Name:

9.1 Translate Figures and Use Vectors

Goal • Use a vector to translate a figure.

Rewrite the Goal as an "I can" statement!

Complete the vocab. with definitions or pictures that make sense to you.

VOCABULARY

Image

Preimage

Isometry

Vector

Initial point

Terminal point

Horizontal component

Vertical component

Component form

Your Notes

You can use *prime notation* to name an image. For example, if the preimage is $\triangle ABC$, then its image is $\triangle A'B'C'$, read as "*triangle A prime, B prime, C prime.*"

Example 1 ***Translate a figure in the coordinate plane***

Graph quadrilateral *ABCD* with vertices $A(-2, 6)$, $B(2, 4)$, $C(2, 1)$, and $D(-2, 3)$. Find the image of each vertex after the translation $(x, y) \to (x + 3, y - 3)$. Then graph the image using prime notation.

Solution

First, draw *ABCD*. Find the translation of each vertex by __________ 3 to its *x*-coordinate and _______________ 3 from its *y*-coordinate. Then graph the image.

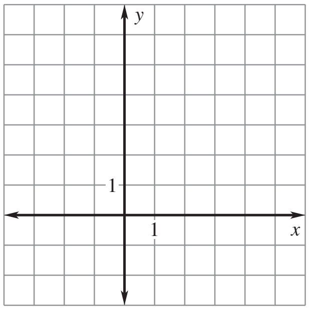
$(x, y) \to (x + 3, y - 3)$

$A(-2, 6) \to A'($______$)$

$B(2, 4) \to B'($______$)$

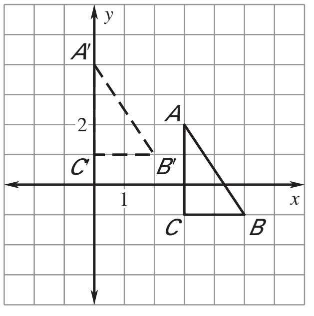
$C(2, 1) \to C'($________$)$

$D(-2, 3) \to D'($______$)$



Example 2 ***Write a translation rule and verify congruence***

Write a rule for the translation of $\triangle ABC$ to $\triangle A'B'C'$. Then verify that the transformation is an isometry.



Solution

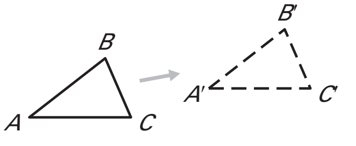
To go from *A* to *A′*, move 3 units ______ and 2 units _____. So, a rule for the translation is $(x, y) \to ($_______________$)$.

Use the SAS Congruence Postulate. Notice that $CB = C'B' =$ ___, and $AC = A'C' =$ ___. The slopes of $\overline{CB}$ and $\overline{C'B'}$ are ___, and the slopes of $\overline{CA}$ and $\overline{C'A'}$ are _____________, so the sides are _________________. Therefore, $\angle C$ and $\angle C'$ are ____________________________. So, $\triangle ABC$ ____ $\triangle A'B'C'$. The translation is an isometry.

Stop and get the teacher's signature before you move on.

THEOREM 9.1: TRANSLATION THEOREM

A translation is an isometry.

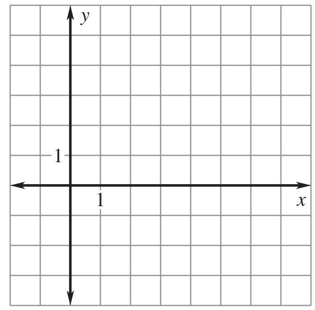


$\triangle ABC \cong \triangle A'B'C'$

Your Notes

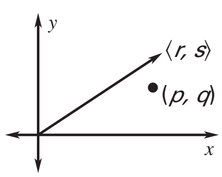
✔ ***Checkpoint*** **Complete the following exercises.**

1. Draw $\triangle PQR$ with vertices $P(4, 2)$, $Q(6, 2)$, and $R(4, -2)$. Find the image of each vertex after the translation $(x, y) \rightarrow (x - 4, y + 1)$. Graph the image using prime notation.



2. In Example 2, write a rule to translate $\triangle A'B'C'$ back to $\triangle ABC$.

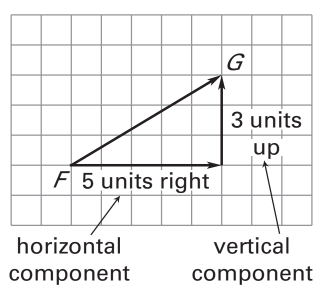
Use brackets to write the component form of the vector $\langle r, s \rangle$. Use parentheses to write the coordinates of the point (p, q).



VECTORS

The diagram shows a vector named $\overrightarrow{FG}$, read as "vector *FG*."

The initial point, or starting point, of the vector is ___.



The terminal point, or ending point, of the vector is ___.

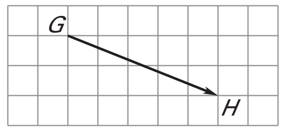
The component form of a vector combines the horizontal and vertical components. So, the component form of $\overrightarrow{FG}$ is _______.

Your Notes

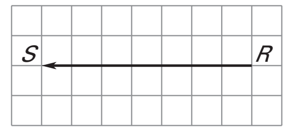
Example 3 *Identify vector components*

Name the vector and write its component form.

a.



b.



Solution

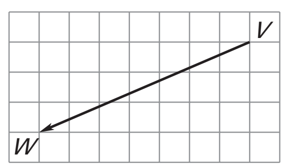
a. The vector is $\overrightarrow{GH}$. From initial point ____ to terminal point ____, you move ____ units ________ and ____ units ________. So, the component form is __________.

b. The vector is $\overrightarrow{RS}$. From initial point ____ to terminal point ____, you move ____ units _______ and ____ units ____________. So, the component form is __________.

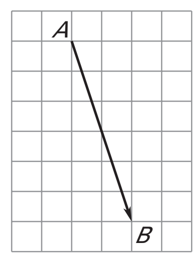
Stop and get the teacher's signature before you move on.

✔ ***Checkpoint*** **Name the vector and write its component form.**

3.



4.



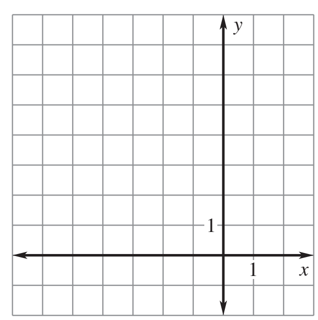
Example 4 *Use a vector to translate a figure*

The vertices of △*ABC* are *A*(0, 4), *B*(2, 3), and *C*(1, 0). Translate △*ABC* using the vector ⟨−4, 1⟩.

