

1.1 Identify Points, Lines, and **Planes**

Goal • Name and sketch geometric figures.

Your Notes

Rewrite the goal as an I can statement:

VOCABULARY	
Undefined term	
Point	
Line	
Plane	
Collinear points	
Coplanar points	
Defined Terms	
Line segment, endpoints	
Ray	
Opposite rays	
Intersection	

UNDEFINED TERMS Point A **point** has dimension. It is represented by a point A **Line** A **line** has dimension. It is represented by a with two arrowheads, but it extends without end. line ℓ , line $AB(\hat{AB})$, Through any points, there is or line $BA(\overrightarrow{BA})$ exactly line. You can use any points on a line to name it. Plane A plane has dimensions. It is represented by a shape that looks like a floor or wall, but it plane Mor plane ABC extends without end. Through any _____ points not on the same line, there is exactly plane. You can use points that are not all on the same line to name a plane.

There is a line through points *L* and *Q* that is not shown in the diagram. Try to imagine what plane *LMQ* would look like if it were shown.

Example 1 Name points, lines, and planes

- **a.** Give two other names for \overrightarrow{LN} . Give another name for plane Z.
- Z N b
- **b.** Name three points that are collinear. Name four points that are coplanar.
- **a.** Other names for \overrightarrow{LN} are ____ and ____. Other names for plane Z are plane and .
- b. Points _____ lie on the same line, so they are collinear. Points _____ lie on the same plane, so they are coplanar.

Stop and get the teacher's signature after each checkpoint, before you move on.

Checkpoint Use the diagram in Example 1.

1. Give two other names for \overrightarrow{MQ} . Name a point that is not coplanar with points L, N, and P.

DEFINED TERMS: SEGMENTS AND RAYS line Line AB (written as) and points A and B are used here to define the terms below. Segment The line segment AB, or segment AB, (written as) consists of the segment endpoints A and B and all points on endpoint endpoint AB that are A and B. Note that \overline{AB} can also be named Ray The ray AB (written as) endpoint consists of the endpoint A and all points on \overrightarrow{AB} that lie on the same endpoint side of as . Note that AB and BA are ravs.

In Example 2, WY and WX have a common

but are not

So they are not opposite rays.

Example 2 Name segments, rays, and opposite rays

- a. Give another name for VX.
- **b.** Name all rays with endpoints *W*. Which of these rays are opposite rays?
- **a.** Another name for *VX* is .
- **b.** The rays with endpoint W are The opposite rays with endpoint W are and .

Stop and get the teacher's signature after each checkpoint, before you move on.

Checkpoint	Use	the	diagram	in	Example	2

- 2. Give another name for YW.
- **3.** Are \overrightarrow{VX} and \overrightarrow{XV} the same ray? Are \overrightarrow{VW} and \overrightarrow{VX} the same ray? Explain.

Example 3

Sketch intersections of lines and planes

- a. Sketch a plane and a line that intersects the plane at more than one point.
- **b.** Sketch a plane and a line that is in the plane. Sketch another line that intersects the line and plane at a point.
- a.

b.

Example 4

Sketch intersections of planes

Sketch two planes that intersect in a line.

- Step 1 Draw one plane as if you are facing it.
- Step 2 Draw a second plane that is Use dashed lines to show where one plane is hidden.
- Step 3 Draw the line of .

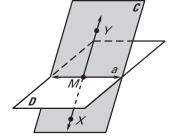
Stop and get the teacher's signature after each

checkpoint, before you

move on.

Checkpoint Complete the following exercises.

- 4. Sketch two different lines that intersect a plane at different points.
- **5.** Name the intersection of \overrightarrow{MX} and line a.
- 6. Name the intersection of plane C and plane D.



Homework



1.2 Use Segments and Congruence

Rewrite the Goal as an "I can" statement.

е

Goal • Use segment postulates to identify congruent segments.

Your Notes

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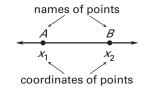
VOCABULARY	
Postulate, axiom	
Theorem	
 Coordinate	
 Distance	
Between	
Congruent segments	

POSTULATE 1 RULER POSTULATE

The points on a line can be matched one to one with real numbers. The real number that corresponds to a point is the

of the point.

The between points A and B, written as AB, is the absolute value of the difference of the coordinates of A and B.



$$A \qquad B$$

$$x_1 \qquad x_2$$

$$AB = |x_2 - x_1|$$

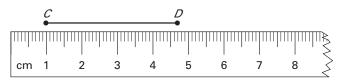
Example 1 Apply the Ruler Postulate

Measure the length of \overline{CD} to the nearest tenth of a centimeter.



