9.1 Translate Figures and Use Vectors

Before

Now

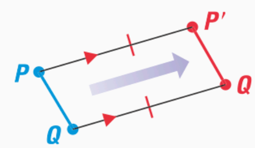
Why? So you can find a distance covered on snowshoes, as in Exs. 35–37.

Key Vocabulary
- **image**
- **preimage**
- **isometry**
- **vector**
  initial point, terminal point, horizontal component, vertical component
- **component form**
- **translation,** *p. 272*

In Lesson 4.8, you learned that a *transformation* moves or changes a figure in some way to produce a new figure called an **image**. Another name for the original figure is the **preimage**.

Recall that a *translation* moves every point of a figure the same distance in the same direction. More specifically, a translation maps, or moves, the points P and Q of a plane figure to the points P' (read "*P prime*") and Q', so that one of the following statements is true:

- $PP' = QQ'$ and $\overline{PP'} \parallel \overline{QQ'}$, or
- $PP' = QQ'$ and $\overline{PP'}$ and $\overline{QQ'}$ are collinear.



EXAMPLE 1 Translate a figure in the coordinate plane

Graph quadrilateral $ABCD$ with vertices $A(-1, 2)$, $B(-1, 5)$, $C(4, 6)$, and $D(4, 2)$. Find the image of each vertex after the translation $(x, y) \to (x + 3, y - 1)$. Then graph the image using prime notation.

Solution

USE NOTATION
You can use *prime notation* to name an image. For example, if the preimage is $\triangle ABC$, then its image is $\triangle A'B'C'$, read as *"triangle A prime, B prime, C prime."*

First, draw $ABCD$. Find the translation of each vertex by adding 3 to its x-coordinate and subtracting 1 from its y-coordinate. Then graph the image.

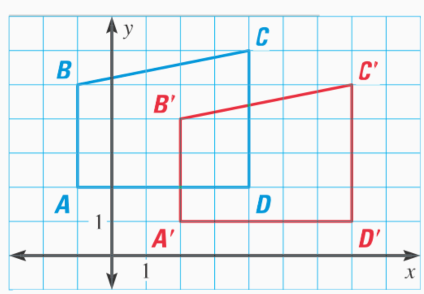
$(x, y) \to (x + 3, y - 1)$

$A(-1, 2) \to A'(2, 1)$

$B(-1, 5) \to B'(2, 4)$

$C(4, 6) \to C'(7, 5)$

$D(4, 2) \to D'(7, 1)$



GUIDED PRACTICE for Example 1

1. Draw $\triangle RST$ with vertices $R(2, 2)$, $S(5, 2)$, and $T(3, 5)$. Find the image of each vertex after the translation $(x, y) \to (x + 1, y + 2)$. Graph the image using prime notation. **See margin for art; $R'(3, 4)$, $S'(6, 4)$, $T'(4, 7)$.**

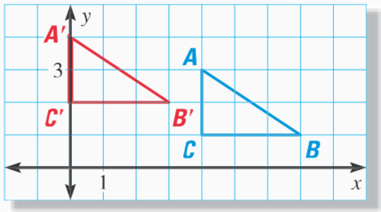
2. The image of $(x, y) \to (x + 4, y - 7)$ is $\overline{P'Q'}$ with endpoints $P'(-3, 4)$ and $Q'(2, 1)$. Find the coordinates of the endpoints of the preimage. **$P(-7, 11)$, $Q(-2, 8)$**

ISOMETRY An **isometry** is a transformation that preserves length and angle measure. Isometry is another word for congruence transformation (page 272).

EXAMPLE 2 Write a translation rule and verify congruence

READ DIAGRAMS
In this book, the preimage is always shown in blue, and the image is always shown in red.

Write a rule for the translation of $\triangle ABC$ to $\triangle A'B'C'$. Then verify that the transformation is an isometry.



Solution

To go from A to A', move 4 units left and 1 unit up. So, a rule for the translation is $(x, y) \rightarrow (x - 4, y + 1)$.

Use the SAS Congruence Postulate. Notice that $CB = C'B' = 3$, and $AC = A'C' = 2$. The slopes of $\overline{CB}$ and $\overline{C'B'}$ are 0, and the slopes of $\overline{CA}$ and $\overline{C'A'}$ are undefined, so the sides are perpendicular. Therefore, $\angle C$ and $\angle C'$ are congruent right angles. So, $\triangle ABC \cong \triangle A'B'C'$. The translation is an isometry.

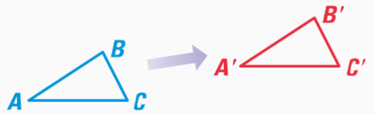
✓ GUIDED PRACTICE for Example 2

3. In Example 2, write a rule to translate $\triangle A'B'C'$ back to $\triangle ABC$.
$(x, y) \rightarrow (x + 4, y - 1)$

THEOREM *For Your Notebook*

THEOREM 9.1 Translation Theorem

A translation is an isometry.