Notice that the vector can have different initial points. The vector describes only the direction and magnitude of the translation.

Solution

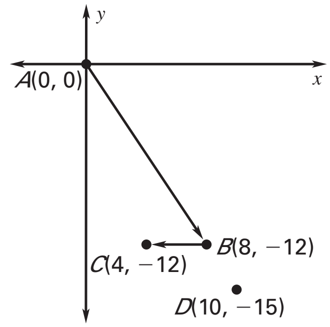
First, graph △*ABC*. Use ⟨−4, 1⟩ to move each vertex ____ units to the ______ and ____ unit _____. Label the image vertices. Draw △*A′B′C′*. Notice that the vectors drawn from preimage to image vertices are __________.



Your Notes

Example 5 ***Solve a multi-step problem***

Construction A car heads out from point *A* toward point *D*. The car encounters construction at *B*, 8 miles east and 12 miles south of its starting point. The detour route leads the car to point *C*, as shown.



a. Write the component form of $\overrightarrow{AB}$.

b. Write the component form of $\overrightarrow{BC}$.

c. Write the component form of the vector that describes the straight line path from the car's current position *C* to its intended destination *D*.

a. The component form of the vector from *A*(0, 0) to *B*(8, −12) is

$\overrightarrow{AB}$ = ____________________ = ___________.

b. The component form of the vector from *B*(8, −12) to *C*(4, −12) is

$\overrightarrow{BC}$ = _________________________ = _________.

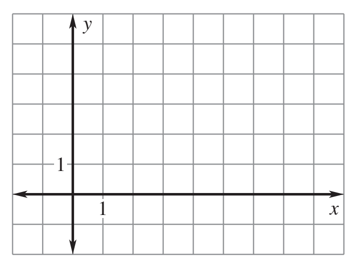
c. The car is currently at point *C* and needs to travel to *D*. The component form of the vector from *C*(4, −12) to *D*(10, −15) is

$\overrightarrow{CD}$ = __________________________ = _________.

Stop and get the teacher's signature before you move on.

Checkpoint **Complete the following exercises.**

5. The vertices of $\triangle ABC$ are $A(-1, -1)$, $B(0, 2)$, and $C(1, -1)$. Translate $\triangle ABC$ using the vector $\langle 5, 2 \rangle$.



6. In Example 5, suppose there is no construction. Write the component form of the vector that describes the straight path from the car's starting point *A* to its final destination *D*.

Homework

9.2 Use Properties of Matrices

Goal • Perform translations using matrix operations.

Rewrite the Goal as an "I can" statement!

Complete the vocab. with definitions or pictures that make sense to you.

VOCABULARY
Matrix
Element
Dimensions

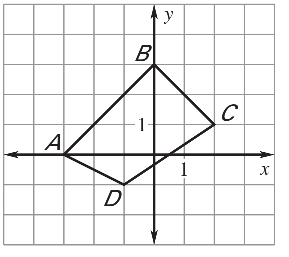
An element of a matrix may also be called an *entry*.

Example 1 ***Represent figures using matrices***

Write a matrix to represent the point or polygon.

a. Point *A*

b. Quadrilateral *ABCD*



Solution

a. Point matrix for *A*

$\begin{bmatrix} \_\_ \\ \_\_ \end{bmatrix}$ ← *x*-coordinate ← *y*-coordinate

The columns in a polygon matrix follow the consecutive order of the vertices of the polygon.

b. Polygon matrix for *ABCD*

$\begin{matrix} A & B & C & D \end{matrix}$

$\begin{bmatrix} \_\_ & \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ & \_\_ \end{bmatrix}$ ← *x*-coordinates ← *y*-coordinates

Stop and get the teacher's signature before you move on.

✔ ***Checkpoint*** **Complete the following exercise.**

1. Write a matrix to represent △*RST* with vertices *R*(−5, −4), *S*(−1, 2), and *T*(3, 1).

Your Notes

Example 2 *Add and subtract matrices*

a. $\begin{bmatrix} 4 & -2 \\ 2 & -3 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ 5 & -6 \end{bmatrix} = \begin{bmatrix} \_\_\_\_\_\_ & \_\_\_\_\_\_ \\ \_\_\_\_\_\_ & \_\_\_\_\_\_ \end{bmatrix}$

$= \begin{bmatrix} \_\_\_ & \_\_\_ \\ \_\_\_ & \_\_\_ \end{bmatrix}$

b. $\begin{bmatrix} 7 & 4 & 5 \\ 1 & -2 & 8 \end{bmatrix} - \begin{bmatrix} 3 & -6 & 5 \\ 0 & 7 & 1 \end{bmatrix}$

$= \begin{bmatrix} \_\_\_\_\_\_ & \_\_\_\_\_\_ & \_\_\_\_\_\_ \\ \_\_\_\_\_\_ & \_\_\_\_\_\_ & \_\_\_\_\_\_ \end{bmatrix}$

$= \begin{bmatrix} \_\_\_ & \_\_\_ & \_\_\_ \\ \_\_\_ & \_\_\_ & \_\_\_ \end{bmatrix}$

Example 3 *Represent a translation using matrices*

The matrix $\begin{bmatrix} 2 & 3 & 4 \\ -3 & 2 & 0 \end{bmatrix}$ represents $\triangle ABC$. Find the image matrix that represents the translation of $\triangle ABC$ 4 units left and 1 unit down. Then graph $\triangle ABC$ and its image.

In order to add two matrices, they must have the same dimensions, so the translation matrix here must have three columns like the polygon matrix.

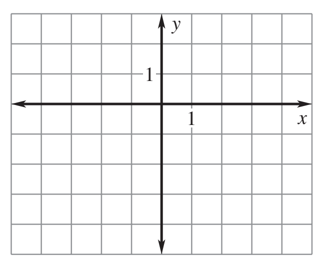
Solution

The translation matrix is $\begin{bmatrix} \_\_\_\_\_\_\_\_ \end{bmatrix}$.

Add this to the polygon matrix for the preimage to find the image matrix.

$\begin{bmatrix} \_\_\_\_\_\_\_\_ \end{bmatrix} + \overset{\quad A \quad B \quad C}{\begin{bmatrix} 2 & 3 & 4 \\ -3 & 2 & 0 \end{bmatrix}} = \overset{A' \quad B' \quad C'}{\begin{bmatrix} \_\_\_\_\_\_\_\_ \end{bmatrix}}$

Translation matrix **Polygon matrix** **Image matrix**



Your Notes

Example 4 *Multiply matrices*

Multiply $\begin{bmatrix} 0 & 4 \\ 5 & 2 \end{bmatrix}\begin{bmatrix} -4 & 1 \\ 8 & -3 \end{bmatrix}$.