Solution

Align one mark of a metric ruler with C. Then estimate the coordinate of D. For example, if you align C with 1, D appears to align with

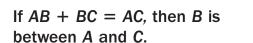


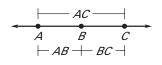
$$CD = | ___ - __| = ___$$
 Ruler postulate

The length of $\overline{\textit{CD}}$ is about centimeters.

POSTULATE 2 SEGMENT ADDITION POSTULATE

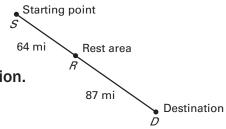
If B is between A and C, then AB + BC = AC.





Example 2 Apply the Segment Addition Postulate

Road Trip The locations
shown lie in a straight line.
Find the distance from the starting point to the destination.



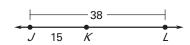
Solution

The rest area lies between the starting point and the destination, so you can apply the Segment Addition Postulate.

The distance from the starting point to the destination is ____ miles.

Example 3 Find a length

Use the diagram to find KL.



Use the Segment Addition Postulate to write an equation. Then solve the equation to find *KL*.

___ = ____ + KL Segment Addition Postulate

$$=$$
 + KL

____ = ____ + *KL* Substitute for ____ and ____.

$$= KL$$

= KL Subtract from each side.

Example 4 Compare segments for congruence

Plot F(4, 5), G(-1, 5), H(3, 3), and J(3, -2) in a coordinate plane. Then determine whether FG and HJ are congruent.

Horizontal segment: Subtract the

_____ of the endpoints.

Vertical segment: Subtract the _____ of the endpoints.

HJ = |____| = ___

 \overline{FG} and \overline{HJ} have the _____ length. So \overline{FG} ____ \overline{HJ} .

Checkpoint Complete the following exercises.

Stop and get the teacher's signature after each checkpoint, before you move on.

Homework

1. Find the length of \overline{AB} to the nearest $\frac{1}{8}$ inch.

2. Find QS and PQ.

3. Consider the points A(-2, -1), B(4, -1), C(3, 0), and D(3, 5). Are \overline{AB} and \overline{CD} congruent?

Use Midpoint and Distance **Formulas**

Rewrite the Goal as an

"I can" statement.

Your Notes

Goal • Find lengths of segments in the coordinate plane.

VO	CA	BU	LA	RY
\mathbf{v}	VA			

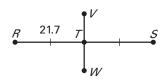
Midpoint

Segment bisector

Example 1

Find segments lengths

Find RS.



Solution

Point is the midpoint of \overline{RS} . So, RT = 21.7.

RS = ____ + ____ **Segment Addition Postulate**

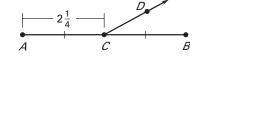
Substitute.

Add.

The length of \overline{RS} is _____.

Checkpoint Complete the following exercise.

1. Find *AB*.

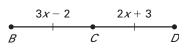


Stop and get the teacher's signature after each checkpoint, before you move on.

Example 2

Use algebra with segment lengths

Point C is the midpoint of \overline{BD} . Find the length of \overline{BC} .



Solution

Step 1 Write and solve an equation.

$$BC = CD$$

Write equation.

Substitute.

Subtract ____ from

$$x =$$

Add to each side.

Step 2 Evaluate the expression for *BC* when x =___.

So, the length of \overline{BC} is _____.

Stop and get the teacher's signature after each checkpoint, before you

move on.

- **Checkpoint** Complete the following exercise.
 - **2.** Point K is the midpoint of \overline{JL} . Find the length of \overline{KL} .

THE MIDPOINT FORMULA

The coordinates of the midpoint of a segment are the averages of the x-coordinates and of the y-coordinates of the endpoints.

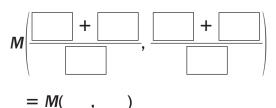
If $A(x_1, y_1)$ and $B(x_2, y_2)$ are points in a coordinate plane, then the midpoint M of \overline{AB} has coordinates

$$\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$$

- a. Find Midpoint The endpoints of PR are P(-2, 5) and R(4, 3). Find the coordinates of the midpoint M.
- **b. Find Endpoint** The midpoint of AC is M(3, 4). One endpoint is A(1, 6). Find the coordinates of endpoint C.

