7.1 Apply the Pythagorean Theorem

Name:

Goal • Find side lengths in right triangles.

Complete the vocab. with definitions or pictures that make sense to you.

Your Notes

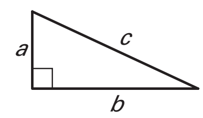
VOCABULARY

Pythagorean triple

Rewrite the Goal as an "I can" statement!

THEOREM 7.1: PYTHAGOREAN THEOREM

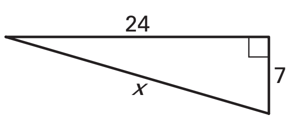
In a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs.



$c^2 = a^2 + b^2$

Example 1 ***Find the length of a hypotenuse***

Find the length of the hypotenuse of the right triangle.



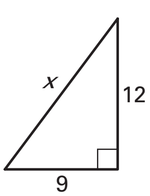
In the equation for the Pythagorean Theorem, "length of hypotenuse" and "length of leg" was shortened to "hypotenuse" and "leg".

Solution

$(\text{hypotenuse})^2 = (\text{leg})^2 + (\text{leg})^2$		**Pythagorean Theorem**
$x^2 =$	$\_\_\_\_^2 + \_\_\_\_^2$	**Substitute.**
$x^2 =$	$\_\_\_\_ + \_\_\_\_$	**Multiply.**
$x^2 =$	$\_\_\_\_$	**Add.**
$x =$	$\_\_\_\_$	**Find the positive square root.**

✔ ***Checkpoint*** **Complete the following exercise.**

1. Find the length of the hypotenuse of the right triangle.

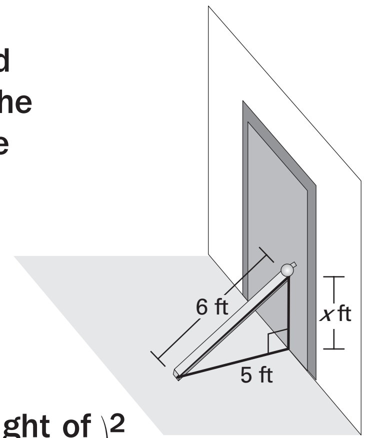


Stop and get the teacher's signature before you move on.

Your Notes

Example 2 *Find the length of a leg*

Door A 6 foot board rests under a doorknob and the base of the board is 5 feet away from the bottom of the door. Approximately how high above the ground is the doorknob?



Solution

$$\left(\text{Length of board}\right)^2 = \left(\text{Distance from door}\right)^2 + \left(\text{Height of doorknob}\right)^2$$

$\_\_\_^2 = \_\_\_^2 + x^2$	**Substitute.**
$\_\_\_\_ = \_\_\_\_ + x^2$	**Multiply.**
$\_\_\_\_ = x^2$	**Subtract ____ from each side.**
$\_\_\_\_\_ = x$	**Find positive square root.**
$\_\_\_\_ \approx x$	**Approximate with a calculator.**

The board is resting against the doorknob at about _____ feet above the ground.

In real-world applications, it is usually appropriate to use a calculator to approximate the square root of a number. Round your answer to the nearest tenth.

Checkpoint Complete the following exercise.

2. A 5 foot board rests under a doorknob and the base of the board is 3.5 feet away from the bottom of the door. Approximately how high above the ground is the doorknob?

Stop and get the teacher's signature before you move on.

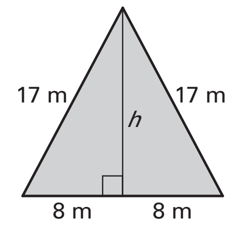
Your Notes

Example 3 *Find the area of an isosceles triangle*

Find the area of the isosceles triangle with side lengths 16 meters, 17 meters, and 17 meters.

Solution

Step 1 **Draw** a sketch. By definition, the length of an altitude is the ________of the triangle. In an isosceles triangle, the altitude to the base is also a perpendicular bisector. So, the altitude divides the triangle into two ______ triangles with the dimensions shown.



Step 2 **Use** the Pythagorean Theorem to find the height of the triangle.

$c^2 = a^2 + b^2$	**Pythagorean Theorem**
$\_\_\_\_^2 = \_\_\_^2 + h^2$	**Substitute.**
$\_\_\_\_\_ = \_\_\_\_ + h^2$	**Multiply.**
$\_\_\_\_\_ = h^2$	**Subtract ____ from each side.**
$\_\_\_\_ = h$	**Find the positive square root.**

Step 3 **Find** the area.

Area $= \frac{1}{2}$(base)(height) $= \frac{1}{2}$(____)(____) = _____

The area of the triangle is _____ square meters.

You may find it helpful to memorize the basic Pythagorean triples, shown in **bold,** for standardized tests.

COMMON PYTHAGOREAN TRIPLES AND SOME OF THEIR MULTIPLES

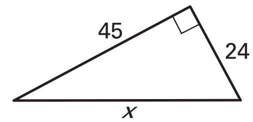
3, 4, 5	**5, 12, 13**	**8, 15, 17**	**7, 24, 25**
6, 8, 10	10, 24, 26	16, 30, 34	14, 48, 50
9, 12, 15	15, 36, 39	24, 45, 51	21, 72, 75
30, 40, 50	50, 120, 130	80, 150, 170	70, 240, 250
$3x, 4x, 5x$	$5x, 12x, 13x$	$8x, 15x, 17x$	$7x, 24x, 25x$

The most common Pythagorean triples are in bold. The other triples are the result of multiplying each integer in a bold face triple by the same factor.

Your Notes

Example 4 ***Find length of a hypotenuse using two methods***

Find the length of the hypotenuse of the right triangle.