Solution

The matrices are both 2 × 2, so their product is defined. Use the following steps to find the elements of the product matrix.

Step 1 **Multiply** the numbers in the __________ of the first matrix by the numbers in the __________ of the second matrix. Put the result in the first row, first column of the product matrix.

$$\begin{bmatrix} 0 & 4 \\ 5 & 2 \end{bmatrix}\begin{bmatrix} -4 & 1 \\ 8 & -3 \end{bmatrix} = \begin{bmatrix} \underline{\qquad\qquad} & ? \\ ? & ? \end{bmatrix}$$

Step 2 **Multiply** the numbers in the __________ of the first matrix by the numbers in the __________ of the second matrix. Put the result in the first row, second column of the product matrix.

$$\begin{bmatrix} 0 & 4 \\ 5 & 2 \end{bmatrix}\begin{bmatrix} -4 & 1 \\ 8 & -3 \end{bmatrix} = \begin{bmatrix} \underline{\qquad\qquad} & \underline{\qquad\qquad} \\ ? & ? \end{bmatrix}$$

Step 3 **Multiply** the numbers in the __________ of the first matrix by the numbers in the __________ of the second matrix. Put the result in the second row, first column of the product matrix.

$$\begin{bmatrix} 0 & 4 \\ 5 & 2 \end{bmatrix}\begin{bmatrix} -4 & 1 \\ 8 & -3 \end{bmatrix} = \begin{bmatrix} \underline{\qquad\qquad} & \underline{\qquad\qquad} \\ \underline{\qquad\qquad} & ? \end{bmatrix}$$

Step 4 **Multiply** the numbers in the __________ of the first matrix by the numbers in the __________ of the second matrix. Put the result in the second row, second column of the product matrix.

$$\begin{bmatrix} 0 & 4 \\ 5 & 2 \end{bmatrix}\begin{bmatrix} -4 & 1 \\ 8 & -3 \end{bmatrix} = \begin{bmatrix} \underline{\qquad\qquad} & \underline{\qquad\qquad} \\ \underline{\qquad\qquad} & \underline{\qquad\qquad} \end{bmatrix}$$

Step 5 **Simplify** the product matrix.

$$\begin{bmatrix} 0 & 4 \\ 5 & 2 \end{bmatrix}\begin{bmatrix} -4 & 1 \\ 8 & -3 \end{bmatrix} = \begin{bmatrix} \underline{\quad} & \underline{\quad} \\ \underline{\quad} & \underline{\quad} \end{bmatrix}$$

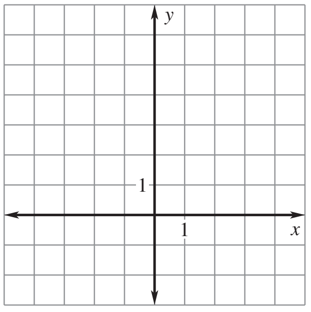
Stop and get the teacher's signature before you move on.

Your Notes

✔ ***Checkpoint*** **Complete the following exercises.**

2. Subtract $\begin{bmatrix} 3 & -5 \\ 8 & -4 \end{bmatrix} - \begin{bmatrix} 9 & 7 \\ -3 & 1 \end{bmatrix}$.

3. The matrix $\begin{bmatrix} -3 & -1 & 0 \\ -1 & 3 & 0 \end{bmatrix}$ represents $\triangle ABC$. Find the image matrix that represents the translation of $\triangle ABC$ 3 units right and 2 units up. Then graph $\triangle ABC$ and its image.



4. Multiply $\begin{bmatrix} 6 & 3 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$.

Your Notes

You could solve this problem arithmetically, multiplying the number of sticks by the price of sticks, and so on, then adding the costs for each team.

Stop and get the teacher's signature before you move on.

Example 5 ***Solve a real-world problem***

Hockey A men's hockey team m needs 7 sticks, 30 pucks, and 4 helmets. A women's team w needs 5 sticks, 25 pucks, and 5 helmets. A hockey stick costs \$30, a puck costs \$4, and a helmet costs \$50. Use matrix multiplication to find the total cost of equipment for each team.

Solution

Write equipment needs and costs per item in matrix form. You will use matrix multiplication, so form the matrices so that the number of columns of the __________ matrix matches the number of rows of the __________ matrix.

Equipment • **Cost** = **Total Cost**

	Sticks	Pucks	Helmets			Dollars			Dollars
m				•	Stick		=	m	?
w					Puck			w	?
					Helmet				

You can find the total cost of equipment for each team by multiplying the equipment matrix by the cost per item matrix. The equipment matrix is ___ × ___ and the cost per item matrix is ___ × ___, so their product is a ___ × ___ matrix.

$$\begin{bmatrix} \quad \\ \quad \end{bmatrix}\begin{bmatrix} \quad \\ \quad \\ \quad \end{bmatrix} = \begin{bmatrix} \quad \\ \quad \end{bmatrix} = \begin{bmatrix} \quad \\ \quad \end{bmatrix}$$

The total cost of equipment for the men's team is ______, and the total cost for the women's team is ______.

✔ ***Checkpoint*** **Complete the following exercise.**

5. In Example 5, find the total costs if a stick costs \$50, a puck costs \$2, and a helmet costs \$70.

Homework

9.3 Perform Reflections

Goal • Reflect a figure in any given line.

Rewrite the Goal as an "I can" statement!

Complete the vocab. with definitions or pictures that make sense to you.

VOCABULARY
Line of reflection

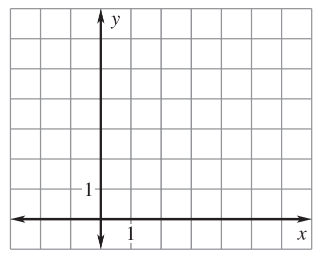
Example 1 ***Graph reflections in horizontal and vertical lines***

The vertices of $\triangle ABC$ are $A(1, 2)$, $B(3, 0)$, and $C(5, 3)$. Graph the reflection of $\triangle ABC$ described.

