SAVE THIS PACKET BECAUSE YOU CAN USE IT ON THE CH.2 SUMMATIVE AS WELL AS THE SEM. 1 FINAL.

# Use Inductive Reasoning Name:

Example 1

**Goal** • Describe patterns and use inductive reasoning.

#### **Your Notes**

"I can" statement!

VOCABULARY	
Conjecture	
Inductive Reasoning	
Counterexample	

Complete the vocab. with definitions or pictures that make sense to you.

Describe how to sketch the fourth figure in the pattern. Then sketch the fourth figure.				
Figure 1	Figure 2	Figure 3		
Solution				
	is divided into regions as the figure the fourth figure by		_	

Describe a visual pattern

Stop and get the teacher's signature before you move on.

**Checkpoint** Complete the following exercise.

dividing the rectangle into \_\_\_\_\_. Shade the section just the horizontal segment at the . .

**1.** Sketch the fifth figure in the pattern in Example 1.

Figure 4

Three dots (. . .) tell you that the pattern continues.

# **Example 2** Describe the number pattern

Describe the pattern in the numbers -1, -4, -16,  $-64, \ldots$  Write the next three numbers in the pattern.

Notice that each number in the pattern is times the previous number.

The next three numbers are \_\_\_\_\_

## **Example 3** *Make a conjecture*

Given five noncollinear points, make a conjecture about the number of ways to connect different pairs of the points.

Make a table and look for a pattern. Notice the pattern in how the number of connections . You can use the pattern to make a conjecture.

Number of points	1	2	3	4	5
Picture	•	•	< <tr> ✓</tr>	$\bowtie$	
Number of connections					_?_
				1	
	+	+	+	+ ?	)

Conjecture You can connect five noncollinear points + , or \_\_\_\_ different ways.

# **Checkpoint** Complete the following exercises.

Stop and get the teacher's signature before you move on.

- 2. Describe the pattern in the numbers 1, 2.5, 4, 5.5, ... and write the next three numbers in the pattern.
- 3. Rework Example 3 if you are given six noncollinear points.

#### Example 4

Make and test a conjecture

Numbers such as 1, 3, and 5 are called consecutive odd numbers. Make and test a conjecture about the sum of any three consecutive odd numbers.

**Step 1 Find** a pattern using groups of small numbers.

**Conjecture** The sum of any three consecutive odd numbers is three times \_\_\_\_\_.

**Step 2 Test** your conjecture using other numbers.

Stop and get the teacher's signature before you move on.

- **Checkpoint** Complete the following exercise.
  - **4.** Make and test a conjecture about the sign of the product of any four negative numbers.
  - 1. What do you think will happen if you multiply four negative numbers? (this is your conjecture)
  - 2. Pick 4 neg. numbers and test it.

# **Example 5** Find a counterexample

A student makes the following conjecture about the difference of two numbers. Find a counterexample to disprove the student's conjecture.

**Conjecture** The difference of any two numbers is always smaller than the larger number.

To find a counterexample, you need to find a difference that is \_\_\_\_\_ than the \_\_\_\_ number.

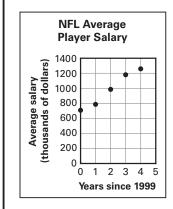
$$8 - (-4) =$$

Because  $\_\_$   $\not<$   $\_$ , a counterexample exists. The conjecture is false.

#### **Example 6**

## Making conjectures from data displays

The scatter plot shows the average salary of players in the National Football League (NFL) since 1999. Make a conjecture based on the graph.



#### Solution

The scatter plot shows that the values \_\_\_\_\_ each year. So, one possible conjecture is that the average player in the NFL is earning \_\_\_\_\_ money today than in 1999.

# **Checkpoint** Complete the following exercises.

Stop and get the teacher's signature before you move on.

**5.** Find a counterexample to show that the following conjecture is false.

**Conjecture** The quotient of two numbers is always smaller than the dividend.

**6.** Use the graph in Example 6 to make a conjecture that *could* be true. Give an explanation that supports your reasoning.