Solution

Method 1: Use a Pythagorean triple.

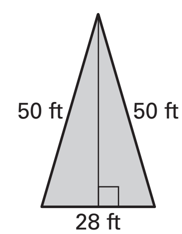
A common Pythagorean triple is 8, 15, ____. Notice that if you multiply the lengths of the legs of the Pythagorean triple by ___, you get the lengths of the legs of this triangle: 8 • ___ = 24 and 15 • ___ = 45. So, the length of the hypotenuse is ____ • ___ = ____.

Method 2: Use the Pythagorean Theorem.

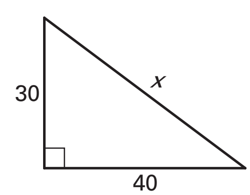
$x^2 = 24^2 + 45^2$	**Pythagorean Theorem**
$x^2 =$ _____ + _____	**Multiply.**
$x^2 =$ _____	**Add.**
$x =$ ____	**Find the positive square root.**

Checkpoint **Complete the following exercises.**

3. Find the area of the triangle.



4. Use a Pythagorean triple to find the unknown side length of the right triangle.



Homework

Stop and get the teacher's signature before you move on.

7.2 Use the Converse of the Pythagorean Theorem

Complete the vocab. with definitions or pictures that make sense to you.

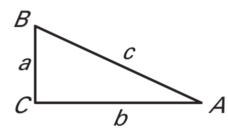
Goal • Use the Converse of the Pythagorean Theorem to determine if a triangle is a right triangle.

Your Notes

Rewrite the Goal as an "I can" statement!

THEOREM 7.2: CONVERSE OF THE PYTHAGOREAN THEOREM

If the square of the length of the longest side of a triangle is equal to the sum of the squares of the lengths of the other two sides, then the triangle is a ______ triangle.

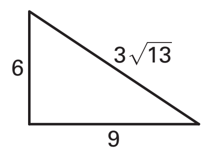


If $c^2 = a^2 + b^2$, then $\triangle ABC$ is a ______ triangle.

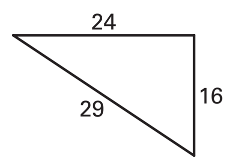
Example 1 ***Verify right triangles***

Tell whether the given triangle is a right triangle.

a.



b.



Solution

Let c represent the length of the longest side of the triangle. Check to see whether the side lengths satisfy the equation $c^2 = a^2 + b^2$.

a. $(\_\_\_\_\_\_)^2 \stackrel{?}{=} \_\_\_^2 + \_\_\_^2$

$\_\_\_ \cdot \_\_\_\_ \stackrel{?}{=} \_\_\_\_ + \_\_\_\_$

$\_\_\_\_\_ = \_\_\_\_\_$ ✓

The triangle ______ a right triangle.

b. $\_\_\_\_^2 \stackrel{?}{=} \_\_\_\_^2 + \_\_\_\_^2$

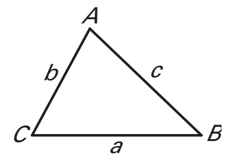
$\_\_\_\_\_ \stackrel{?}{=} \_\_\_\_\_ + \_\_\_\_\_$

$\_\_\_\_\_ \neq \_\_\_\_\_$

The triangle ______ a right triangle.

Your Notes

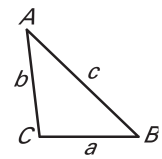
THEOREM 7.3



If the square of the length of the longest side of a triangle is less than the sum of the squares of the lengths of the other two sides, then the triangle *ABC* is an ________ triangle.

If $c^2 < a^2 + b^2$, then the triangle *ABC* is ________.

THEOREM 7.4



If the square of the length of the longest side of a triangle is greater than the sum of the squares of the lengths of the other two sides, then the triangle *ABC* is an ________ triangle.

If $c^2 > a^2 + b^2$, then the triangle *ABC* is ________.

Example 2 *Classify triangles*

Can segments with lengths of 2.8 feet, 3.2 feet, and 4.2 feet form a triangle? If so, would the triangle be *acute*, *right*, or *obtuse*?

Solution

The Triangle Inequality Theorem states that the sum of the lengths of any two sides of a triangle is greater than the length of the third side.

Step 1 **Use** the Triangle Inequality Theorem to check that the segments can make a triangle.

2.8 + 3.2 = ____	2.8 + 4.2 = ____	3.2 + 4.2 = ____
____ > 4.2	____ > 3.2	____ > 2.8

Step 2 **Classify** the triangle by comparing the square of the length of the longest side with the sum of squares of the lengths of the shorter sides.

$c^2 \ \underline{?}\ a^2 + b^2$	**Compare c^2 with $a^2 + b^2$.**
$\_\_\_\_^2 \ \underline{?}\ \_\_\_\_^2 + \_\_\_\_^2$	**Substitute.**
________ $\underline{?}$ ________ + ________	**Simplify.**
________ ____ ________	**c^2 is ________ than $a^2 + b^2$.**

The side lengths 2.8 feet, 3.2 feet, and 4.2 feet form an ________ triangle.

Your Notes

Example 3 ***Use the Converse of the Pythagorean Theorem***

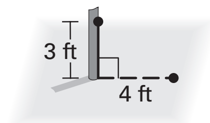
Lights You are helping install a light pole in a parking lot. When the pole is positioned properly, it is perpendicular to the pavement. How can you check that the pole is perpendicular using a tape measure?

Solution

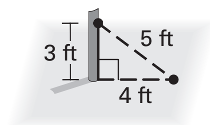
To show a line is perpendicular to a plane you must show that the line is perpendicular to __________ in the plane.