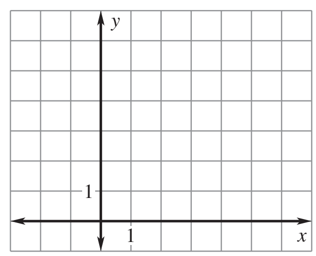
a. In the line n: $x = 2$ **b.** In the line m: $y = 3$

Solution

a. Point A is 1 unit _____ of n, so its reflection A' is 1 unit ______ of n at (___, ___). Also, B' is 1 unit _____ of n at (___, ___), and C' is 3 units _____ of n at (_____, ___).



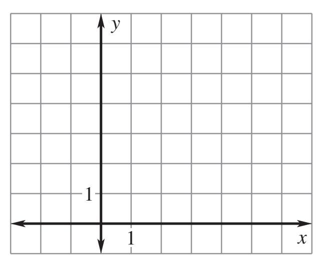
b. Point A is 1 unit _______ m, so A' is 1 unit ________ m at (___, ___). Also, B' is 3 units ________ m at (___, ___). Because point C is on line m, you know that C = ____.



Stop and get the teacher's signature before you move on.

Checkpoint **Complete the following exercise.**

1. Graph the reflection of $\triangle ABC$ from Example 1 in the line $y = 2$.



Your Notes

The product of the slopes of perpendicular lines is -1.

Stop and get the teacher's signature before you move on.

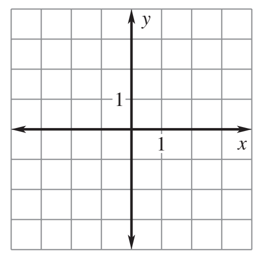
Example 2 ***Graph a reflection in $y = x$***

The endpoints of $\overline{CD}$ are $C(-2, 2)$ and $D(1, 2)$. Reflect the segment in the line $y = x$. Graph the segment and its image.

Solution

The slope of $y = x$ is ___. The segment from C to its image, $\overline{CC'}$, is _______________ to the line of reflection $y = x$, so the slope of $\overline{CC'}$ will be _____ (because $1(-1) =$ _____). From C, move ___ units right and ___ units down to $y = x$. From that point, move ___ units right and ___ units down to locate C'(___, _____).

The slope of $\overline{DD'}$ will also be _____. From D, move _____ units right and _____ units down to $y = x$. Then move _____ units right and _____ units down to locate D'(___, ___).



COORDINATE RULES FOR REFLECTIONS

- If (a, b) is reflected in the x-axis, its image is the point (___, _____).
- If (a, b) is reflected in the y-axis, its image is the point (_____, ___).
- If (a, b) is reflected in the line $y = x$, its image is the point (___, ___).
- If (a, b) is reflected in the line $y = -x$, its image is the point (_____, _____).

Your Notes

Stop and get the teacher's signature before you move on.

Example 3 *Graph a reflection in $y = -x$*

Reflect $\overline{CD}$ from Example 2 in the line $y = -x$. Graph $\overline{CD}$ and its image.

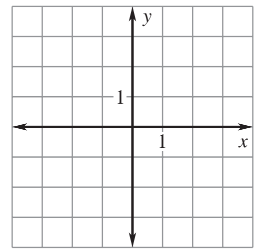
Solution

Use the coordinate rule for reflecting in the line $y = -x$.

$(a, b) \to (-b, -a)$

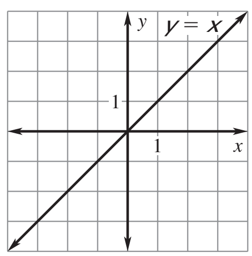
$C(-2, 2) \to C'(\_\_\_\_\_, \_\_\_)$

$D(1, 2) \to D'(\_\_\_\_\_, \_\_\_\_\_)$

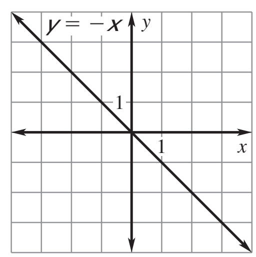


Checkpoint **The endpoints of $\overline{JK}$ are $J(-1, -2)$ and $K(1, -2)$. Reflect the segment in the given line. Graph the segment and its image.**

2. $y = x$

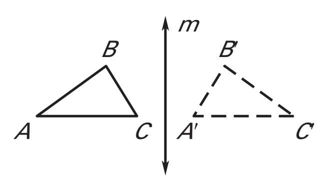


3. $y = -x$



THEOREM 9.2: REFLECTION THEOREM

A reflection is an isometry.



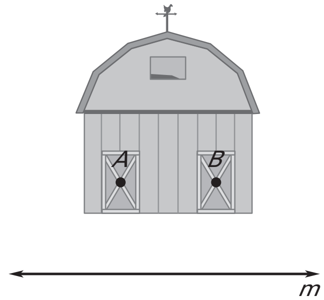
$\triangle ABC \cong \triangle A'B'C'$

Your Notes

Stop and get the teacher's signature before you move on.

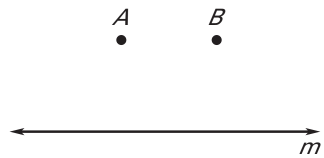
Example 4 ***Find a minimum distance***

Tools Workers are retrieving tools that they need for a project. One will enter the building at point A and the other at point B. Where should they park on driveway m to minimize the distance they will walk?



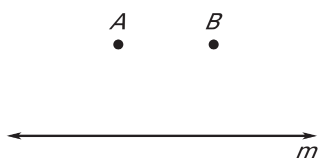
Solution

Reflect B in line m to obtain B'. Then draw $\overline{AB'}$. Label the ______________ of $\overline{AB'}$ and m as C. Because AB' is the __________ distance between A and B' and BC = _____, park at point ___ to minimize the combined distance, $AC + BC$, they have to walk.



Checkpoint Complete the following exercise.

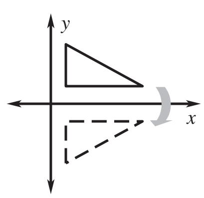
4. In Example 4, reflect A in line m. What do you notice?



REFLECTION MATRICES

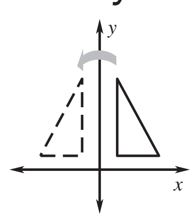
Reflection in the x-axis.

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$



Reflection in the y-axis.

$$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$



Your Notes

Example 5 *Use matrix multiplication to reflect a polygon*

The vertices of $\triangle DEF$ are $D(1, 2)$, $E(2, 3)$, and $F(4, 1)$. Find the reflection of $\triangle DEF$ in the y-axis using matrix multiplication. Graph $\triangle DEF$ and its image.