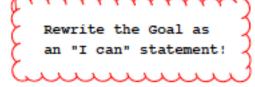
Look at the graph and tell me what you think will happen after year 4.

### **Homework**

# 2.2 Analyze Conditional **Statements**

**Goal** • Write definitions as conditional statements.

#### **Your Notes**



VOCABULARY	
Conditional statement	
If-then form	
Hypothesis	
Conclusion	
Negation	
Converse	
Inverse	
Contrapositive	
Equivalent statements	
Perpendicular lines (dra	w a picture)
Biconditional statement	

Complete the vocab. with definitions or pictures that make sense to you.

#### Example 1

### Rewrite a statement in if-then form

### Rewrite the conditional statement in if-then form.

All vertebrates have a backbone.

### Solution

First, identify the hypothesis and the conclusion. When you rewrite the statement in if-then form, you may need to reword the hypothesis or conclusion.

All vertebrates have a backbone.

lf	, then	
	-	

Stop and get the teacher's signature before you move on.

- Checkpoint Write the conditional statement in if-then form.
  - 1. All triangles have 3 sides.
- **2.** When x = 2,  $x^2 = 4$ .

# **Example 2** Write four related conditional statements

Write the if-then form, the converse, the inverse, and the contrapositive of the conditional statement "Olympians are athletes." Decide whether each statement is *true* or *false*.

## **Solution**

If-then form \_\_\_\_\_

Converse

Inverse \_\_\_\_

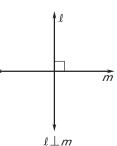
Contrapositive \_\_\_\_\_

DEDD			INICC
PERP	ENU	AK L	IIVE3

**Definition** If two lines intersect to form a angle, then they are perpendicular lines.

The definition can also be written using the converse: If any two lines are perpendicular lines, then they intersect to form a angle.

You can write "line  $\ell$  is perpendicular to line m" as  $\ell \perp m$ .



### **Example 3** Use definitions

Decide whether each statement about the diagram is true. Explain your answer using the definitions you have learned.

- a.  $\overrightarrow{AC} \perp \overrightarrow{BD}$
- **b.**  $\angle AED$  and  $\angle BEC$  are a linear pair.



#### Solution

- a. The statement is . The right angle symbol indicates that the lines intersect to form a angle. So you can say the lines are
- **b.** The statement is . Because  $\angle AED$  and  $\angle BEC$ are not \_\_\_\_\_ angles,  $\angle \textit{AED}$  and  $\angle \textit{BEC}$  are not

## **Example 4** Write a biconditional

Write the definition of parallel lines as a biconditional.

**Definition:** If two lines lie in the same plane and do not intersect, then they are parallel.

### Solution

Biconditional:

Stop and get the teacher's signature before you move on.

Square: A 4-sided polygon that has 4 right angles and 4 congruent sides.

Rectangle: A 4-sided polygon that has 4 right angles.

# **Checkpoint** Complete the following exercises.

3. Write the if-then form, the converse, the inverse, and the contrapositive of the conditional statement "Squares are rectangles." Decide whether each statement is true or false.

if-then:

converse:

inverse:

contrapositive:

4. Decide whether each statement about the diagram is true. Explain your answer using the definitions you have learned.

- a.  $\angle GLK$  and  $\angle JLK$  are supplementary.
- **b.**  $\overrightarrow{GJ} \perp \overrightarrow{HK}$

5. Write the statement below as a biconditional.

converse

Statement: If a student is a boy, he will be in group A. If a student is in group A, the student must be a boy.

A biconditional does not start with the word "if".

#### Homework

# 23 Apply Deductive Reasoning

**Goal** • Form logical arguments using deductive reasoning.

#### **Your Notes**

The Law of Detachment is also called a direct argument. The Law of Syllogism is sometimes called the chain rule.

When you use the 'Law of Detachment' you will restate the conclusion from the original conditional.