Solution

a. Use the Midpoint Formula.



M(?,?)R(4, 3)

 $y \mid A(1, 6)$

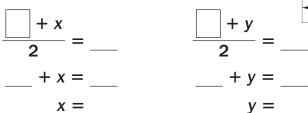
M(3, 4)

C(x, y)

The coordinates of the midpoint of \overline{PR} are

- Multiply each side of the equation by the denominator to clear the fraction.
- **b.** Let (x, y) be the coordinates of endpoint C. Use the Midpoint Formula to find *x* and *y*.

Step 1 Find x. Step 2 Find y.



The coordinates of endpoint C are .

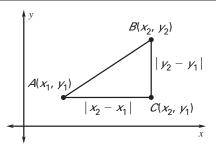
Stop and get the teacher's signature after each checkpoint, before you move on.

Checkpoint Complete the following exercises.

- **3.** The endpoints of CD are C(-8, -1) and D(2, 4). Find the coordinates of the midpoint M.
- **4.** The midpoint of \overline{XZ} is M(5, -6). One endpoint is X(-3, 7). Find the coordinates of endpoint Z.

THE DISTANCE FORMULA

If $A(x_1, y_1)$ and $B(x_2, y_2)$ are points in a coordinate plane, then the distance between A and B is

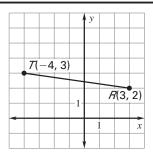


$$AB = \sqrt{()^2 + ()^2}$$

Example 4

Use the Distance Formula

What is the approximate length of \overline{RT} , with endpoints R(3, 2) and T(-4, 3)?



Solution

Use the Distance Formula.

Distance Formula

Substitute.

Subtract.

Evaluate powers.

Add.

Use a calculator.

The symbol \approx means "is approximately equal

The length of \overline{RT} is about .

Checkpoint Complete the following exercise.

Stop and get the teacher's Homework signature after each checkpoint, before you move on.

5. What is the approximate length of \overline{GH} , with endpoints G(5, -1) and H(-3, 6)?



Measure and Classify Angles

Rewrite the Goal as an

"I can" statement.

Your Notes

е

Goal • Name, measure, and classify angles.

Angle	
Sides of an angle	
Vertex of an angle	
Measure of an angle	
Acute angle	
Right angle	
Obtuse angle	
Straight angle	
Congruent angles	
Angle bisector	

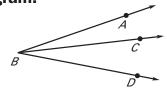
You should not name any of these angles B because all three angles have B as their

Example 1 Name angles

Name the three angles in the diagram.

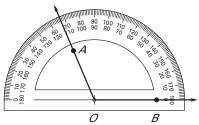
, or , or

__, or ____



POSTULATE 3: PROTRACTOR POSTULATE

Consider OB and point A on one side of OB. The rays of the form OA can be matched one to one with the real numbers from 0 to



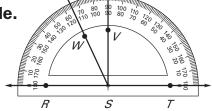
The measure of is equal to between the real numbers

for OA and OB.

Measure and classify angles Example 2

Use the diagram to find the measure of the indicated angle. Then classify the angle.

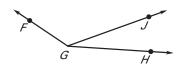
- a. / WSR
- b. / TSW
- c. $\angle RST$ d. $\angle VST$



- a. \overrightarrow{SR} is lined up with the 0° on the _____ scale of the protractor. SW passes through ____ on the ____ scale. So, $m \angle WSR =$ ____. It is _____ angle.
- **b.** \overrightarrow{ST} is lined up with the 0° on the _____ scale of the protractor. \overrightarrow{SW} passes through _____ on the ____ scale. So, $m \angle TSW =$ ____. It is _____
- **c.** $m \angle RST =$ ____. It is _____ angle.
- **d.** $m \angle VST =$ ____. It is _____ angle.

Checkpoint Complete the following exercises.

1. Name all the angles in the diagram at the right.



2. What type of angles do the *x*-axis and *y*-axis form in a coordinate plane?

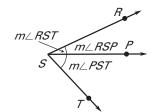
Stop and get the teacher's signature after each checkpoint, before you move on.

A point is in the interior of an angle if it is between points that lie on each side of the angle.



POSTULATE 4: ANGLE ADDITION POSTULATE

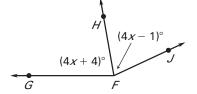
Words If P is in the interior of $\angle RST$, then the measure of $\angle RST$ is equal to the sum of the measures of \angle and \angle .