Solution

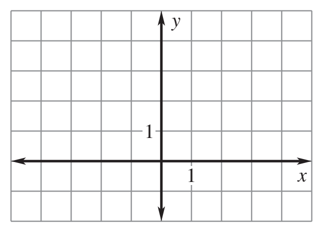
Step 1 **Multiply** the polygon matrix by the matrix for a reflection in the y-axis.

$$\text{Reflection matrix} \rightarrow \begin{bmatrix} \_\_ & \_\_ \\ \_\_ & \_\_ \end{bmatrix} \overset{\quad D \qquad E \qquad F}{\begin{bmatrix} \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \end{bmatrix}} \leftarrow \text{Polygon matrix}$$

$$= \begin{bmatrix} \_\_\_\_\_\_ & \_\_\_\_\_\_ & \_\_\_\_\_\_ \\ \_\_\_\_\_\_ & \_\_\_\_\_\_ & \_\_\_\_\_\_ \end{bmatrix}$$

$$= \overset{\quad D' \quad E' \quad F'}{\begin{bmatrix} \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \end{bmatrix}}$$

Step 2 **Graph** $\triangle DEF$ and $\triangle D'E'F'$.



Stop and get the teacher's signature before you move on.

Checkpoint **Complete the following exercise.**

5. The vertices of $\triangle QRS$ are $Q(-1, 4)$, $R(0, 1)$, and $S(2, 3)$. Find the reflection of $\triangle QRS$ in the x-axis using matrix multiplication.

Homework

9.4 Perform Rotations

Goal • Rotate figures about a point.

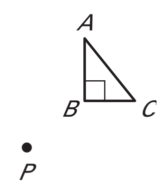
Rewrite the Goal as an "I can" statement!

VOCABULARY
Center of rotation
Angle of rotation

Complete the vocab. with definitions or pictures that make sense to you.

Example 1 ***Draw a rotation***

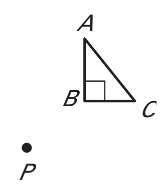
Draw a 150° rotation of △*ABC* about *P*.



Solution

Step 1 **Draw** a segment from *A* to *P*.

Step 2 **Draw** a ray to form a 150° angle with $\overline{PA}$.



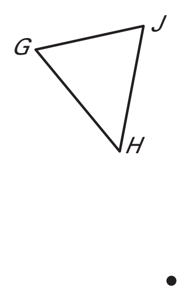
Step 3 **Draw** *A*′ so that *PA*′ = *PA*.

Step 4 **Repeat** Steps 1–3 for each vertex. Draw △*A*′*B*′*C*′.

Stop and get the teacher's signature before you move on.

✓ ***Checkpoint*** **Complete the following exercise.**

1. Draw a 60° rotation of △*GHJ* about *P*.



Your Notes

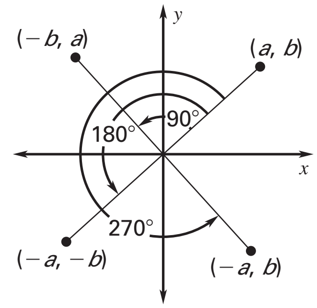
COORDINATE RULES FOR ROTATIONS ABOUT THE ORIGIN

When a point (a, b) is rotated counterclockwise about the origin, the following are true:

1. For a rotation of 90°, (a, b) → (_____, ___).

2. For a rotation of 180°, (a, b) → (_____, _____).

3. For a rotation of 270°, (a, b) → (___, _____).



Example 2 ***Rotate a figure using the coordinate rules***

Graph quadrilateral *KLMN* with vertices *K*(3, 2), *L*(4, 2), *M*(4, −3), and *N*(2, −1). Then rotate the quadrilateral 270° about the origin.

Solution

Graph *KLMN*. Use the coordinate rule for a 270° rotation to find the images of the vertices.

(a, b) → (b, $-a$)

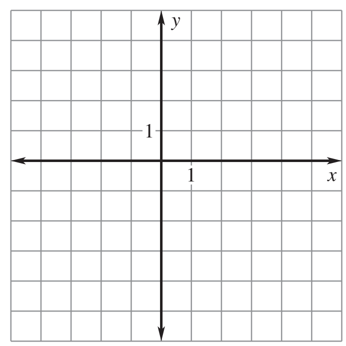
K(3, 2) → *K*′(___, _____)

L(4, 2) → *L*′(___, _____)

M(4, −3) → *M*′(_____, _____)

N(2, −1) → *N*′(_____, _____)

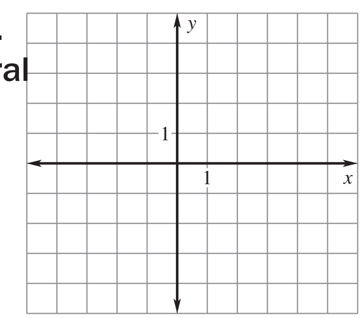
Graph the image *K*′*L*′*M*′*N*′.



Stop and get the teacher's signature before you move on.

Checkpoint **Complete the following exercise.**

2. Graph *KLMN* in Example 2. Then rotate the quadrilateral 90° about the origin.

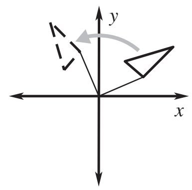


Your Notes

ROTATION MATRICES (COUNTERCLOCKWISE)

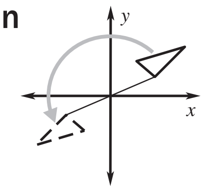
90° rotation

$$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$



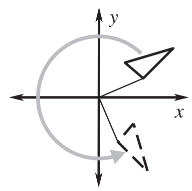
180° rotation

$$\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$



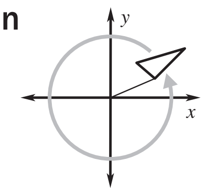
270° rotation

$$\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$



360° rotation

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$



> Notice that a 360° rotation returns the figure to its original position. The matrix that represents this rotation is called the *identity matrix*.

Example 3 *Use matrices to rotate a figure*

Trapezoid *DEFG* has vertices *D*(−1, 3), *E*(1, 3), *F*(2, 1), and *G*(1, 0). Find the image matrix for a 180° rotation of *DEFG* about the origin. Graph *DEFG* and its image.