### VOCABULARY

**Deductive Reasoning** 

LAWS OF LOGIC

**Law of Detachment** If the hypothesis of a true conditional statement is true, then the is also true.

Law of Syllogism

If hypothesis p, then conclusion q. are true,

If hypothesis q, then conclusion r.

If hypothesis p, then conclusion r.  $\leftarrow$ 

If these statements

then this statement is true.

# **Example 1** Use the Law of Detachment

Use the Law of Detachment to make a valid conclusion in the true situation.

- a. If two angles have the same measure, then they are congruent. You know that  $m \angle A = m \angle B$ .
- **b.** Jesse goes to the gym every weekday. Today is Monday.

#### Solution

- **a.** Because  $m \angle A = m \angle B$  satisfies the hypothesis of a true conditional statement, the conclusion is also true. So, .
- b. First, identify the hypothesis and the conclusion of the first statement. The hypothesis is " ," and the conclusion is "

"Today is Monday" satisfies the hypothesis of the conditional statement, so you can conclude that

Complete

with

you.

the vocab.

definitions or pictures that make sense to

When you use the 'Law of Syllogism' you will create a new if-then statement.

> The order in which the statements are given does not affect whether you can use the Law of Syllogism.

Stop and get the teacher's signature before you move on.

Example 2 Use the Law of Syllogism

If possible, use the Law of Syllogism to write the conditional statement that follows from the pair of true statements.

- a. If Ron eats lunch today, then he will eat a sandwich. If Ron eats a sandwich, then he will drink a glass of milk.
- **b.** If  $x^2 > 36$ , then  $x^2 > 30$ . If x > 6, then  $x^2 > 36$ .
- c. If a triangle is equilateral, then all of its sides are congruent. If a triangle is equilateral, then all angles in the interior of the triangle are congruent.

#### **Solution**

a. The conclusion of the first statement is the hypothesis of the second statement, so you can write the following.

If Ron eats lunch today, then

**b.** Notice that the conclusion of the second statement is the hypothesis of the first statement, so you can write the following.

If x > 6, then .

c. Neither statement's conclusion is the same as the other statement's . You cannot use the Law of Syllogism to write a new conditional statement.

# **Checkpoint** Complete the following exercises.

- **1.** If  $0^{\circ} < m \angle A < 90^{\circ}$ , then A is acute. The measure of  $\angle A$  is 38°. Using the Law of Detachment, what statement can you make?
- 2. State the law of logic that is illustrated below. If you do your homework, then you can watch TV. If you watch TV, then you can watch your favorite show. If you do your homework, then you can watch your favorite show.

#### Example 3

Use inductive and deductive reasoning

What conclusion can you make about the sum of an odd integer and an odd integer?

#### Solution

**Step 1 Look** for a pattern in several examples. Use inductive reasoning to make a conjecture.

$$-3 + 5 =$$
\_\_\_,  $-1 + 5 =$ \_\_\_,  $3 + 5 =$ \_\_\_,  $-3 + (-5) =$ \_\_\_,  $1 + (-5) =$ \_\_\_,  $3 + (-5) =$ 

Conjecture: Odd integer + Odd integer = integer

**Step 2 Let** *n* and *m* each be any integer. Use deductive reasoning to show the conjecture is true.

2 <i>n</i> and	2m are		integers	because	any
integer	multiplied	by 2	is	_•	

$$2n - 1$$
 and  $2m + 1$  are \_\_\_\_ integers because  $2n$  and  $2m$  are \_\_\_\_ integers.

$$(2n - \underline{1}) + (2m + \underline{\hspace{1cm}})$$
 represents the sum of an  $\underline{\hspace{1cm}}$  integer  $2n - \underline{\hspace{1cm}}$  and an  $\underline{\hspace{1cm}}$  integer  $2m + \underline{\hspace{1cm}}$ .

$$(2n - \underline{\hspace{1cm}}) + (2m + \underline{\hspace{1cm}}) = \underline{\hspace{1cm}}(n + m)$$

The result is the product of and an integer n + m. So, (n + m) is an integer.