Symbols If P is in the interior of $\angle RST$, then $m\angle RST = m\angle ___ + m\angle ___$.

Example 3 Find angle measures

Given that $m \angle GFJ = 155^{\circ}$, find $m \angle GFH$ and $m \angle HFJ$.



Solution

Step 1 Write and solve an equation to find the value of x.

 $m \angle GFJ = m \angle \underline{\hspace{1cm}} + m \angle \underline{\hspace{1cm}}$

Angle Addition Postulate

_____ = (_____)° + (_____)°

Substitute.

____=__

Combine like terms.

____ = ___

Subtract ____ from each side.

____ = x

Divide each side by .

Step 2 Evaluate the given expressions when $x = ____.$

$$m \angle GFH = (_____)^{\circ} = (____)^{\circ} = ___.$$

 $m \angle HFJ = (___)^{\circ} = (___)^{\circ} = __.$

So, $m \angle GFH = \underline{\hspace{1cm}}$ and $m \angle HFJ = \underline{\hspace{1cm}}$.

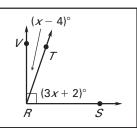
Checkpoint Complete the following exercise.

teacher's signature after each checkpoint, before you

Stop and get the

move on.

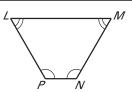
3. Given that $\angle VRS$ is a right angle, find $m\angle VRT$ and $\angle TRS$.



Example 4

Identify congruent angles

Identify all pairs of congruent angles in the diagram. If $m\angle P = 120^{\circ}$, what is $m \angle N$?



Solution

There are two pairs of congruent angles:

$$\angle P \cong ___$$
 and $\angle L \cong ___$

Because
$$\angle P \cong \underline{\hspace{1cm}}, m \angle P = \underline{\hspace{1cm}}.$$

So,
$$m \angle N =$$
 .

Example 5

Double an angle measure

In the diagram at the right, WY bisects $\angle XWZ$, and $m\angle XWY = 29^{\circ}$. Find $m \angle XWZ$.



Solution

By the Angle Addition Postulate,

$$m \angle XWZ = +$$
 .

Because \overrightarrow{WY} bisects $\angle XWZ$, you know

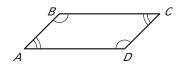
So, ____ = ____, and you can write
$$m\angle XWZ$$
 = ____ + ____
= __ + ___ = __.

Stop and get the teacher's signature after each checkpoint, before you move on.

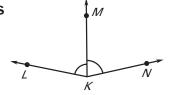


Checkpoint Complete the following exercises.

4. Identify all pairs of congruent angles in the diagram. If $m \angle B = 135^{\circ}$, what is $m \angle D$?



5. In the diagram below, *KM* bisects $\angle LKN$ and $m\angle LKM = 78^{\circ}$. Find $m \angle LKN$.





1.5 Describe Angle Pair **Relationships**

Rewrite the Goal as an "I can" statement.

Goal • Use special angle relationships to find angle measures.

Your Notes

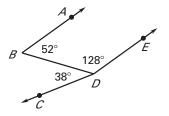
Complementary angles		
Supplementary angles		
Adjacent angles		
Linear pair		
Vertical angles		

In Example 1, $\angle BDE$ and $\angle CDE$ share a common vertex. But they share common

points, so they are not adjacent angles.

Example 1 Identify complements and supplements

In the figure, name a pair of complementary angles, a pair of supplementary angles, and a pair of adjacent angles.



Solution

Because ____ + ___ = 90°, ___ and ____ are _____ angles.

Because ____ + ___ = 180°, ____ and ____ are _____ angles.

Because _____ and ____ share a common vertex and side, they are angles.

Angles are sometimes named with numbers. An angle measure in a diagram has a degree symbol. An angle name does not.

Example 2 Find measures of complements and supplements

- a. Given that $\angle 1$ is a complement of $\angle 2$ and $m \angle 2 = 57^{\circ}$, find $m \angle 1$.
- **b.** Given that $\angle 3$ is a supplement of $\angle 4$ and $m \angle 4 = 41^{\circ}$, find $m \angle 3$.

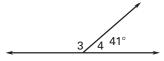
Solution

a. You can draw a diagram with complementary adjacent angles to illustrate the relationship.



$$m \angle 1 = - = - =$$

b. You can draw a diagram with supplementary adjacent angles to illustrate the relationship.



Stop and get the teacher's signature after each checkpoint, before you move on.

Checkpoint Complete the following exercises.