Solution

Step 1 **Write** the polygon matrix:

$$\begin{array}{c} \begin{matrix} D & E & F & G \end{matrix} \\ \begin{bmatrix} \_\_ & \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ & \_\_ \end{bmatrix} \end{array}$$

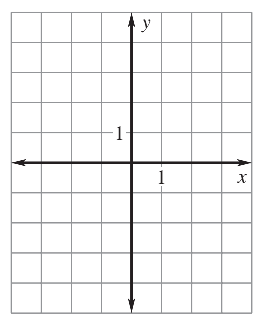
Step 2 **Multiply** by the matrix for a 180° rotation.

$$\begin{bmatrix} \_\_ & \_\_ \\ \_\_ & \_\_ \end{bmatrix} \overset{\begin{matrix} D & E & F & G \end{matrix}}{\begin{bmatrix} \_\_ & \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ & \_\_ \end{bmatrix}} = \overset{\begin{matrix} D' & E' & F' & G' \end{matrix}}{\begin{bmatrix} \_\_ & \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ & \_\_ \end{bmatrix}}$$

Rotation matrix   **Polygon matrix**   **Image matrix**

> Because matrix multiplication is not commutative, always write the rotation matrix first, then the polygon matrix.

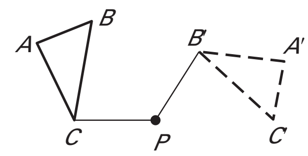
Step 3 **Graph** the preimage *DEFG*. Graph the image *D′E′F′G′*.



Your Notes

THEOREM 9.3: ROTATION THEOREM

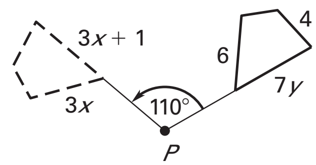
A rotation is an isometry.



$\triangle ABC \cong \triangle A'B'C'$

Example 4 *Find side lengths in a rotation*

The quadrilateral is rotated about *P*. Find the value of *y*.



Solution

By Theorem 9.3, the rotation is an __________, so corresponding side lengths are ________. Then $3x =$ ___, so $x =$ ___. Now set up an equation to solve for *y*.

___$y =$ ________ Corresponding lengths in an isometry are equal.

___$y =$ __________ Substitute ___ for *x*.

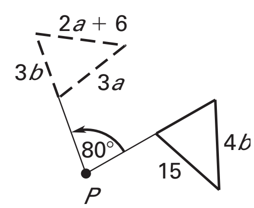
$y =$ ___ Solve for *y*.

Stop and get the teacher's signature before you move on.

Checkpoint **Complete the following exercises.**

3. Use the quadrilateral in Example 3. Find the image matrix after a 270° rotation about the origin.

4. The triangle is rotated about *P*. Find the value of *b*.



Homework

9.5 Apply Compositions of Transformations

Goal • Perform combinations of two or more transformations.

Rewrite the Goal as an "I can" statement!

Complete the vocab. with definitions or pictures that make sense to you.

VOCABULARY
Glide reflection
Composition of transformations

Example 1 ***Find the image of a glide reflection***

The vertices of △*ABC* are *A*(2, 1), *B*(5, 3), and *C*(6, 2). Find the image of △*ABC* after the glide reflection.

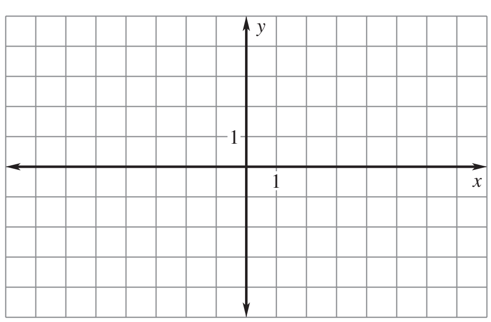
Translation: $(x, y) \rightarrow (x - 8, y)$

Reflection: in the x-axis

The line of reflection must be parallel to the direction of the translation to be a glide reflection.

Solution

Begin by graphing △*ABC*. Then graph △*A′B′C′* after a translation 8 units _____. Finally, graph △*A″B″C″* after a reflection in the x-axis.



Your Notes

THEOREM 9.4: COMPOSITION THEOREM

The composition of two (or more) isometries is an isometry.

Example 2 *Find the image of a composition*

Unless you are told otherwise, do the transformations in the order given.

The endpoints of $\overline{CD}$ are $C(-2, 6)$ and $D(-1, 3)$. Graph the image of $\overline{CD}$ after the composition.

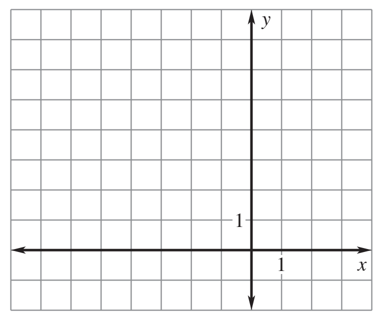
Reflection: in the y-axis
Rotation: 90° about the origin

Solution

Step 1 Graph $\overline{CD}$.

Step 2 Reflect $\overline{CD}$ in the y-axis. $\overline{C'D'}$ has endpoints C'(___, ___) and D'(___, ___).

Step 3 Rotate $\overline{C'D'}$ 90° about the origin. $\overline{C''D''}$ has endpoints C''(_____, ___) and D''(_____, ___).

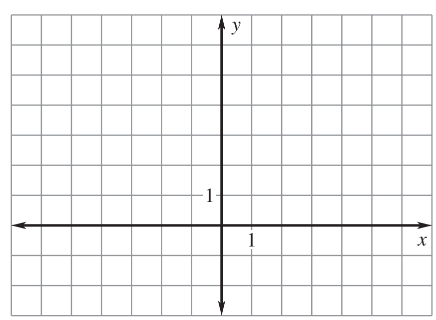


Stop and get the teacher's signature before you move on.

Checkpoint Complete the following exercises.

1. Suppose $\triangle ABC$ in Example 1 is translated 5 units down, then reflected in the y-axis. What are the coordinates of the vertices of the image?

2. Graph $\overline{CD}$ from Example 2. Do the rotation first, followed by the reflection.

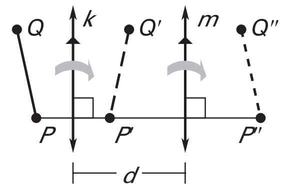


Your Notes

THEOREM 9.5: REFLECTIONS IN PARALLEL LINES THEOREM

If lines k and m are parallel, then a reflection in line k followed by a reflection in line m is the same as a ____________.

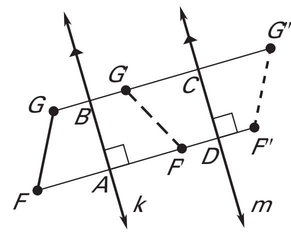
If P'' is the image of P, then:



1. $\overline{PP''}$ is perpendicular to k and m, and
2. $PP'' = 2d$, where d is the distance between k and m.