The sum of an odd integer and an odd integer is an integer.

Stop and get the teacher's signature before you move on.

Do this problem like example 3. You will probably need more space. **Checkpoint** Complete the following exercise.

3. Use inductive reasoning to make a conjecture about the sum of a negative integer and itself. Then use deductive reasoning to show the conjecture is true.

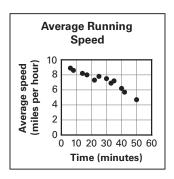
Inductive: use patterns and examples.

Deductive: use algebra and variables.

#### Example 4

## Reasoning from a graph

Tell whether the statement is the result of *inductive* reasoning or deductive reasoning. Explain your choice.



- a. The runner's average speed decreases as time spent running increases.
- **b.** The runner's average speed is slower when running for 40 minutes than when running for 10 minutes.

#### **Solution**

- reasoning, because it is based on a pattern in the data
- reasoning, because you are comparing values that are given on the graph

Stop and get the teacher's signature before you move on.

# Checkpoint Complete the following exercises.

4. Use inductive reasoning to write another statement about the graph in Example 4.

5. Use deductive reasoning to write another statement about the graph in Example 4. Homework

# 4. Use Postulates and Diagrams

**Goal** • Use postulates involving points, lines, and planes.

#### **Your Notes**

# **VOCABULARY**

Line perpendicular to a plane (draw a picture)

Complete the vocab. with definitions or pictures that make sense to you.

POINT, LIN	E, AND PLANE POSTULATES
Postulate 5	Through any two points there exists exactly one
Postulate 6	A line contains at least two
Postulate 7	If two lines intersect, then their intersection is exactly
Postulate 8	Through any three points there exists exactly one plane.
Postulate 9	A plane contains at least three points.
Postulate 10	If two points lie in a plane, then the line containing them
Postulate 11	If two planes intersect, then their intersection is a

# **Example 1** Identify a postulate illustrated by a diagram

State the postulate illustrated by the diagram.



#### Solution

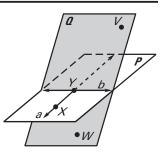
Postulate Through any three points there exists exactly one plane.

#### Example 2 Identify postulates from a diagram

Use the diagram to write examples of Postulates 9 and 11.

Postulate 9 Plane contains at least three noncollinear points,

**Postulate 11** The intersection of plane P and plane Q is \_\_\_\_\_.



Checkpoint Use the diagram in Example 2 to complete the following exercises.

Stop and get the teacher's signature before you move on.

- 1. Which postulate allows you to say that the intersection of line a and line b is a point?
- 2. Write examples of Postulates 5 and 6.

You CAN Assume.

#### CONCEPT SUMMARY: INTERPRETING A DIAGRAM

When you interpret a diagram, you can only assume information about size or measure if it is marked.

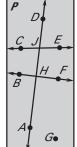
#### YOU CANNOT ASSUME

All points shown are .

∠AHB and \_\_\_\_\_ are a linear pair. ∠AHF and \_\_\_\_\_ are vertical angles.

A, H, J, and D are \_\_\_\_\_.

 $\overrightarrow{AD}$  and  $\overrightarrow{BF}$  intersect at .



#### YOU CANNOT ASSUME

G, F, and E are collinear.

 $\overrightarrow{BF}$  and  $\overrightarrow{CE}$  intersect.

 $\overrightarrow{BF}$  and  $\overrightarrow{CE}$  do not intersect.

 $\angle BHA \cong \angle CJA$ 

 $\overrightarrow{AD} \perp \overrightarrow{BF}$  or  $m \angle AHB = 90^{\circ}$ 

#### Example 3

# Use given information to sketch a diagram

Sketch a diagram showing  $\overline{RS}$  perpendicular to  $\overline{TV}$ , intersecting at point X.

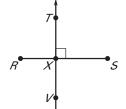
Notice that the picture was drawn so that X does not look like a midpoint of  $\overline{RS}$ .