1. In the figure, name a pair of complementary angles, a pair of supplementary angles, and a pair of adjacent angles.





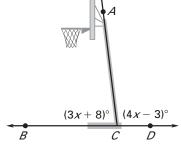
- **2.** Given that $\angle 1$ is a complement of $\angle 2$ and $m \angle 1 = 73^{\circ}$, find $m \angle 2$.
- **3.** Given that $\angle 3$ is a supplement of $\angle 4$ and $m \angle 4 = 37^{\circ}$, find $m \angle 3$.

In a diagram, you can assume that a line that looks straight is straight. In Example 3, B, C, and D lie on \overrightarrow{BD} . So, $\angle BCD$ is a

angle.

Example 3 Find angle measures

Basketball The basketball pole forms a pair of supplementary angles with the ground. Find $m \angle BCA$ and $m \angle DCA$.



Solution

Step 1 Use the fact that is the sum of the measures of supplementary angles.

$$m \angle BCA + m \angle DCA =$$
 Write equation.

(______) $^{\circ}$ + (______) $^{\circ}$ = _____ Substitute.

= ____ Combine like terms.

= ____ Subtract.

= ____ Divide.

Step 2 Evaluate the original expressions when x =.

$$m \angle BCA = (_____)^{\circ} = (____)^{\circ} = ___.$$

 $m \angle DCA = (___)^{\circ} = (___)^{\circ} = ___.$

The angle measures are ____ and ____.

Stop and get the teacher's signature after each checkpoint, before you move on.

Checkpoint Complete the following exercise.

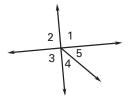
4. In Example 3, suppose the angle measures are $(5x + 1)^{\circ}$ and $(6x + 3)^{\circ}$. Find $m \angle BCA$ and $m \angle DCA$.

In the diagram, one side of $\angle 1$ and one side of $\angle 4$ are opposite rays. But the angles are not a linear pair because they are not

Example 4

Identify angle pairs

Identify all of the linear pairs and all of the vertical angles in the figure at the right.



Solution

To find vertical angles, look for angles formed

and are vertical angles.

To find linear pairs, look for adjacent angles whose noncommon sides are ______.

and ____ are a linear pair. and are a

linear pair.



5. Identify all of the linear pairs and all of the vertical angles in the figure.



You may find it

Example 5 Find angle measures in a linear pair

Two angles form a linear pair. The measure of one angle is 4 times the measure of the other. Find the measure of each angle.

useful to draw a diagram to represent a word problem like the

one in Example 5.

6	A	ution
J	v	utivii

Let x° be the measure of one angle. The measure of the other angle is . Then use the fact that the angles of a linear pair are to write an equation.

+ ____ = ____ Write an equation.

 $\underline{}$ = $\underline{}$ Combine like terms.

___ = ___ Divide each side by ___.

The measures of the angles are and

___ = ____,

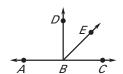
Stop and get the teacher's signature after each checkpoint, before you move on.

Checkpoint Complete the following exercise.

6. Two angles form a linear pair. The measure of one angle is 3 times the measure of the other. Find the meaure of each angle.

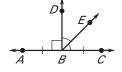
CONCEPT SUMMARY: INTERPRETING A DIAGRAM

There are some things you can conclude from a diagram, and some you cannot. For example, here are some things that you can conclude from the diagram at the right.



- All points shown are ______.
- Points A, B, and C are , and B is between A and C.
- \overrightarrow{AC} , \overrightarrow{BD} , and \overrightarrow{BE} _____ at point B.
- \angle DBE and \angle EBC are angles, and \angle ABC is a .
- Point E lies in the ____ of $\angle DBC$.

In the diagram above, you cannot conclude that $AB \cong BC$, that $\angle DBE \cong \angle EBC$, or that $\angle ABD$ is a right angle. This information must be indicated, as shown at the right.



Homework

1.7

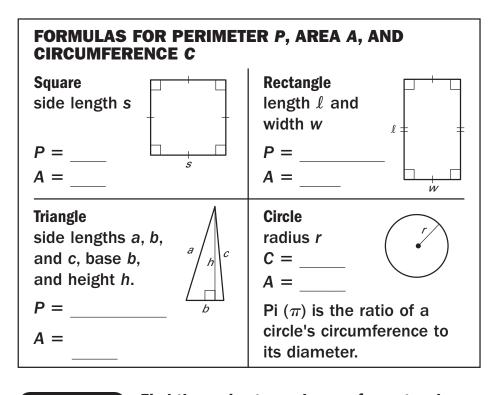
Find Perimeter, Circumference, and Area

Rewrite the Goal as an "I can" statement.