Example 3 *Use Theorem 9.5*

In the diagram, a reflection in line k maps $\overline{GF}$ to $\overline{G'F'}$. A reflection in line m maps $\overline{G'F'}$ to $\overline{G''F''}$. Also, $FA = 6$ and $DF'' = 3$.



a. Name any segments congruent to each segment: $\overline{GF}$, $\overline{FA}$, and $\overline{GB}$.

b. Does $AD = BC$? *Explain.*

c. What is the length of $\overline{GG''}$?

a. $\overline{GF} \cong$ ______, and $\overline{GF} \cong$ ______. $\overline{FA} \cong$ _____.
$\overline{GB} \cong$ ______.

b. _____, AD ____ BC because $\overline{GG''}$ and $\overline{FF''}$ are ________________ to both k and m, so $\overline{BC}$ and $\overline{AD}$ are opposite sides of a ____________.

c. By the properties of reflections, $F'A =$ ___ and $F'D =$ ___. Theorem 9.5 implies that $GG'' = FF'' =$ ___ • _____, so the length of $\overline{GG''}$ is ___(___ + ___), or ____ units.

Stop and get the teacher's signature before you move on.

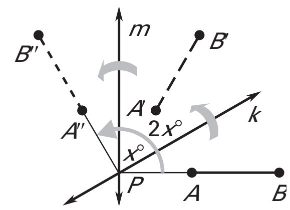
✔ ***Checkpoint*** **Complete the following exercise.**

3. In Example 3, suppose you are given that $BC = 10$ and $G'F' = 6$. What is the perimeter of quadrilateral $GG''F''F$?

Your Notes

THEOREM 9.6: REFLECTIONS IN INTERSECTING LINES THEOREM

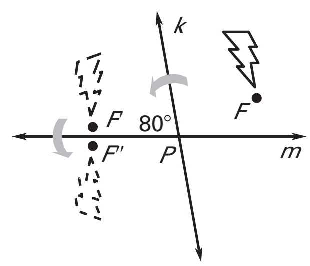
If lines k and m intersect at point P, then a reflection in k followed by a reflection in m is the same as a __________ about ___.



The angle of rotation is $2x°$, where $x°$ is the measure of the acute or right angle formed by k and m.

Example 4 ***Use Theorem 9.6***

In the diagram, the figure is reflected in line *k*. The image is then reflected in line *m*. Describe a single transformation that maps *F* to *F″*.



Solution

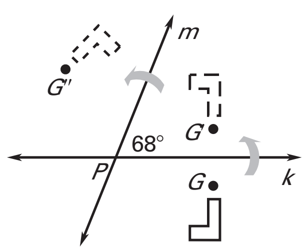
The measure of the acute angle formed between lines k and m is _____. So, by Theorem 9.6, a single transformation that maps F to F'' is a ______ rotation about _________.

You can check that this is correct by tracing lines k and m and point F, then rotating the point ______.

Stop and get the teacher's signature before you move on.

Checkpoint **Complete the following exercise.**

4. In the diagram below, the preimage is reflected in line k, then in line m. Describe a single transformation that maps G to G''.



Homework

9.6 Identify Symmetry

Goal • Identify line and rotational symmetries of a figure.

Rewrite the Goal as an "I can" statement!

VOCABULARY

Line symmetry

Line of symmetry

Rotational symmetry

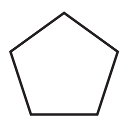
Center of symmetry

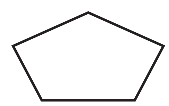
Complete the vocab. with definitions or pictures that make sense to you.

Example 1 ***Identify lines of symmetry***

How many lines of symmetry does the figure have?

a. 

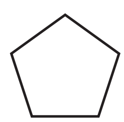
b. 

c. 

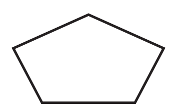
Solution

a. ______ lines of symmetry

b. ______ lines of symmetry



c. ______ line of symmetry



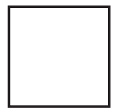
Notice that the lines of symmetry are also lines of reflection.

Your Notes

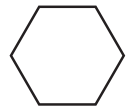
Example 2 ***Identify rotational symmetry***

Does the figure have rotational symmetry? If so, *describe* any rotations that map the figure onto itself.

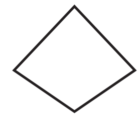
a. Square



b. Regular hexagon

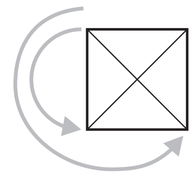


c. Kite

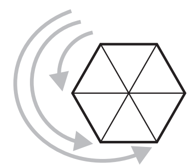


Solution

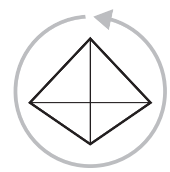
a. The square _____ rotational symmetry. The center is the intersection of the diagonals. Rotations of _____ or _____ about the center map the square onto itself.



b. The regular hexagon _____ rotational symmetry. The center is the intersection of the diagonals. Rotations of _____, _____, or _____ about the center all map the hexagon onto itself.

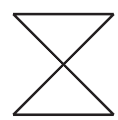


c. The kite ______________ rotational symmetry because no rotation of _____ or less maps the kite onto itself.



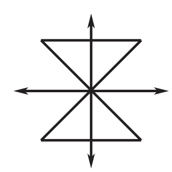
Example 3 ***Identify symmetry***

Identify the line symmetry and rotational symmetry of the figure at the right.

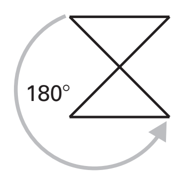


Solution

The figure _____ line symmetry. _____ lines of symmetry can be drawn for the figure.



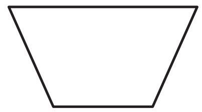
For a figure with *s* lines of symmetry, the smallest rotation that maps the figure onto itself has the measure _____. So, the figure has _____, or _____ rotational symmetry.

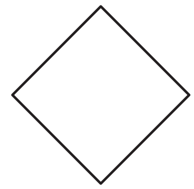


Stop and get the teacher's signature before you move on.

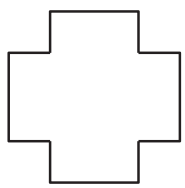
Your Notes

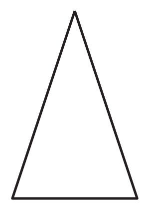
✔ *Checkpoint* How many lines of symmetry does the figure have?

1. 