#### Solution

**Step 1 Draw**  $\overline{RS}$  and label points R and S.

Step 2 Draw a point X R and S.

**Step 3 Draw**  $\overrightarrow{TV}$  through X so that it is to RS.



#### Example 4

## Interpret a diagram in three dimensions

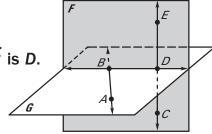
Which of the following statements cannot be assumed from the diagram?

E, D, and C are collinear.

The intersection of  $\overrightarrow{BD}$  and  $\overrightarrow{EC}$  is D.

 $\overrightarrow{BD} \perp \overrightarrow{EC}$ 

 $\overrightarrow{EC} \perp \text{plane } G$ 



## **Solution**

With no right angles marked, you cannot assume that

Stop and get the teacher's signature before you move on.

# **Homework**

# Checkpoint Complete the following exercises.

- 3. In Example 3, if the given information indicated that RX and XS are congruent, how would the diagram change?
- 4. In the diagram for Example 4, can you assume that BD is the intersection of plane F and plane G?

# Reason Using Properties from Algebra

**Goal** • Use algebraic properties in logical arguments.

#### **Your Notes**

Rev	rit	te tl	he	Goal	as	
an	"I	can	" 5	state	ment!	

# **ALGEBRAIC PROPERTIES OF EQUALITY**

Let a, b, and c be real numbers.

If a = b, then \_\_\_\_\_. **Addition Property** 

**Subtraction Property** If a = b, then

**Multiplication Property** If a = b, then .

If a = b and  $c \neq 0$ , then **Division Property** 

**Substitution Property** If a = b, then \_\_\_\_\_

# **Example 1** Write reasons for each step

Solve 2x + 3 = 9 - x. Write a reason for each step.

**Explanation Equation** Reason

2x + 3 = 9 - xWrite Given original

equation.

2x + 3 + = 9 - x + Add to each side.

+ 3 = \_\_\_ Combine

like terms.

Subtract from

each side. Divide each side by .

The value of x is x.

x =

## **DISTRIBUTIVE PROPERTY**

a(b + c) =, where a, b, and c are real numbers.

# **Example 2** Use the Distributive Property

Solve -4(6x + 2) = 64. Write a reason for each step.

#### Solution

Equation	Explanation	Reason
-4(6x + 2) = 64	Write original equation.	Given
= 64	Multiply.	
=	Add to each side.	Property of Equality
=	Divide each side by .	Property of Equality

**Checkpoint** Complete the following exercises.

**1.** Solve x - 5 = 7 + 2x. Write a reason for each step.

Stop and get the teacher's signature before you move on.

**2.** Solve 4(5 - x) = -2x. Write a reason for each step.

Speed A motorist travels 5 miles per hour slower than the speed limit s for 3.5 hours. The distance traveled d can be determined by the formula d = 3.5(s - 5). Solve for s.

Equation	Explanation	Reason
d = 3.5(s - 5)	Write original equation.	Given
d =	Multiply.	
d + =	Add to each side.	Property of Equality
$\frac{d + \square}{\square} = s$	Divide each side by	Property of Equality

REFLEXIVE PR	ROPERTY OF EQUALITY
<b>Real Numbers</b>	For any real number a,
Segment Length	For any segment AB,
Angle Measure	For any angle <i>A</i> ,
SYMMETRIC F	PROPERTY OF EQUALITY
Real Numbers	For any real numbers $a$ and $b$ , if $a = b$ , then
Segment Length	For any segments $AB$ and $CD$ , if $AB = CD$ , then
Angle Measure	For any angles A and B, if $m\angle A = m\angle B$ , then
TRANSITIVE P	ROPERTY OF EQUALITY
Real Numbers	For any real numbers $a$ , $b$ , and $c$ , if $a = b$ and $b = c$ , then
Segment Length	For any segments $AB$ , $CD$ , and $EF$ , if $AB = CD$ and $CD = EF$ , then
Angle Measure	For any angles $A$ , $B$ , and $C$ , if $m\angle A = m\angle B$ and $m\angle B = m\angle C$ , then

#### Use properties of equality Example 4

Show that CF = AD.