$\triangle ABC \cong \triangle A'B'C'$

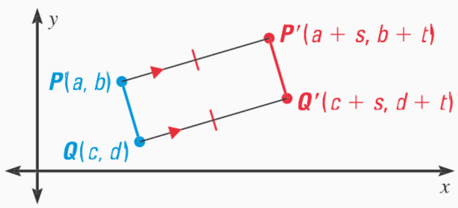
Proof: below; Ex. 46, p. 579

PROOF Translation Theorem

A translation is an isometry.

GIVEN ▶ $P(a, b)$ and $Q(c, d)$ are two points on a figure translated by $(x, y) \rightarrow (x + s, y + t)$.

PROVE ▶ $PQ = P'Q'$



The translation maps $P(a, b)$ to $P'(a + s, b + t)$ and $Q(c, d)$ to $Q'(c + s, d + t)$.

Use the Distance Formula to find PQ and $P'Q'$. $PQ = \sqrt{(c - a)^2 + (d - b)^2}$.

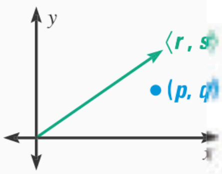
$$P'Q' = \sqrt{[(c + s) - (a + s)]^2 + [(d + t) - (b + t)]^2}$$
$$= \sqrt{(c + s - a - s)^2 + (d + t - b - t)^2}$$
$$= \sqrt{(c - a)^2 + (d - b)^2}$$

Therefore, $PQ = P'Q'$ by the Transitive Property of Equality.

VECTORS Another way to describe a translation is by using a vector. A **vector** is a quantity that has both direction and *magnitude,* or size. A vector is represented in the coordinate plane by an arrow drawn from one point to another.

USE NOTATION

Use brackets to write the component form of the vector $\langle r, s\rangle$. Use parentheses to write the coordinates of the point (p, q).

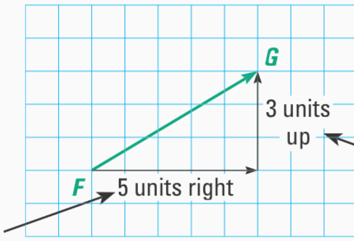


KEY CONCEPT *For Your Notebook*

Vectors

The diagram shows a vector named $\overrightarrow{FG}$, read as "vector FG."

The **initial point**, or starting point, of the vector is F.



The **terminal point**, or ending point, of the vector is G.

vertical component

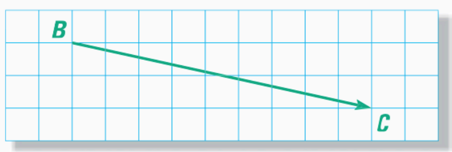
horizontal component

The **component form** of a vector combines the horizontal and vertical components. So, the component form of $\overrightarrow{FG}$ is $\langle 5, 3\rangle$.

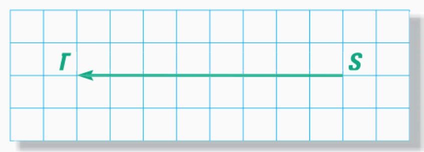
EXAMPLE 3 Identify vector components

Name the vector and write its component form.

a.



b.



Solution

a. The vector is $\overrightarrow{BC}$. From initial point B to terminal point C, you move 9 units right and 2 units down. So, the component form is $\langle 9, -2\rangle$.

b. The vector is $\overrightarrow{ST}$. From initial point S to terminal point T, you move 8 units left and 0 units vertically. The component form is $\langle -8, 0\rangle$.

EXAMPLE 4 Use a vector to translate a figure

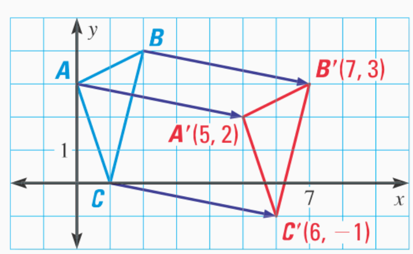
The vertices of $\triangle ABC$ are $A(0, 3)$, $B(2, 4)$, and $C(1, 0)$. Translate $\triangle ABC$ using the vector $\langle 5, -1\rangle$.

USE VECTORS

Notice that the vector can have different initial points. The vector describes only the direction and magnitude of the translation.

Solution

First, graph $\triangle ABC$. Use $\langle 5, -1\rangle$ to move each vertex 5 units to the right and 1 unit down. Label the image vertices. Draw $\triangle A'B'C'$. Notice that the vectors drawn from preimage to image vertices are parallel.



4. $\overrightarrow{RS}$, $\langle 5, 0\rangle$

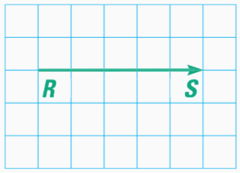
5. $\overrightarrow{TX}$, $\langle 0, 3\rangle$

6. $\overrightarrow{BK}$, $\langle -5, 2\rangle$

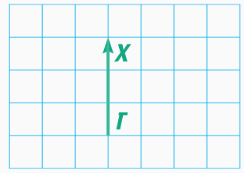
✓ GUIDED PRACTICE for Examples 3 and 4

Name the vector and write its component form.

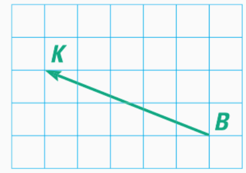
4.



5.