Think of the pole as a line and the pavement as a plane. Use a 3-4-5 right triangle and the Converse of the Pythagorean Theorem to show that the pole is perpendicular to different lines on the pavement.

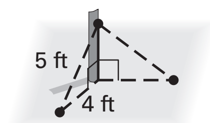
First mark 3 feet up the pole and mark on the pavement 4 feet from the pole.



Use the tape measure to check that the distance between the two marks is ___ feet. The pole makes ________ angle with the line on the pavement.

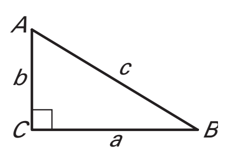


Finally, repeat the procedure to show that the pole is _______________ to another line on the pavement.



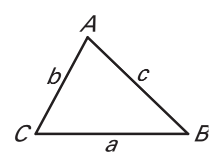
METHODS FOR CLASSIFYING A TRIANGLE BY ANGLES USING ITS SIDE LENGTHS

Theorem 7.2



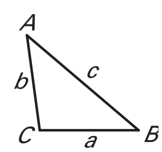
If $c^2 = a^2 + b^2$, then $m\angle C = 90°$ and $\triangle ABC$ is a ______ triangle.

Theorem 7.3



If $c^2 < a^2 + b^2$, then $m\angle C < 90°$ and $\triangle ABC$ is an _______ triangle.

Theorem 7.4

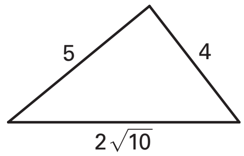


If $c^2 > a^2 + b^2$, then $m\angle C > 90°$ and $\triangle ABC$ is an ________ triangle.

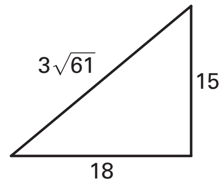
Your Notes

✔ ***Checkpoint*** **In Exercises 1 and 2, tell whether the triangle is a right triangle.**

1.



2.



3. Can segments with lengths of 6.1 inches, 9.4 inches, and 11.3 inches form a triangle? If so, would the triangle be *acute*, *right*, or *obtuse*?

4. In Example 3, could you use triangles with side lengths 50 inches, 120 inches, and 130 inches to verify that you have perpendicular lines? *Explain*.

Homework

Stop and get the teacher's signature before you move on.

7.3 Use Similar Right Triangles

Goal • Use properties of the altitude of a right triangle.

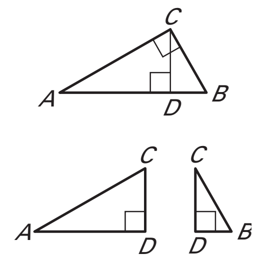
Complete the vocab. with definitions or pictures that make sense to you.

Your Notes

Rewrite the Goal as an "I can" statement!

THEOREM 7.5

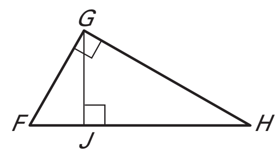
If the altitude is drawn to the hypotenuse of a right triangle, then the two triangles formed are _________ to the original triangle and to each other.



$\triangle CBD$ ____ $\triangle ABC$, $\triangle ACD$ ____ $\triangle ABC$, and $\triangle CBD$ ____ $\triangle ACD$.

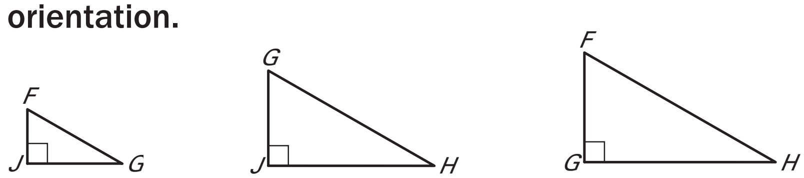
Example 1 ***Identify similar triangles***

Identify the similar triangles in the diagram.



Solution

Sketch the three similar right triangles so that the corresponding angles and sides have the same orientation.

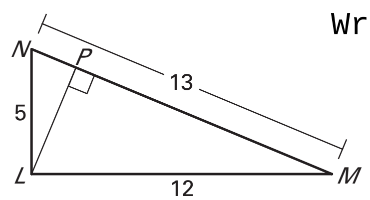


$\triangle$______ ~ $\triangle$______ ~ $\triangle$______

✔ ***Checkpoint*** **Complete the following exercise.**

1. Identify the similar triangles in the diagram.

Write a similarity statement.

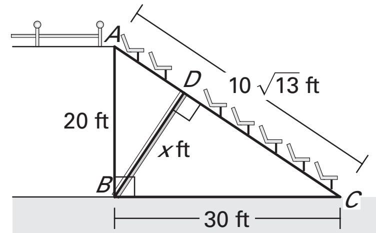


Stop and get the teacher's signature before you move on.

Your Notes

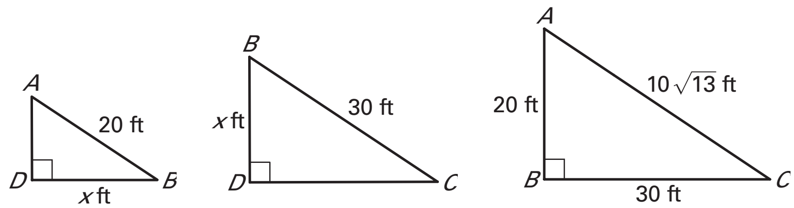
Example 2 ***Find the length of the altitude to the hypotenuse***

Stadium A cross section of a group of seats at a stadium shows a drainage pipe $\overline{BD}$ that leads from the seats to the inside of the stadium. What is the length of the pipe?



Solution

Step 1 **Identify** the similar triangles and sketch them.