•	•	•	<del>- +</del>	-	—•
Α	В	С	D	Ε	F

**Equation** 

Reason

$$AC = AB + BC$$

DF = +

$$DF = BC + AB$$

**Equality** 

$$DF + CD =$$
\_\_\_\_ +  $CD$ 

**Substitution Property of Equality** 

**Checkpoint** Complete the following exercises. In Exercises 4-6, name the property of equality that the statement illustrates.

Stop and get the teacher's signature before you move on. **3.** Suppose the equation in Example 3 is d = 5(s + 3). Solve for s. Write a reason for each step.

4. If GH = JK, then JK = GH.

#### **Homework**

**5.** If 
$$r = s$$
, and  $s = 44$ , then  $r = 44$ .

6. 
$$m \angle N = m \angle N$$

# **Prove Statements about Segments and Angles**

**Goal** • Write proofs using geometric theorems.

#### **Your Notes**

Rewrite the Goal as an "I can" statement!

## **VOCABULARY**

**Proof** 

Two-column proof

Theorem

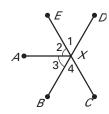
Complete the vocab. with definitions or pictures that make sense to you.

## **Example 1** Write a two-column proof

Use the diagram to prove  $m \angle 1 = m \angle 4$ .

Given  $m \angle 2 = m \angle 3$ ,  $m \angle AXD = m \angle AXC$ 

Prove  $m \angle 1 = m \angle 4$ 



Writing a twocolumn proof is a formal way of organizing your reasons to show a statement is true.

#### **Statements**

- **1.**  $m\angle AXC = m\angle AXD$
- **2.** *m*∠*AXD*  $= m \angle + m \angle$
- 3.  $m\angle AXC$  $= m \angle + m \angle$
- 4.  $m \angle 1 + m \angle 2$  $= m \angle 3 + m \angle 4$
- **5.**  $m \angle 2 = m \angle 3$
- **6.**  $m \angle 1 + m \angle$  $= m \angle 3 + m \angle 4$
- 7.  $m \angle 1 = m \angle 4$

#### Reasons

- 2. Angle Addition Postulate
- 3. Angle Addition Postulate

- **6.** Substitution Property of **Equality**

## **THEOREM 2.1 CONGRUENCE OF SEGMENTS**

Segment congruence is reflexive, symmetric, and transitive.

For any segment AB, \_\_\_\_\_\_. Reflexive

Symmetric If  $\overline{AB}\cong\overline{CD}$ , then .

**Transitive** If  $\overline{AB} \cong \overline{CD}$  and  $\overline{CD} \cong \overline{EF}$ , then

### THEOREM 2.2 CONGRUENCE OF ANGLES

Angle congruence is reflexive, symmetric, and transitive.

**Reflexive** For any angle A, \_\_\_\_\_.

Symmetric If  $\angle A \cong \angle B$ , then .

**Transitive** If  $\angle A \cong \angle B$  and  $\angle B \cong \angle C$ , then

# **Example 2** Name the property shown

Name the property illustrated by the statement.

If  $\angle 5 \cong \angle 3$ , then  $\angle 3 \cong \angle 5$ .

**Checkpoint** Complete the following exercises.

**1.** Three steps of a proof are shown. Give the reasons for the last two steps.

Given BC = AB

Prove AC = AB + AB

Statements	Reasons
1. $BC = AB$	1. Given
2. AC = AB + BC	2
3. AC = AB + AB	<b>3.</b>

before you move on.

teacher's signature

Stop and get the

**2.** Name the property illustrated by the statement. If  $\angle H \cong \angle T$  and  $\angle T \cong \angle B$ , then  $\angle H \cong \angle B$ .

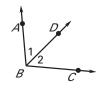
Before writing a proof, organize your reasoning by copying or drawing a diagram for the situation described. Then identify the **GIVEN and PROVE** statements.