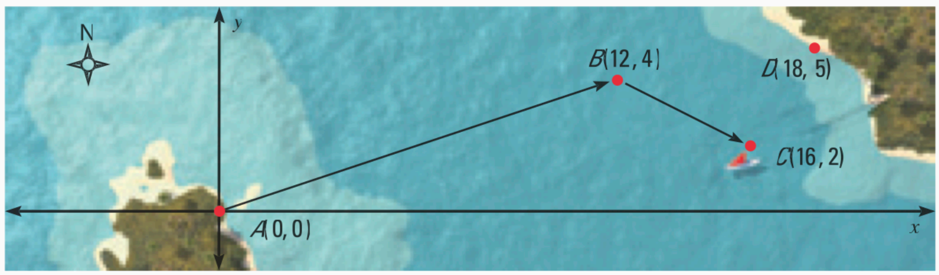
6.



7. The vertices of $\triangle LMN$ are $L(2, 2)$, $M(5, 3)$, and $N(9, 1)$. Translate $\triangle LMN$ using the vector $\langle -2, 6\rangle$. $L'(0, 8)$, $M'(3, 9)$, $N'(7, 7)$

EXAMPLE 5 Solve a multi-step problem

NAVIGATION A boat heads out from point A on one island toward point D on another. The boat encounters a storm at B, 12 miles east and 4 miles north of its starting point. The storm pushes the boat off course to point C, as shown.



a. Write the component form of $\overrightarrow{AB}$.

b. Write the component form of $\overrightarrow{BC}$.

c. Write the component form of the vector that describes the straight line path from the boat's current position C to its intended destination D.

Solution

a. The component form of the vector from $A(0, 0)$ to $B(12, 4)$ is

$$\overrightarrow{AB} = \langle 12 - 0, 4 - 0\rangle = \langle 12, 4\rangle.$$

b. The component form of the vector from $B(12, 4)$ to $C(16, 2)$ is

$$\overrightarrow{BC} = \langle 16 - 12, 2 - 4\rangle = \langle 4, -2\rangle.$$

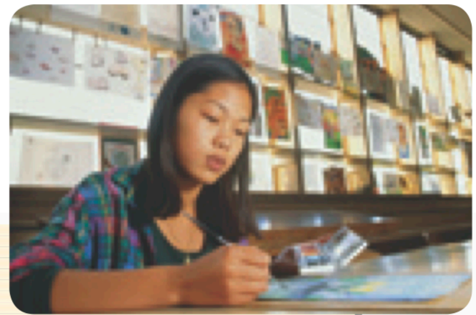
c. The boat is currently at point C and needs to travel to D.
The component form of the vector from $C(16, 2)$ to $D(18, 5)$ is

$$\overrightarrow{CD} = \langle 18 - 16, 5 - 2\rangle = \langle 2, 3\rangle.$$

✓ GUIDED PRACTICE for Example 5

8. WHAT IF? In Example 5, suppose there is no storm. Write the component form of the vector that describes the straight path from the boat's starting point A to its final destination D. $\langle 18, 5\rangle$

9.2 Use Properties of Matrices



Before

Now

Why 36.

Key Vocabulary
- **matrix**
- **element**
- **dimensions**

A **matrix** is a rectangular arrangement of numbers in rows and columns. (The plural of matrix is *matrices*.) Each number in a matrix is called an **element**.

$$\begin{bmatrix} 5 & 4 & 4 & 9 \\ -3 & 5 & 2 & 6 \\ 3 & -7 & 8 & 7 \end{bmatrix}$$

column; row; ← The element in the second row and third column is 2.

READ VOCABULARY
An element of a matrix may also be called an *entry*.

The **dimensions** of a matrix are the numbers of rows and columns. The matrix above has three rows and four columns, so the dimensions of the matrix are 3×4 (read "3 by 4").

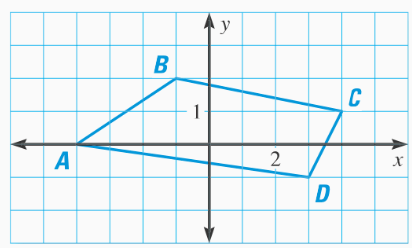
You can represent a figure in the coordinate plane using a matrix with two rows. The first row has the *x*-coordinate(s) of the vertices. The second row has the corresponding *y*-coordinate(s). Each column represents a vertex, so the number of columns depends on the number of vertices of the figure.

EXAMPLE 1 Represent figures using matrices

Write a matrix to represent the point or polygon.

a. Point *A*

b. Quadrilateral *ABCD*



Solution

a. Point matrix for *A*

$$\begin{bmatrix} -4 \\ 0 \end{bmatrix}$$ ← *x*-coordinate; ← *y*-coordinate

b. Polygon matrix for *ABCD*

$$\begin{array}{cccc} A & B & C & D \end{array}$$
$$\begin{bmatrix} -4 & -1 & 4 & 3 \\ 0 & 2 & 1 & -1 \end{bmatrix}$$ ← *x*-coordinates; ← *y*-coordinates

AVOID ERRORS
The columns in a polygon matrix follow the consecutive order of the vertices of the polygon.