$\triangle$______ ~ $\triangle$______ ~ $\triangle$______

Notice that if you tried to write a proportion using $\triangle ADB$ and $\triangle BDC$, there would be two unknowns, so you would not be able to solve for *x*.

Step 2 **Find** the value of *x*. Use the fact that $\triangle BDC \sim \triangle ABC$ to write a proportion.

$\frac{BD}{\square} = \frac{\square}{AC}$ **Corresponding side lengths of similar triangles are in proportion.**

$\frac{x}{\square} =$ ______ **Substitute.**

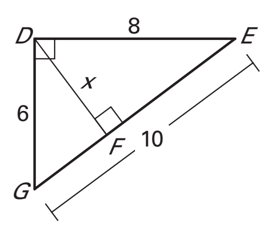
______ $x =$ ______ **Cross Products Property**

$x \approx$ ______ **Approximate.**

The length of the pipe is about ______ feet.

Checkpoint **Complete the following exercise.**

2. Identify the similar triangles. Then find the value of *x*.



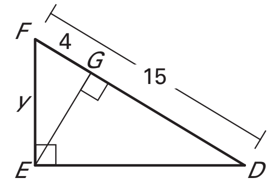
Stop and get the teacher's signature before you move on.

Your Notes

Notice that $\triangle FEG$ and $\triangle FDE$ both contain the side with length y, so these are the similar pair of triangles to use to solve for y.

Example 3 *Use a geometric mean*

Find the value of y. Write your answer in simplest radical form.



Solution

Write a proportion.

$$\frac{\boxed{}}{\text{length of hyp. of } \triangle FEG} = \frac{\text{length of shorter leg of } \triangle FDE}{\boxed{}}$$

$\frac{\square}{y} = \frac{y}{\square}$ **Substitute.**

$\_\_\_\_ = y^2$ **Cross Products Property**

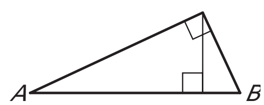
$\sqrt{\_\_\_\_} = y$ **Take positive square roots.**

$\_\_\_\sqrt{\_\_\_\_} = y$ **Simplify.**

THEOREM 7.6: GEOMETRIC MEAN (ALTITUDE) THEOREM

In a right triangle, the altitude from the right angle to the hypotenuse divides the hypotenuse into two segments.

The length of the altitude is the ________________ of the lengths of the two segments.

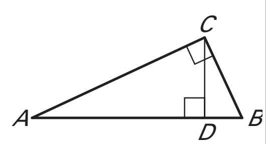


$$\frac{BD}{\square} = \frac{\square}{AD}$$

THEOREM 7.7: GEOMETRIC MEAN (LEG) THEOREM

In a right triangle, the altitude from the right angle to the hypotenuse divides the hypotenuse into two segments.

The length of each leg of the right triangle is the geometric mean of the lengths of the hypotenuse and the segment of the hypotenuse that is __________ to the leg.



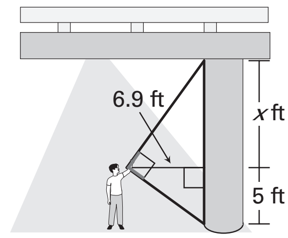
$\frac{AB}{CB} = \frac{CB}{\square}$ and

$\frac{AB}{AC} = \frac{AC}{\square}$

Your Notes

Example 4 *Find a height using indirect measurement*

Overpass To find the clearance under an overpass, you need to find the height of a concrete support beam.



You use a cardboard square to line up the top and bottom of the beam. Your friend measures the vertical distance from the ground to your eye and the distance from you to the beam. Approximate the height of the beam.

Solution

By Theorem 7.6, you know that _____ is the geometric mean of ___ and ___.

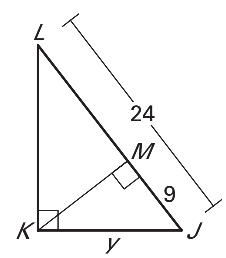
$\frac{\_\_\_\_\_}{\_\_\_\_\_} = \frac{\_\_\_\_\_}{\_\_\_\_\_}$ **Write a proportion.**

$x \approx$ _____ **Solve for x.**

So, the clearance under the overpass is $5 + x \approx 5 +$ _____ = ______ feet.

Checkpoint Complete the following exercises.

3. Find the value of y. Write your answer in simplest radical form.



4. The distance from the ground to Larry's eyes is 4.5 feet. How far from the beam in Example 4 would he have to stand in order to measure its height?

Homework

Stop and get the teacher's signature before you move on.

7.4 Special Right Triangles

Goal • Use the relationships among the sides in special right triangles.

Complete the vocab. with definitions or pictures that make sense to you.

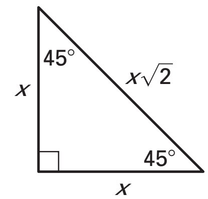
Your Notes

The extended ratio of the side lengths of a 45°-45°-90° triangle is $1:1:\sqrt{2}$.

THEOREM 7.8: 45°-45°-90° TRIANGLE THEOREM

In a 45°-45°-90° triangle, the hypotenuse is _____ times as long as each leg.

hypotenuse = leg • _____

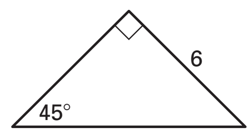


Rewrite the Goal as an "I can" statement!

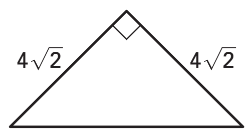
Example 1 ***Find hypotenuse length in a 45°-45°-90° triangle***

Find the length of the hypotenuse.

a.



b.