Your Notes

е

Goal • Find dimensions of polygons.



Tennis The in-bounds portion of a singles tennis court is shown. Find its perimeter and area. Perimeter Area $P = 2\ell + 2w$ $= 2(\underline{}) + 2(\underline{})$ $= \underline{}$ The perimeter is $\underline{}$ ft and the area is $\underline{}$ ft².

Stop and get the teacher's signature after each checkpoint, before you move on.

- **Checkpoint** Complete the following exercise.
 - 1. In Example 1, the width of the in-bounds rectangle increases to 36 feet for doubles play. Find the perimeter and area of the in-bounds rectangle.

The approximations 3.14 and $\frac{22}{7}$ are commonly used as approximations for the irrational number π . Unless told otherwise, use 3.14 for π .

Example 2

Find the circumference and area of a circle

Archery The smallest circle on an Olympic target is 12 centimeters in diameter. Find the approximate circumference and area of the smallest circle.

Solution

First find the radius. The diameter is 12 centimeters, so the radius is $\frac{1}{2}$ (____) = ___ centimeters.

Then find the circumference and area. Use 3.14 for π .

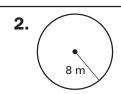
$$P = 2\pi r \approx 2(____)(__) = ____$$

 $A = \pi r^2 \approx ___(__)^2 = _____$

Stop and get the teacher's signature after each checkpoint, before you

move on.

Checkpoint Find the approximate circumference and area of the circle.

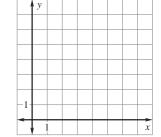


Example 3 Using a coordinate plane

Triangle JKL has vertices J(1, 6), K(6, 6), and L(3, 2). Find the approximate perimeter of triangle JKL.

Solution

First draw triangle JKL in a coordinate plane. Then find the side lengths. Because JK is horizontal, use the



_____to find JK. Use the to find

JL and LK.

$$JK = |\underline{} - \underline{}| = \underline{}$$
 units

$$JK = | __ - __ | = __ units$$
 $JL = \sqrt{(_ - 1)^2 + (2 - __)^2} = \sqrt{_ } \approx __ units$
 $LK = \sqrt{(_ - 3)^2 + (_ - 2)^2} = \sqrt{_ } = __ units$

$$LK = \sqrt{(_{-} - 3)^2 + (_{-} - 2)^2} = \sqrt{_{-}} = _{-}$$
 units

Then find the perimeter.
$$P = JK + JL + LK \approx __ + __ + __ = __ units.$$

Write down your calculations to make sure you do not make a mistake substituting values in the Distance Formula.

Lawn care You are using a roller to smooth a lawn. You can roll about 125 square yards in one minute. About how many minutes does it take to roll a lawn that is 120 feet long and 75 feet wide?

Solution

You can roll the lawn at a rate of 125 square yards per minute. So, the amount of time it takes you to roll the lawn depends on its .

Step 1 Find the area of the rectangular lawn.

Area =
$$\ell w$$
 = () = ft^2

The rolling rate is in square yards per minute. Rewrite the area of the lawn in square yards. There are feet in 1 yard, and 2 = square feet in one square yard.

9000
$$\text{ft}^2 \cdot \frac{1 \text{ yd}^2}{\text{ft}^2} = \underline{\qquad} \text{yd}^2$$
 Use unit analysis.

Step 2 Write a verbal model to represent the situation. Then write and solve an equation based on the verbal model.

> Let t represent the total time (in minutes) needed to roll the lawn.

It takes about ___ minutes to roll the lawn.

Stop and get the

signature after each

checkpoint, before you

teacher's

move on.

The base of a triangle is 24 feet. Its area is 216 square feet. Find the height of the triangle.

Solution

$$A = Are$$

Area of a triangle

Substitute.

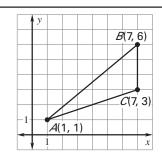
Multiply.

Solve for h.

The height is feet.

Checkpoint Complete the following exercises.

3. Find the perimeter of the triangle shown at the right.



4. Suppose a lawn is half as long and half as wide as the lawn in Example 4. Will it take half the time to roll the lawn? Explain.

Homework

5. The area of a triangle is 96 square inches, and its height is 8 inches. Find the length of its base.