✓ GUIDED PRACTICE for Example 1

1. Write a matrix to represent $\triangle ABC$ with vertices $A(3, 5)$, $B(6, 7)$ and $C(7, 3)$.

2. How many rows and columns are in a matrix for a hexagon? **2 rows, 6 columns**

1. $\begin{array}{ccc} A & B & C \end{array}$ $\begin{bmatrix} 3 & 6 & 7 \\ 5 & 7 & 3 \end{bmatrix}$

ADDING AND SUBTRACTING To add or subtract matrices, you add or subtract corresponding elements. The matrices must have the same dimensions.

EXAMPLE 2 Add and subtract matrices

a. $$\begin{bmatrix} 5 & -3 \\ 6 & -6 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ 3 & -4 \end{bmatrix} = \begin{bmatrix} 5+1 & -3+2 \\ 6+3 & -6+(-4) \end{bmatrix} = \begin{bmatrix} 6 & -1 \\ 9 & -10 \end{bmatrix}$$

b. $$\begin{bmatrix} 6 & 8 & 5 \\ 4 & 9 & -1 \end{bmatrix} - \begin{bmatrix} 1 & -7 & 0 \\ 4 & -2 & 3 \end{bmatrix} = \begin{bmatrix} 6-1 & 8-(-7) & 5-0 \\ 4-4 & 9-(-2) & -1-3 \end{bmatrix} = \begin{bmatrix} 5 & 15 & 5 \\ 0 & 11 & -4 \end{bmatrix}$$

TRANSLATIONS You can use matrix addition to represent a translation in the coordinate plane. The image matrix for a translation is the sum of the translation matrix and the matrix that represents the preimage.

EXAMPLE 3 Represent a translation using matrices

The matrix $\begin{bmatrix} 1 & 5 & 3 \\ 1 & 0 & -1 \end{bmatrix}$ represents $\triangle ABC$. Find the image matrix that represents the translation of $\triangle ABC$ 1 unit left and 3 units up. Then graph $\triangle ABC$ and its image.

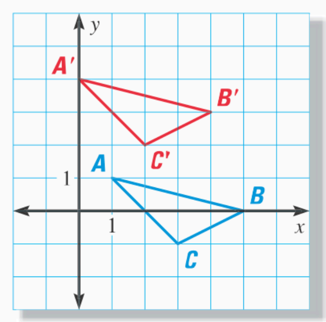
AVOID ERRORS
In order to add two matrices, they must have the same dimensions, so the translation matrix here must have three columns like the polygon matrix.

Solution

The translation matrix is $\begin{bmatrix} -1 & -1 & -1 \\ 3 & 3 & 3 \end{bmatrix}$.

Add this to the polygon matrix for the preimage to find the image matrix.

$$\underset{\text{Translation matrix}}{\begin{bmatrix} -1 & -1 & -1 \\ 3 & 3 & 3 \end{bmatrix}} + \underset{\text{Polygon matrix}}{\overset{A \quad B \quad C}{\begin{bmatrix} 1 & 5 & 3 \\ 1 & 0 & -1 \end{bmatrix}}} = \underset{\text{Image matrix}}{\overset{A' \quad B' \quad C'}{\begin{bmatrix} 0 & 4 & 2 \\ 4 & 3 & 2 \end{bmatrix}}}$$



✓ GUIDED PRACTICE for Examples 2 and 3

In Exercises 3 and 4, add or subtract.

3. $\begin{bmatrix} -3 & 7 \end{bmatrix} + \begin{bmatrix} 2 & -5 \end{bmatrix}$ $\begin{bmatrix} -1 & 2 \end{bmatrix}$

4. $\begin{bmatrix} 1 & -4 \\ 3 & -5 \end{bmatrix} - \begin{bmatrix} 2 & 3 \\ 7 & 8 \end{bmatrix}$ $\begin{bmatrix} -1 & -7 \\ -4 & -13 \end{bmatrix}$

5. The matrix $\begin{bmatrix} 1 & 2 & 6 & 7 \\ 2 & -1 & 1 & 3 \end{bmatrix}$ represents quadrilateral *JKLM*. Write the translation matrix and the image matrix that represents the translation of *JKLM* 4 units right and 2 units down. Then graph *JKLM* and its image. **See margin.**

MULTIPLYING MATRICES The product of two matrices A and B is defined only when the number of columns in A is equal to the number of rows in B. If A is an $m \times n$ matrix and B is an $n \times p$ matrix, then the product AB is an $m \times p$ matrix.

USE NOTATION
Recall that the dimensions of a matrix are always written as rows × columns.

$$A \quad \cdot \quad B \quad = \quad AB$$

$$(m \text{ by } n) \quad \cdot \quad (n \text{ by } p) \quad = \quad (m \text{ by } p)$$

equal (the two n's) — **dimensions of *AB*** (m by p)

You will use matrix multiplication in later lessons to represent transformations.

EXAMPLE 4 Multiply matrices

Multiply $\begin{bmatrix} 1 & 0 \\ 4 & 5 \end{bmatrix}\begin{bmatrix} 2 & -3 \\ -1 & 8 \end{bmatrix}$.

Solution

The matrices are both 2 × 2, so their product is defined. Use the following steps to find the elements of the product matrix.