Solution

a. By the Triangle Sum Theorem, the measure of the third angle must be _____. Then the triangle is a _____-_____-90° triangle, so by Theorem 7.8, the hypotenuse is _____ times as long as each leg.

hypotenuse = leg • _____   **_____-_____-90° Triangle Theorem**

= _____   **Substitute.**

b. By the Base Angles Theorem and the Corollary to the Triangle Sum Theorem, the triangle is a 45°-45°-90° triangle.

hypotenuse = leg • _____   **45°-45°-90° Triangle Theorem**

= _____ • _____   **Substitute.**

= ___ • ___   **Product of square roots**

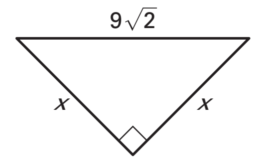
= ___   **Simplify.**

Remember the following properties of radicals: $\sqrt{a} \cdot \sqrt{b} = \sqrt{a \cdot b}$; $\sqrt{a \cdot a} = a$

Your Notes

Example 2 *Find leg lengths in a 45°-45°-90° triangle*

Find the lengths of the legs in the triangle.



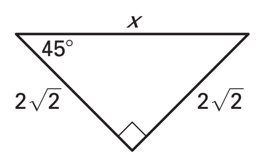
Solution

By the Base Angles Theorem and the Corollary to the Triangle Sum Theorem, the triangle is a 45°-45°-90° triangle.

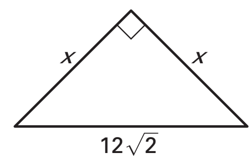
hypotenuse = leg • ____	**45°-45°-90° Triangle Theorem**
____ = x • ____	**Substitute.**
$\frac{\_\_\_\_}{\_\_\_\_} = \frac{x\,\square}{\square}$	**Divide each side by ____.**
____ = x	**Simplify.**

✓ ***Checkpoint*** **Find the value of the variable.**

1.



2.



Stop and get the teacher's signature before you move on.

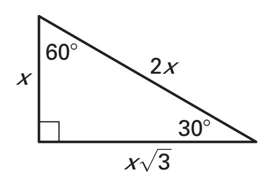
The extended ratio of the side lengths of a 30°-60°-90° triangle is $1:\sqrt{3}:2$.

THEOREM 7.9: 30°-60°-90° TRIANGLE THEOREM

In a 30°-60°-90° triangle, the hypotenuse is ________ as long as the shorter leg, and the longer leg is _____ times as long as the shorter leg.

hypotenuse = ____ • shorter leg

longer leg = shorter leg • _____



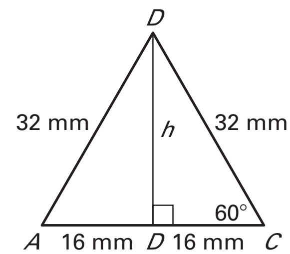
Your Notes

Remember that in an equilateral triangle, the altitude to a side is also the median to that side. So, altitude $\overline{BD}$ __________ $\overline{AC}$.

Example 3 ***Find the height of an equilateral triangle***

Music You make a guitar pick that resembles an equilateral triangle with side lengths of 32 millimeters. What is the approximate height of the pick?

Solution



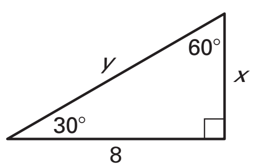
Draw the equilateral triangle described. Its altitude forms the longer leg of two _____-_____-90° triangles. The length h of the altitude is approximately the height of the pick.

longer leg = shorter leg • _____

h = _____ • _____ ≈ ______ mm

Example 4 ***Find lengths in a 30°-60°-90° triangle***

Find the values of x and y. Write your answer in simplest radical form.



Solution

Step 1 **Find** the value of x.

longer leg = shorter leg • _____

___ = x_____ **Substitute.**

$\frac{\quad}{\_\_\_\_\_}$ = x **Divide each side by _____.**

$\frac{\quad}{\_\_\_\_\_}$ • $\frac{\quad}{\_\_\_\_\_}$ = x **Multiply numerator and denominator by _____.**

$\frac{\quad}{\_\_\_\_\_}$ = x **Multiply fractions.**

Step 2 **Find** the value of y.

hypotenuse = ___ • shorter leg

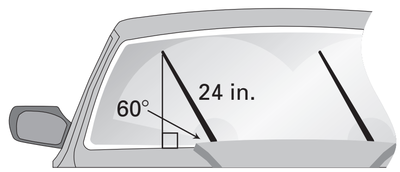
y = ___ • $\frac{\quad}{\_\_\_\_\_}$ = $\frac{\quad}{\_\_\_\_\_\_}$

Your Notes

Example 5 ***Find a height***

Windshield wipers A car is turned off while the windshield wipers are moving. The 24 inch wipers stop, making a 60° angle with the bottom of the windshield. How far from the bottom of the windshield are the ends of the wipers?

Solution



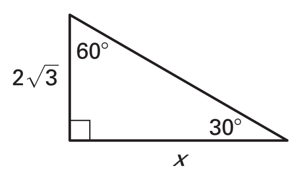
The distance *d* is the length of the longer leg of a _____-_____-90° triangle.
The length of the hypotenuse is _____ inches.

hypotenuse = ___ • shorter leg	**_____-_____-90° Triangle Theorem**
_____ = ___ • *s*	**Substitute.**
_____ = *s*	**Divide each side by ___.**
longer leg = shorter leg • _____	**_____-_____-90° Triangle Theorem**
d = _______	**Substitute.**
d ≈ _______	**Approximate.**

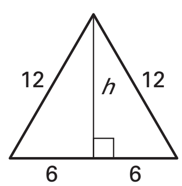
The ends of the wipers are about ______ inches from the bottom of the windshield.