**Example 3** Use properties of equality

If you know that BD bisects  $\angle ABC$ , prove that  $m \angle ABC$  is two times  $m \angle 1$ .

Given  $\overrightarrow{BD}$  bisects  $\angle ABC$ .

Prove  $m \angle ABC = 2 \cdot m \angle 1$ 



### **Statements**

- **1.** BD bisects  $\angle ABC$ .

- **4.**  $m \angle 1 + m \angle 2 = m \angle ABC$
- **5.**  $m\angle 1 + m\angle = m\angle ABC$

#### Reasons

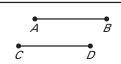
- 2. Definition of angle bisector
- 3. Definition of congruent angles
- 5. Substitution Property of Equality
- 6. Distributive Property

## CONCEPT SUMMARY: WRITING A TWO-COLUMN **PROOF**

**Proof of the Symmetric Property** of Segment Congruence

Given  $\overline{AB} \cong \overline{CD}$ 

Prove  $CD \cong \overline{AB}$ 



Copy or draw diagrams and label information to help develop proofs.

Statements based on facts that you know or conclusions from deductive reasoning

#### **Statements** Reasons

- **1.**  $\overline{AB} \cong \overline{CD}$

- 4.  $\overline{CD} \cong \overline{AB}$

The number of statements will vary.

- 1.
- 2. Definition of congruent segments
- 3. Symmetric Property of Equality
- 4. Definition of congruent segments

Remember to give a reason for the last statement.

**Definitions**, postulates, or proven theorems that allow you to state the corresponding statement.

## **Example 4** Solve a multi-step problem

**Interstate** There are two exits between rest areas on a stretch of interstate. The Rice exit is halfway between rest area A and the Mason exit. The distance between rest area B and the Mason exit is the same as the distance between rest area A and the Rice exit. Prove that the Mason exit is halfway between the Rice exit and rest area B.

#### Solution

Step 1 Draw a diagram.

**Step 2 Draw** diagrams showing relationships.

Step 3 Write a proof.

Given R is the midpoint of  $\overline{AM}$ ,  $\overline{MB} = AR$ .

**Prove** M is the midpoint of  $\overline{RB}$ .

# **Statements** Reasons **1.** R is the midpoint of AM, MB = AR. 2. \_\_\_\_\_ 2. Definition of midpoint 3. Definition of congruent segments 4. MB = RM**5.** Definition of congruent segments **6.** *M* is the midpoint of *RB*.

Stop and get the teacher's signature before you move on.

#### **Homework**

# **Checkpoint** Complete the following exercise.

3. In Example 4, there are rumble strips halfway between the Rice and Mason exits. What other two places are the same distance from the rumble strips?

# **27** Prove Angle Pair Relationships

**Goal** • Use properties of special pairs of angles.

#### **Your Notes**

The given information in Example 1 is about perpendicular lines. You must then use deductive reasoning to show that the angles are right angles.

### THEOREM 2.3 RIGHT ANGLES CONGRUENCE **THEOREM**

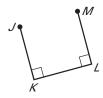
All right angles are .

**Example 1** Use right angle congruence

Write a proof.

Given  $\overline{JK} \perp \overline{KL}$ ,  $\overline{ML} \perp \overline{KL}$ 

Prove  $\angle K \cong \angle L$ 



**Statements** 

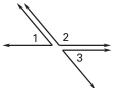
- **1.**  $\overline{JK} \perp \overline{KL}$ ,  $\overline{ML} \perp \overline{KL}$
- **3.** ∠K ≅ ∠L

Reasons

- 2. Definition of perpendicular lines

### THEOREM 2.4 CONGRUENT SUPPLEMENTS THEOREM

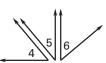
If two angles are supplementary to the same angle (or to congruent angles), then they are .



If  $\angle 1$  and  $\angle 2$  are supplementary and  $\angle 3$  and  $\angle 2$ are supplementary, then .