STEP 1 **Multiply** the numbers in the first row of the first matrix by the numbers in the first column of the second matrix. Put the result in the first row, first column of the product matrix.

$$\begin{bmatrix} 1 & 0 \\ 4 & 5 \end{bmatrix}\begin{bmatrix} 2 & -3 \\ -1 & 8 \end{bmatrix} = \begin{bmatrix} 1(2) + 0(-1) & ? \\ ? & ? \end{bmatrix}$$

STEP 2 **Multiply** the numbers in the first row of the first matrix by the numbers in the second column of the second matrix. Put the result in the first row, second column of the product matrix.

$$\begin{bmatrix} 1 & 0 \\ 4 & 5 \end{bmatrix}\begin{bmatrix} 2 & -3 \\ -1 & 8 \end{bmatrix} = \begin{bmatrix} 1(2) + 0(-1) & 1(-3) + 0(8) \\ ? & ? \end{bmatrix}$$

STEP 3 **Multiply** the numbers in the second row of the first matrix by the numbers in the first column of the second matrix. Put the result in the second row, first column of the product matrix.

$$\begin{bmatrix} 1 & 0 \\ 4 & 5 \end{bmatrix}\begin{bmatrix} 2 & -3 \\ -1 & 8 \end{bmatrix} = \begin{bmatrix} 1(2) + 0(-1) & 1(-3) + 0(8) \\ 4(2) + 5(-1) & ? \end{bmatrix}$$

STEP 4 **Multiply** the numbers in the second row of the first matrix by the numbers in the second column of the second matrix. Put the result in the second row, second column of the product matrix.

$$\begin{bmatrix} 1 & 0 \\ 4 & 5 \end{bmatrix}\begin{bmatrix} 2 & -3 \\ -1 & 8 \end{bmatrix} = \begin{bmatrix} 1(2) + 0(-1) & 1(-3) + 0(8) \\ 4(2) + 5(-1) & 4(-3) + 5(8) \end{bmatrix}$$

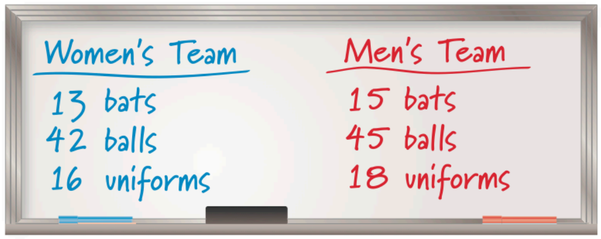
STEP 5 **Simplify** the product matrix.

$$\begin{bmatrix} 1(2) + 0(-1) & 1(-3) + 0(8) \\ 4(2) + 5(-1) & 4(-3) + 5(8) \end{bmatrix} = \begin{bmatrix} 2 & -3 \\ 3 & 28 \end{bmatrix}$$

Animated Geometry at classzone.com

EXAMPLE 5 Solve a real-world problem

SOFTBALL Two softball teams submit equipment lists for the season. A bat costs \$20, a ball costs \$5, and a uniform costs \$40. Use matrix multiplication to find the total cost of equipment for each team.



Solution

First, write the equipment lists and the costs per item in matrix form. You will use matrix multiplication, so you need to set up the matrices so that the number of columns of the equipment matrix matches the number of rows of the cost per item matrix.

EQUIPMENT · COST = TOTAL COST

$$\begin{array}{r}\\ \textbf{Women}\\ \textbf{Men}\end{array}\begin{array}{c}\textbf{Bats}\quad\textbf{Balls}\quad\textbf{Uniforms}\\ \begin{bmatrix}13 & 42 & 16\\ 15 & 45 & 18\end{bmatrix}\end{array} \cdot \begin{array}{r}\\ \textbf{Bats}\\ \textbf{Balls}\\ \textbf{Uniforms}\end{array}\begin{array}{c}\textbf{Dollars}\\ \begin{bmatrix}20\\ 5\\ 40\end{bmatrix}\end{array} = \begin{array}{r}\\ \textbf{Women}\\ \textbf{Men}\end{array}\begin{array}{c}\textbf{Dollars}\\ \begin{bmatrix}?\\ ?\end{bmatrix}\end{array}$$

You can find the total cost of equipment for each team by multiplying the equipment matrix by the cost per item matrix. The equipment matrix is 2×3 and the cost per item matrix is 3×1, so their product is a 2×1 matrix.

$$\begin{bmatrix}13 & 42 & 16\\ 15 & 45 & 18\end{bmatrix}\begin{bmatrix}20\\ 5\\ 40\end{bmatrix} = \begin{bmatrix}13(20) + 42(5) + 16(40)\\ 15(20) + 45(5) + 18(40)\end{bmatrix} = \begin{bmatrix}1110\\ 1245\end{bmatrix}$$

▶ The total cost of equipment for the women's team is \$1110, and the total cost for the men's team is \$1245.

ANOTHER WAY

You could solve this problem arithmetically, multiplying the number of bats by the price of bats, and so on, then adding the costs for each team.