✔ ***Checkpoint*** **In Exercises 3 and 4, find the value of the variable.**

3.



4.



5. In Example 5, how far from the bottom of the windshield are the ends of the wipers if they make a 30° angle with the bottom of the windshield?

Homework

Stop and get the teacher's signature before you move on.

7.5 Apply the Tangent Ratio

Goal • Use the tangent ratio for indirect measurement.

Complete the vocab. with definitions or pictures that make sense to you.

Your Notes

Rewrite the Goal as an "I can" statement!

VOCABULARY

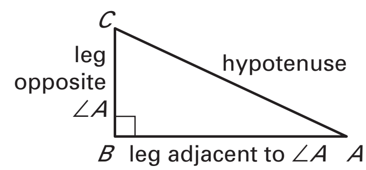
Trigonometric ratio

Tangent

TANGENT RATIO

Remember these abbreviations:
tangent → tan
opposite → opp.
adjacent → adj.

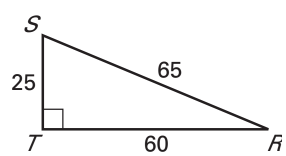
Let $\triangle ABC$ be a right triangle with acute $\angle A$. The tangent of $\angle A$ (written as tan A) is defined as follows:



$$\tan A = \frac{\text{length of leg opposite } \angle A}{\text{length of leg adjacent to } \angle A} = \frac{\square}{\square}$$

Example 1 ***Find tangent ratios***

Find tan S and tan R. Write each answer as a fraction and as a decimal rounded to four places, if necessary.



Solution

Unless told otherwise, round values of trigonometric ratios to the ten-thousandths' place and round lengths to the tenths' place.

$$\tan S = \frac{\text{opp. } \angle S}{\text{adj. to } \angle S} = \frac{\square}{\square} = \frac{\square}{\square} = \frac{\square}{\square} = \_\_\_\_$$

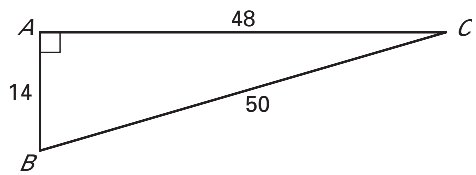
$$\tan R = \frac{\text{opp. } \angle R}{\text{adj. to } \angle R} = \frac{\square}{\square} = \frac{\square}{\square} = \frac{\square}{\square} \approx \_\_\_\_$$

Your Notes

Stop and get the teacher's signature before you move on.

✔ ***Checkpoint*** **Find tan B and tan C. Write each answer as a fraction and as a decimal rounded to four places.**

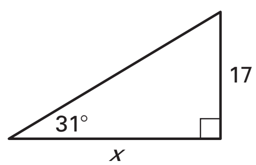
1.



Example 2 *Find a leg length*

Find the value of x.

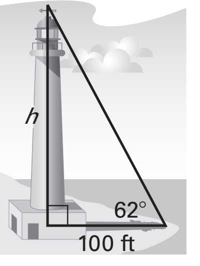
Use the tangent of an acute angle to find a leg length.



tan 31° = ____	**Write ratio for tangent of 31°.**
tan 31° = ____	**Substitute.**
____ • tan 31° = ____	**Multiply each side by ____.**
x = ____	**Divide each side by ____.**
$x \approx$ ____	**Use a calculator to find ____.**
$x \approx$ ____	**Simplify.**

Example 3 *Estimate height using tangent*

Lighthouse Find the height h of the lighthouse to the nearest foot.



____ = $\frac{\text{opp.}}{\text{adj.}}$	**Write ratio for ____.**
____ = ____	**Substitute.**
____ • ____ = h	**Multiply each side by ____.**
____ $\approx h$	**Use a calculator and simplify.**

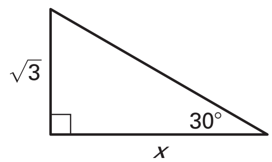
Your Notes

Example 4 *Use a special right triangle to find a tangent*

Use a special right triangle to find the tangent of a 30° angle.

Solution

Step 1 Choose ____ as the length of the shorter leg to simplify calculations. Use the 30°-60°-90° Triangle Theorem to find the length of the longer leg.



longer leg = ______________

x = ________ = ___

Step 2 Find tan 30°.

$\tan 30° = \frac{\_\_\_\_}{\_\_\_\_}$ **Write ratio for tangent of 30°.**

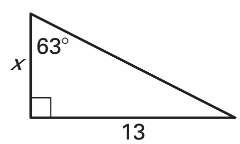
$\tan 30° = \frac{\_\_\_\_}{\_\_\_\_}$ **Substitute.**

The tangent of any 30° angle is $\frac{\_\_\_}{\_\_\_}$ ≈ ________.

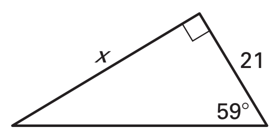
The tangents of all 30° angles are the same constant ratio. Any right triangle with a 30° angle can be used to determine this value.

Checkpoint **In Exercises 2 and 3, find the value of *x*. Round to the nearest tenth.**

2.



3.



4. In Example 4, suppose the length of the shorter leg is 1 instead of $\sqrt{3}$. Show that the tangent of 30° is still equal to $\frac{\sqrt{3}}{3}$.

Homework

Stop and get the teacher's signature before you move on.

7.6 Apply the Sine and Cosine Ratios

Goal • Use the sine and cosine ratios.

Complete the vocab. with definitions or pictures that make sense to you.

Your Notes

Rewrite the Goal as an "I can" statement!