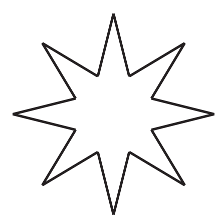
2. 

In Exercises 3 and 4, does the figure have rotational symmetry? If so, *describe* any rotations that map the figure onto itself.

3. 

4. 

5. Describe the lines of symmetry and rotational symmetry of the figure at the right.



Homework

9.7 Identify and Perform Dilations

Goal • Use drawing tools and matrices to draw dilations.

Complete the vocab. with definitions or pictures that make sense to you.

Rewrite the Goal as an "I can" statement!

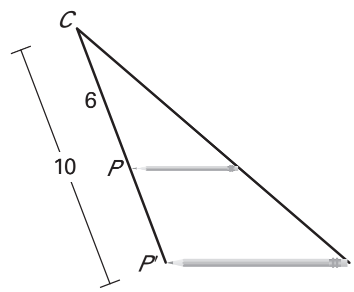
VOCABULARY

Scalar multiplication

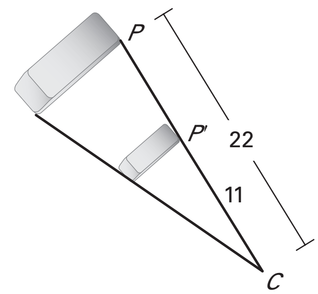
Example 1 *Identify dilations*

Find the scale factor of the dilation. Then tell whether the dilation is a *reduction* or an *enlargement*.

a.



b.



Solution

a. Because $\frac{CP'}{CP} = \_\_\_\_$, the scale factor is $k = \_\_\_$.
The image P' is an ________________.

b. Because $\frac{CP'}{CP} = \_\_\_\_$, the scale factor is $k = \_\_\_$.
The image P' is a _____________.

Stop and get the teacher's signature before you move on.

✔ ***Checkpoint*** **Complete the following exercise.**

1. In a dilation, $CP' = 4$ and $CP = 20$. Tell whether the dilation is a *reduction* or an *enlargement* and find its scale factor.

Your Notes

Stop and get the teacher's signature before you move on.

Example 2 ***Draw a dilation***

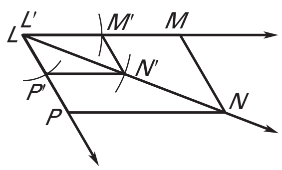
Draw and label ▱*LMNP*. Then construct a dilation of ▱*LMNP* with point *L* as the center of dilation and a scale factor of $\frac{1}{2}$.

Solution

Step 1 **Draw** *LMNP*. Draw rays from *L* through vertices *M*, *N*, and *P*.

Step 2 **Open** the compass to the length of $\overline{LM}$. Locate M' on $\overrightarrow{LM}$ so $LM' =$ ____ (LM). Locate N' and P' the same way.

Step 3 **Add** a second label L' to point *L*. Draw the sides of $L'M'N'P'$.



Checkpoint **Complete the following exercise.**

2. Draw and label △*PQR*. Then construct a dilation of △*PQR* with *P* as the center of dilation and a scale factor of 2.

Your Notes

Stop and get the teacher's signature before you move on.

Example 3 *Scalar multiplication*

Simplify the product: $3\begin{bmatrix} 0 & 5 & 4 \\ 2 & -2 & -1 \end{bmatrix}$.

Solution

$3\begin{bmatrix} 0 & 5 & 4 \\ 2 & -2 & -1 \end{bmatrix} = \begin{bmatrix} \_\_\_ & \_\_\_ & \_\_\_ \\ \_\_\_ & \_\_\_ & \_\_\_ \end{bmatrix}$ Multiply each element in the matrix by ___.

$= \begin{bmatrix} \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \end{bmatrix}$ Simplify.

Checkpoint Simplify the product.

3. $4\begin{bmatrix} -6 & 3 & 2 \\ 5 & -1 & 4 \end{bmatrix}$	**4.** $-3\begin{bmatrix} 5 & -1 & -2 \\ -2 & 0 & 4 \end{bmatrix}$

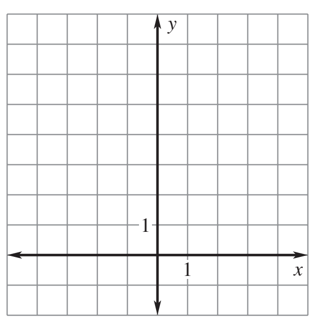
Example 4 *Use scalar multiplication in a dilation*

The vertices of quadrilateral *ABCD* are *A*(−3, 0), *B*(0, 6), *C*(3, 6), and *D*(3, 3). Use scalar multiplication to find the image of *ABCD* after a dilation with its center at the origin and a scale factor of $\frac{1}{3}$. Graph *ABCD* and its image.

Solution

$\_\_\_ \begin{bmatrix} A & B & C & D \\ & & & \end{bmatrix} = \begin{bmatrix} A' & B' & C' & D' \\ & & & \end{bmatrix}$

Scale factor　Polygon matrix　Image matrix



Your Notes

Stop and get the teacher's signature before you move on.

Example 5 *Find the image of a composition*

The vertices of △*KLM* are *K*(−3, 0), *L*(−2, 1), and *M*(−1, −1). Find the image of △*KLM* after the given composition.

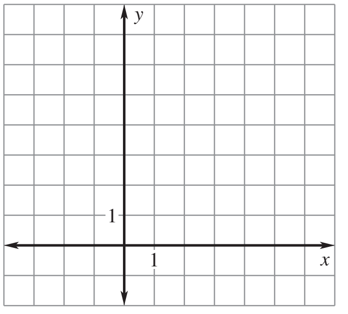
Translation: $(x, y) \rightarrow (x + 4, y + 2)$
Dilation: centered at the origin with a scale factor of 2

Solution

Step 1 **Graph** the preimage △*KLM* in the coordinate plane.

Step 2 **Translate** △*KLM* 4 units to the _______ and 2 units _____. Label it △*K′L′M′*.

Step 3 **Dilate** △*K′L′M′* using the ________ as the center and a scale factor of 2 to find △*K″L″M″*.



Checkpoint **Complete the following exercises.**

5. The vertices of △*RST* are *R*(−4, 3), *S*(−1, −2), and *T*(2, 1). Use scalar multiplication to find the vertices of △*R′S′T′* after a dilation with its center at the origin and a scale factor of 2.

6. A segment has the endpoints *C*(−2, 2) and *D*(2, 2). Find the image of $\overline{CD}$ after a 90° rotation about the origin followed by a dilation with its center at the origin and a scale factor of 2.

Homework