✓ GUIDED PRACTICE for Examples 4 and 5

Use the matrices below. Is the product defined? *Explain.*

$$A = \begin{bmatrix}-3\\ 4\end{bmatrix} \qquad B = \begin{bmatrix}2 & 1\end{bmatrix} \qquad C = \begin{bmatrix}6.7 & 0\\ -9.3 & 5.2\end{bmatrix}$$

6. AB

7. BA Yes; the number of columns in B is equal to the number of rows in A.

8. AC No; the number of columns in A is not equal to the number of rows in C.

Multiply.

9. $\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}\begin{bmatrix}3 & 8\\ -4 & 7\end{bmatrix}$

10. $\begin{bmatrix}5 & 1\end{bmatrix}\begin{bmatrix}-3\\ -2\end{bmatrix}$ $[-17]$

11. $\begin{bmatrix}5 & 1\\ 1 & -1\end{bmatrix}\begin{bmatrix}2 & -4\\ 5 & 1\end{bmatrix}$

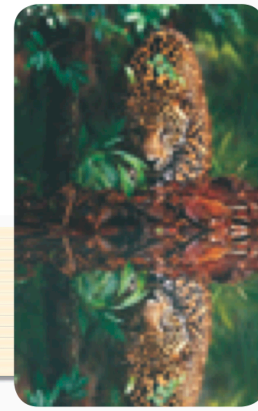
12. **WHAT IF?** In Example 5, find the total cost if a bat costs \$25, a ball costs \$4, and a uniform costs \$35. \$2238

6. Yes; the number of columns in A is equal to the number of rows in B.

9. $\begin{bmatrix}3 & 8\\ 4 & -7\end{bmatrix}$

11. $\begin{bmatrix}15 & -19\\ -3 & -5\end{bmatrix}$

9.3 Perform Reflections



Before x- or y-axis.

Now

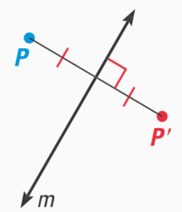
Why?

Key Vocabulary
- **line of reflection**
- **reflection,** *p. 272*

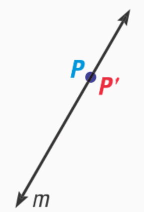
In Lesson 4.8, you learned that a *reflection* is a transformation that uses a line like a mirror to reflect an image. The mirror line is called the **line of reflection**.

A reflection in a line m maps every point P in the plane to a point P', so that for each point one of the following properties is true:

- If P is not on m, then m is the perpendicular bisector of $\overline{PP'}$, or
- If P is on m, then $P = P'$.



Point *P* not on *m*



Point *P* on *m*

EXAMPLE 1 Graph reflections in horizontal and vertical lines

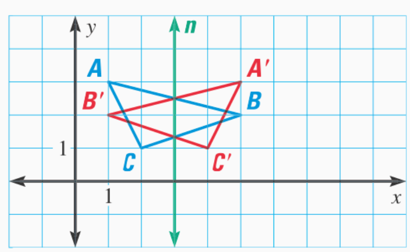
The vertices of $\triangle ABC$ are $A(1, 3)$, $B(5, 2)$, and $C(2, 1)$. Graph the reflection of $\triangle ABC$ described.

a. In the line n: $x = 3$

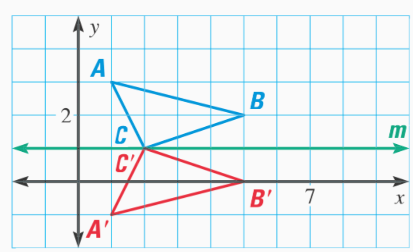
b. In the line m: $y = 1$

Solution

a. Point A is 2 units left of n, so its reflection A' is 2 units right of n at $(5, 3)$. Also, B' is 2 units left of n at $(1, 2)$, and C' is 1 unit right of n at $(4, 1)$.



b. Point A is 2 units above m, so A' is 2 units below m at $(1, -1)$. Also, B' is 1 unit below m at $(5, 0)$. Because point C is on line m, you know that $C = C'$.



✓ GUIDED PRACTICE for Example 1

Graph a reflection of $\triangle ABC$ from Example 1 in the given line. 1–3. See margin.

1. $y = 4$

2. $x = -3$

3. $y = 2$

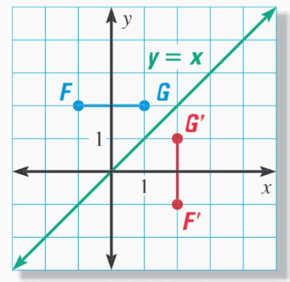
EXAMPLE 2 Graph a reflection in $y = x$

The endpoints of $\overline{FG}$ are $F(-1, 2)$ and $G(1, 2)$. Reflect the segment in the line $y = x$. Graph the segment and its image.

REVIEW SLOPE
The product of the slopes of perpendicular lines is -1.

Solution

The slope of $y = x$ is 1. The segment from F to its image, $\overline{FF'}$, is perpendicular to the line of reflection $y = x$, so the slope of $\overline{FF'}$ will be -1 (because $1(-1) = -1$). From F, move 1.5 units right and 1.5 units down to $y = x$. From that point, move 1.5 units right and 1.5 units down to locate $F'(3, -1)$.



The slope of $\overline{GG'}$ will also be -1. From G, move 0.5 units right and 0.5 units down to $y = x$. Then move 0.5 units right and 0.5 units down to locate $G'(2, 1)$.