VOCABULARY

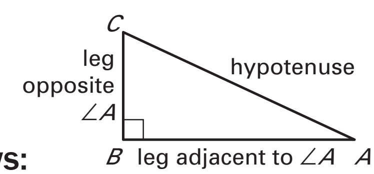
Sine, cosine

Angle of elevation

Angle of depression

SINE AND COSINE RATIOS

Let $\triangle ABC$ be a right triangle with acute $\angle A$. The sine of $\angle A$ and cosine of $\angle A$ (written sin A and cos A) are defined as follows:



$$\sin A = \frac{\text{length of leg opposite } \angle A}{\text{length of hypotenuse}} = \frac{\square}{\square}$$

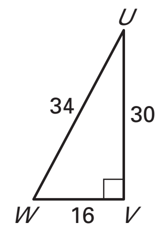
$$\cos A = \frac{\text{length of leg adjacent to } \angle A}{\text{length of hypotenuse}} = \frac{\square}{\square}$$

Remember these abbreviations:
sine → sin
cosine → cos
hypotenuse → hyp

Your Notes

Example 1 ***Find sine ratios***

Find sin *U* and sin *W*. Write each answer as a fraction and as a decimal rounded to four places.



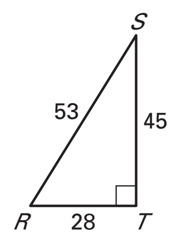
Solution

$\sin U = \frac{\text{opp. } \angle U}{\text{hyp.}} = \frac{\square}{\square} = \frac{\square}{\square} = \frac{\square}{\square} \approx$ ________

$\sin W = \frac{\text{opp. } \angle W}{\text{hyp.}} = \frac{\square}{\square} = \frac{\square}{\square} = \frac{\square}{\square} \approx$ ________

Example 2 ***Find cosine ratios***

Find cos *S* and cos *R*. Write each answer as a fraction and as a decimal rounded to four places.



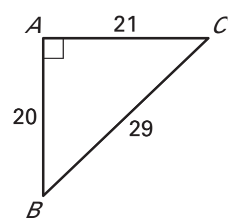
Solution

$\cos S = \frac{\text{adj. to } \angle S}{\text{hyp.}} = \frac{\square}{\square} = \frac{\square}{\square} \approx$ ________

$\cos R = \frac{\text{adj. to } \angle R}{\text{hyp.}} = \frac{\square}{\square} = \frac{\square}{\square} \approx$ ________

✔ ***Checkpoint*** **Find sin *B*, sin *C*, cos *B*, and cos *C*. Write each answer as a fraction and as a decimal rounded to four places.**

1.

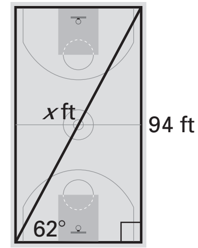


Stop and get the teacher's signature before you move on.

Your Notes

Example 3 *Use a trigonometric ratio to find a hypotenuse*

Basketball You walk from one corner of a basketball court to the opposite corner. Write and solve a proportion using a trigonometric ratio to approximate the distance of the walk.



Solution

sin 62° = ______ **Write ratio for sine of 62°.**

sin 62° = ______ **Substitute.**

___ • _________ = _____ **Multiply each side by ___.**

x = _________ **Divide each side by _________.**

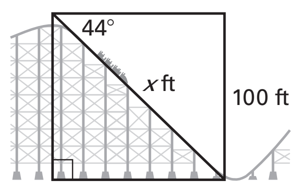
$x \approx$ _________ **Use a calculator to find _________.**

$x \approx$ _______ **Simplify.**

The distance of the walk is about _______ feet.

Example 4 *Find a hypotenuse using an angle of depression*

Roller Coaster You are at the top of a roller coaster 100 feet above the ground. The angle of depression is 44°. About how far do you ride down the hill?



sin 44° = ______ **Write ratio for sine of 44°.**

sin 44° = ______ **Substitute.**

x • _________ = ______ **Multiply each side by ___.**

x = _________ **Divide each side by _________.**

$x \approx$ _________ **Use a calculator to find _________.**

$x \approx$ ________ **Simplify.**

You ride about _____ feet down the hill.

Your Notes

✔ ***Checkpoint*** **Complete the following exercises.**

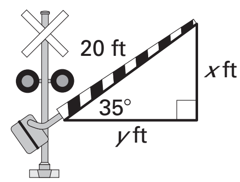
2. In Example 3, use the cosine ratio to approximate the width of the basketball court.

3. Suppose the angle of depression in Example 4 is 72°. About how far would you ride down the hill?

Stop and get the teacher's signature before you move on.

Example 5 ***Find leg lengths using an angle of elevation***

Railroad A railroad crossing arm that is 20 feet long is stuck with an angle of elevation of 35°. Find the lengths x and y.



Solution

Step 1 **Find x.**

$\_\_\_\_\_ = \frac{\text{opp.}}{\text{hyp.}}$ **Write ratio for _____ of _____.**

$\_\_\_\_\_ = \frac{\_\_\_}{\_\_\_}$ **Substitute.**

$\_\_\_\_\_ = x$ **Multiply each side by _____.**

$\_\_\_\_\_ \approx x$ **Use a calculator to simplify.**

Step 2 **Find y.**

$\_\_\_\_\_ = \frac{\text{adj.}}{\text{hyp.}}$ **Write ratio for _____ of _____.**

$\_\_\_\_\_ = \frac{\_\_\_}{\_\_\_}$ **Substitute.**

$\_\_\_\_\_ = y$ **Multiply each side by _____.**

$\_\_\_\_\_ \approx y$ **Use a calculator to simplify.**

Your Notes

Example 6 *Use a special right triangle to find a sin and cos*

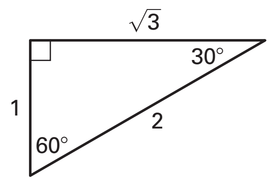
Use a special right triangle to find the sine and cosine of a 30° angle.

