other angle is $5x^\circ$. Then use the fact that the angles of a linear pair are supplementary to write an equation.

$$x^\circ + 5x^\circ = 180^\circ \quad \text{Write an equation.}$$

$$6x = 180 \quad \text{Combine like terms.}$$

$$x = 30 \quad \text{Divide each side by 6.}$$

▶ The measures of the angles are 30° and $5(30^\circ) = 150^\circ$.



DRAW DIAGRAMS

You may find it useful to draw a diagram to represent a word problem like the one in Example 5.

GUIDED PRACTICE for Examples 4 and 5

- Do any of the numbered angles in the diagram at the right form a linear pair? Which angles are vertical angles? Explain.
- The measure of an angle is twice the measure of its complement. Find the measure of each angle.