COORDINATE RULES You can use coordinate rules to find the images of points reflected in four special lines.

KEY CONCEPT *For Your Notebook*

Coordinate Rules for Reflections

- If (a, b) is reflected in the x-axis, its image is the point $(a, -b)$.
- If (a, b) is reflected in the y-axis, its image is the point $(-a, b)$.
- If (a, b) is reflected in the line $y = x$, its image is the point (b, a).
- If (a, b) is reflected in the line $y = -x$, its image is the point $(-b, -a)$.

EXAMPLE 3 Graph a reflection in $y = -x$

Reflect $\overline{FG}$ from Example 2 in the line $y = -x$. Graph $\overline{FG}$ and its image.

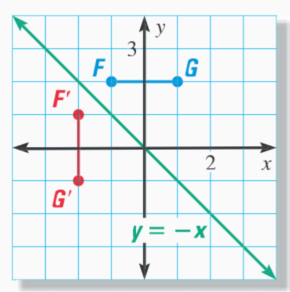
Solution

Use the coordinate rule for reflecting in $y = -x$.

$$(a, b) \to (-b, -a)$$

$F(-1, 2) \to F'(-2, 1)$

$G(1, 2) \to G'(-2, -1)$



Animated Geometry at classzone.com

GUIDED PRACTICE for Examples 2 and 3

4. Graph $\triangle ABC$ with vertices $A(1, 3)$, $B(4, 4)$, and $C(3, 1)$. Reflect $\triangle ABC$ in the lines $y = -x$ and $y = x$. Graph each image. **See margin.**

5. In Example 3, *verify* that $\overline{FF'}$ is perpendicular to $y = -x$.

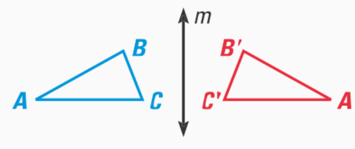
5. Slope of $y = -x$ is -1. The slope of $\overline{FF'}$ is 1. The product of their slopes is -1 making them perpendicular.

REFLECTION THEOREM You saw in Lesson 9.1 that the image of a translation is congruent to the original figure. The same is true for a reflection.

THEOREM *For Your Notebook*

THEOREM 9.2 Reflection Theorem

A reflection is an isometry.

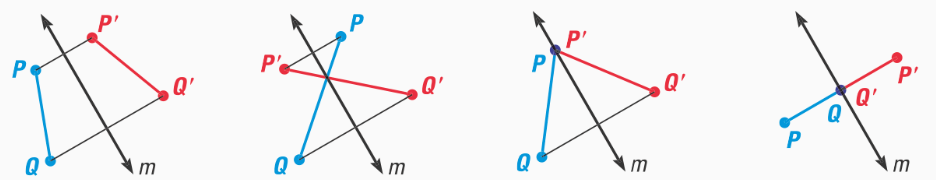


$\triangle ABC \cong \triangle A'B'C'$

Proof: Exs. 35–38, p. 595

WRITE PROOFS
Some theorems, such as the Reflection Theorem, have more than one case. To prove this type of theorem, each case must be proven.

PROVING THE THEOREM To prove the Reflection Theorem, you need to show that a reflection preserves the length of a segment. Consider a segment $\overline{PQ}$ that is reflected in a line m to produce $\overline{P'Q'}$. There are four cases to prove:



Case 1 P and Q are on the same side of m.

Case 2 P and Q are on opposite sides of m.

Case 3 P lies on m, and $\overline{PQ}$ is not $\perp$ to m.

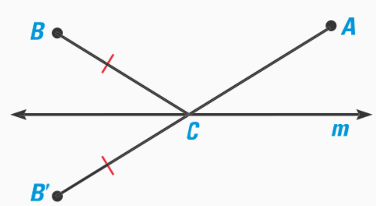
Case 4 Q lies on m, and $\overline{PQ} \perp m$.

EXAMPLE 4 Find a minimum distance

PARKING You are going to buy books. Your friend is going to buy CDs. Where should you park to minimize the distance you both will walk?

Solution

Reflect B in line m to obtain B'. Then draw $\overline{AB'}$. Label the intersection of $\overline{AB'}$ and m as C. Because AB' is the shortest distance between A and B' and $BC = B'C$, park at point C to minimize the combined distance, $AC + BC$, you both have to walk.



GUIDED PRACTICE for Example 4

6. Look back at Example 4. Answer the question by using a reflection of point A instead of point B. **Reflect A in line m obtaining A'. Then draw $\overline{A'B}$. Label the intersection of m and $\overline{A'B}$ as C. Because $A'B$ is the shortest distance between A' and B and $AC = A'C$, park at point C to minimize the combined distance, $AC + BC$, you both have to walk.**

REFLECTION MATRIX You can find the image of a polygon reflected in the x-axis or y-axis using matrix multiplication. Write the reflection matrix to the *left* of the polygon matrix, then multiply.

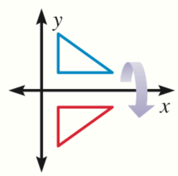
Notice that because matrix multiplication is not commutative, the order of the matrices in your product is important. The reflection matrix must be first followed by the polygon matrix.