Solution

Use the 30°-60°-90° Triangle Theorem to draw a right triangle with side lengths of 1, $\sqrt{3}$, and ____. Then set up sine and cosine ratios for the 30° angle.

$\sin 30° = \frac{\_\_\_\_}{\_\_\_\_} = \frac{\_\_\_\_}{\_\_\_\_} = \_\_\_\_\_\_$

$\cos 30° = \frac{\_\_\_\_}{\_\_\_\_} = \frac{\_\_\_\_}{\_\_\_\_} \approx \_\_\_\_\_\_$



Checkpoint **Complete the following exercises.**

4. In Example 5, suppose the angle of elevation is 40°. What are the new lengths x and y?

5. Use a special right triangle to find the sine and cosine of a 60° angle.

Homework

Stop and get the teacher's signature before you move on.

Complete the vocab. with definitions or pictures that make sense to you.

7.7 Solve Right Triangles

Goal • Use inverse tangent, sine, and cosine ratios.

Your Notes

VOCABULARY

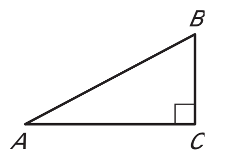
Solve a right triangle

Rewrite the Goal as an "I can" statement!

The expression "$\tan^{-1} x$" is read as "the inverse tangent of x."

INVERSE TRIGONOMETRIC RATIOS

Let $\angle A$ be an acute angle.



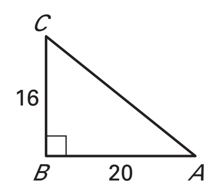
Inverse Tangent If $\tan A = x$, then $\tan^{-1} x = m\angle A$. $\tan^{-1} \frac{BC}{AC} = m\angle A$

Inverse Sine If $\sin A = y$, then $\sin^{-1} y = m\angle A$. $\sin^{-1} \frac{BC}{AB} = m\angle A$

Inverse Cosine If $\cos A = z$, then $\cos^{-1} z = m\angle A$. $\cos^{-1} \frac{AC}{AB} = m\angle A$

Example 1 ***Use an inverse tangent to find an angle measure***

Use a calculator to approximate the measure of $\angle A$ to the nearest tenth of a degree.



Because $\tan A =$ ____ = ____ = ____,

$\tan^{-1}$ ____ $= m\angle A$. Using a calculator,

$\tan^{-1}$ ____ $\approx$ ____________.

So, the measure of $\angle A$ is approximately ______.

✔ ***Checkpoint*** **Complete the following exercise.**

1. In Example 1, use a calculator and an inverse tangent to approximate $m\angle C$ to the nearest tenth of a degree.

Stop and get the teacher's signature before you move on.

Your Notes

Example 2 *Use an inverse sine and an inverse cosine*

Let ∠A and ∠B be acute angles in two right triangles. Use a calculator to approximate the measures of ∠A and ∠B to the nearest tenth of a degree.

a. $\sin A = 0.76$ **b.** $\cos B = 0.17$

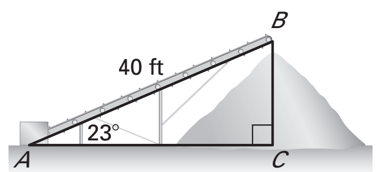
Solution

a. $m\angle A =$ ____________ $\approx$ _______

b. $m\angle B =$ ____________ $\approx$ _______

Example 3 *Solve a right triangle*

Solve the right triangle. Round decimal answers to the nearest tenth.



Solution

Step 1 **Find** $m\angle B$ by using the Triangle Sum Theorem.

______ $= 90° + 23° + m\angle B$

_____ $= m\angle B$

Step 2 **Approximate** *BC* using a ______ ratio.

_________ $= \frac{BC}{40}$ **Write ratio for _________.**

_____________ $= BC$ **Multiply each side by _____.**

_____________ $\approx BC$ **Approximate _________.**

______ $\approx BC$ **Simplify and round answer.**

Step 3 **Approximate** *AC* using a ________ ratio.

_________ $= \frac{AC}{40}$ **Write ratio for _________.**

_____________ $= AC$ **Multiply each side by _____.**

_____________ $\approx AC$ **Approximate _________.**

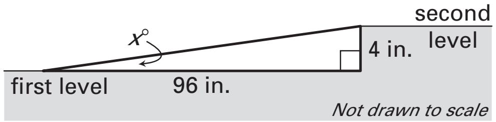
______ $\approx AC$ **Simplify and round answer.**

The angle measures are _____, _____, and _____. The side lengths are ____ feet, about ______ feet, and about ______ feet.

Your Notes

Example 4 ***Solve a real-world problem***

Model Train You are building a track for a model train. You want the track to incline from the first level to the second level, 4 inches higher, in 96 inches. Is the angle of elevation less than 3°?



Solution

Use the tangent and inverse tangent ratios to find the degree measure x of the incline.

$\tan x° =$ ______ $=$ ______ $\approx$ ________

$x \approx$ ______________ $\approx$ _____

The incline is about _____, so it _____________ 3°.

Checkpoint Complete the following exercises.

2. Find $m\angle D$ to the nearest tenth of a degree if $\sin D = 0.48$.

3. Solve a right triangle that has a 50° angle and a 15 inch hypotenuse.

4. In Example 4, suppose another incline rises 8 inches in 120 inches. Is the incline less than 3°?

Homework

Stop and get the teacher's signature before you move on.