## THEOREM 2.5 CONGRUENT COMPLEMENTS **THEOREM**

If two angles are complementary to the same angle (or to congruent angles), then they are \_\_\_\_\_.



If  $\angle 4$  and  $\angle 5$  are complementary and  $\angle 6$  and  $\angle 5$  are complementary, then . . .

Write a proof.

Example 2

Given  $\angle 1$  and  $\angle 2$  are supplements.

 $\angle$ 1 and  $\angle$ 4 are supplements.

Prove  $m \angle 4 = 45^{\circ}$ 

 $m\angle 2 = 45^{\circ}$ 

Statements	Reasons		
<ol> <li>∠1 and ∠2 are supplements. ∠1 and ∠4 are supplements.</li> </ol>	1		
2	2. Congruent Supplements Theorem		
3. $m\angle 2 = m\angle 4$	3		
4. $m\angle 2 = 45^{\circ}$	4		
<b>5.</b>	5. Substitution Property of Equality		

Stop and get the teacher's signature before you move on.

- **Checkpoint** Complete the following exercises.
- **1.** In Example 1, suppose you are given that  $\angle K \cong \angle L$ . Can you use the Right Angles Congruence Theorem to prove that  $\angle K$  and  $\angle L$  are right angles? *Explain*.

**2.** Suppose  $\angle A$  and  $\angle B$  are complements, and  $\angle A$ and  $\angle C$  are complements. Can  $\angle B$  and  $\angle C$  be supplements? Explain.

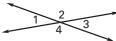
POSTULATE 12 LINEAR PAIR POSTULATE

If two angles form a linear pair, then they are .

 $\angle 1$  and  $\angle 2$  form a linear pair, so  $\angle 1$  and  $\angle 2$  are supplementary and  $m \angle 1 + m \angle 2 =$ .

THEOREM 2.6 VERTICAL ANGLES CONGRUENCE **THEOREM** 

Vertical angles are \_\_\_\_\_.



**Example 3** Use the Vertical Angles Congruence Theorem

You can use information labeled in a diagram in your proof.

Write a proof.

Given  $\angle 4$  is a right angle.

**Prove**  $\angle 2$  and  $\angle 4$  are supplementary.

**Statements** 

**1.**  $\angle$ 4 is a right angle.

**3.** ∠**2** ≅ ∠**4** 

5.  $m\angle 2 = 90^{\circ}$ 

Reasons

**1**.

2. Definition of a right angle

4. Definition of congruent angles

6.  $m \angle 2 + m \angle 4 = 180^{\circ}$ 

Stop and get the teacher's signature before you move on. Checkpoint In Exercises 3 and 4, use the diagram.

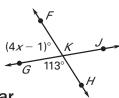
3. If  $m \angle 4 = 63^\circ$ , find  $m \angle 1$  and  $m \angle 2$ .

**4.** If  $m \angle 3 = 121^\circ$ , find  $m \angle 1$ ,  $m \angle 2$ , and  $m \angle 4$ .



**Example 4** Find angle measures

Write and solve an equation to find x. Use x to find  $m \angle FKG$ .



#### **Solution**

Because  $m \angle FKG$  and  $m \angle GKH$  form a linear pair, the sum of their measures is . .

$$(4x - 1)^{\circ} + 113^{\circ} =$$
 Write equation.  
 $4x +$  Simplify.

$$4x =$$
 Subtract \_\_\_\_ from each side.

$$x =$$
 \_\_\_\_ Divide each side by 4.

Use 
$$x =$$
 to find  $m \angle FKG$ .

$$m \angle FKG = (4x - 1)^{\circ}$$
 Write equation.  
 $= [4(\underline{\hspace{1cm}}) - 1]^{\circ}$  Substitute \_\_\_\_ for x.  
 $= [\underline{\hspace{1cm}} - 1]^{\circ}$  Multiply.  
 $= \underline{\hspace{1cm}}$  Simplify.

The measure of  $\angle FKG$  is .

**Checkpoint** Complete the following exercise.

Stop and get the teacher's signature before you move on.

**Homework** 