KEY CONCEPT *For Your Notebook*

Reflection Matrices

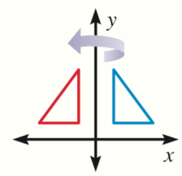
Reflection in the x-axis

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$



Reflection in the y-axis

$$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$



EXAMPLE 5 Use matrix multiplication to reflect a polygon

The vertices of $\triangle DEF$ are $D(1, 2)$, $E(3, 3)$, and $F(4, 0)$. Find the reflection of $\triangle DEF$ in the y-axis using matrix multiplication. Graph $\triangle DEF$ and its image.

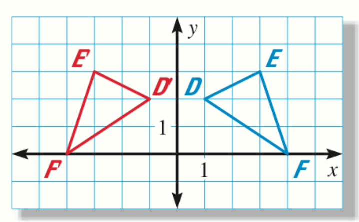
Solution

STEP 1 **Multiply** the polygon matrix by the matrix for a reflection in the y-axis.

$$\underset{\text{Reflection matrix}}{\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}} \underset{\text{Polygon matrix}}{\overset{\;D\;\;E\;\;F}{\begin{bmatrix} 1 & 3 & 4 \\ 2 & 3 & 0 \end{bmatrix}}} = \begin{bmatrix} -1(1)+0(2) & -1(3)+0(3) & -1(4)+0(0) \\ 0(1)+1(2) & 0(3)+1(3) & 0(4)+1(0) \end{bmatrix}$$

$$= \overset{D' \;\; E' \;\; F'}{\begin{bmatrix} -1 & -3 & -4 \\ 2 & 3 & 0 \end{bmatrix}} \quad \text{Image matrix}$$

STEP 2 **Graph** $\triangle DEF$ and $\triangle D'E'F'$.



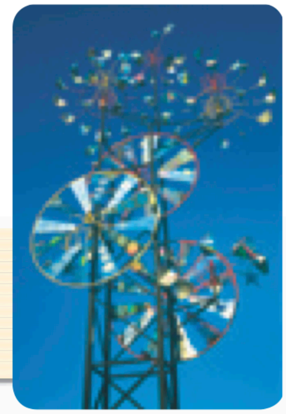
✓ GUIDED PRACTICE for Example 5

The vertices of $\triangle LMN$ are $L(-3, 3)$, $M(1, 2)$, and $N(-2, 1)$. Find the described reflection using matrix multiplication.

7. Reflect $\triangle LMN$ in the x-axis.
$L'(-3, -3)$, $M'(1, -2)$, $N'(-2, -1)$

8. Reflect $\triangle LMN$ in the y-axis.
$L'(3, 3)$, $M'(-1, 2)$, $N'(2, 1)$

9.4 Perform Rotations



Before

Now

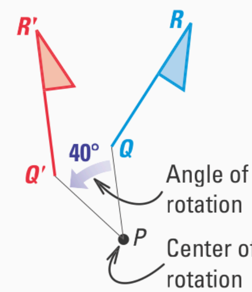
Why?

Key Vocabulary
- **center of rotation**
- **angle of rotation**
- **rotation,** *p. 272*

Recall from Lesson 4.8 that a *rotation* is a transformation in which a figure is turned about a fixed point called the **center of rotation**. Rays drawn from the center of rotation to a point and its image form the **angle of rotation**.

A rotation about a point P through an angle of $x°$ maps every point Q in the plane to a point Q' so that one of the following properties is true:

- If Q is not the center of rotation P, then $QP = Q'P$ and $m\angle QPQ' = x°$, or
- If Q is the center of rotation P, then the image of Q is Q.



A 40° counterclockwise rotation is shown at the right. Rotations can be *clockwise* or *counterclockwise*. In this chapter, all rotations are counterclockwise.

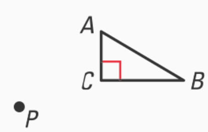
DIRECTION OF ROTATION

clockwise

counterclockwise

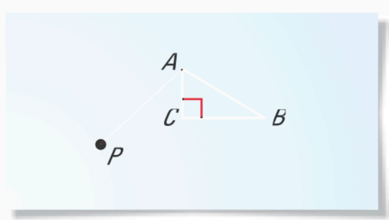
EXAMPLE 1 Draw a rotation

Draw a 120° rotation of $\triangle ABC$ about P.

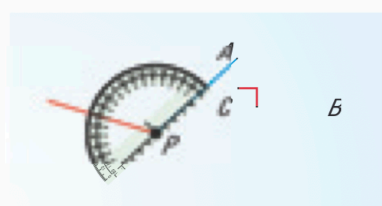


Solution

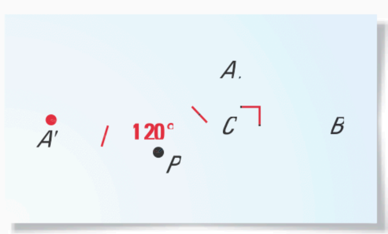
STEP 1 **Draw** a segment from A to P.



STEP 2 **Draw** a ray to form a 120° angle with $\overline{PA}$.



STEP 3 **Draw** A' so that $PA' = PA$.



STEP 4 **Repeat** Steps 1–3 for each vertex. Draw $\triangle A'B'C'$.