

8.1

Find Angle Measures in Polygons

Goal • Find angle measures in polygons.

Complete the vocab. with definitions or pictures that make sense to you.

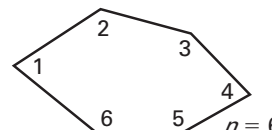
Rewrite the Goal as an "I can" statement!

VOCABULARY

Diagonal

THEOREM 8.1: POLYGON INTERIOR ANGLES THEOREM

The sum of the measures of the interior angles of a convex n -gon is $(n - \underline{\quad}) \cdot \underline{\quad}$.



$$m\angle 1 + m\angle 2 + \dots + m\angle n = (n - \underline{\quad}) \cdot \underline{\quad}$$

COROLLARY TO THEOREM 8.1: INTERIOR ANGLES OF A QUADRILATERAL

The sum of the measures of the interior angles of a quadrilateral is $\underline{\quad}$.

Example 1 Find the sum of angle measures in a polygon

Find the sum of the measures of the interior angles of a convex hexagon.



Solution

A hexagon has $\underline{\quad}$ sides. Use the Polygon Interior Angles Theorem.

$$\begin{aligned} (n - \underline{\quad}) \cdot \underline{\quad} &= (\underline{\quad} - \underline{\quad}) \cdot \underline{\quad} && \text{Substitute} \\ & && \text{for } n. \\ &= \underline{\quad} \cdot \underline{\quad} && \text{Subtract.} \\ &= \underline{\quad} && \text{Multiply.} \end{aligned}$$

The sum of the measures of the interior angles of a hexagon is $\underline{\quad}$.

Your Notes

Example 2 Find the number of sides of a polygon

The sum of the measures of the interior angles of a convex polygon is 1260° . Classify the polygon by the number of sides.

Solution

Use the Polygon Interior Angles Theorem to write an equation involving the number of sides n . Then solve the equation to find the number of sides.

$$(n - \underline{\quad}) \cdot \underline{\quad} = \underline{\quad} \quad \text{Polygon Interior Angles Theorem}$$

$$n - \underline{\quad} = \underline{\quad} \quad \text{Divide each side by } \underline{\quad}.$$

$$n = \underline{\quad} \quad \text{Add } \underline{\quad} \text{ to each side.}$$

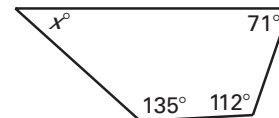
The polygon has $\underline{\quad}$ sides. It is a $\underline{\quad}$.

Example 3 Find an unknown interior angle measure

Find the value of x in the diagram shown.

Solution

The polygon is a quadrilateral. Use the Corollary to the Polygon Interior Angles Theorem to write an equation involving x . Then solve the equation.



$$x^\circ + \underline{\quad} + \underline{\quad} + \underline{\quad} = \underline{\quad} \quad \text{Corollary to Theorem 8.1}$$

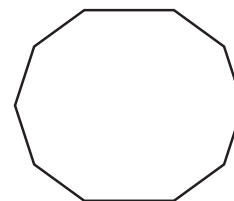
$$x + \underline{\quad} = \underline{\quad} \quad \text{Combine like terms.}$$

$$x = \underline{\quad} \quad \text{Subtract } \underline{\quad} \text{ from each side.}$$

Stop and get the teacher's signature before you move on.

✓ **Checkpoint** Complete the following exercise.

1. Find the sum of the measures of the interior angles of the convex decagon.

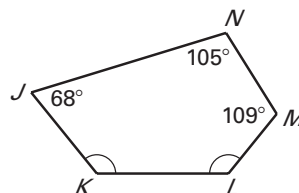


Your Notes

Checkpoint Complete the following exercises.

2. The sum of the measures of the interior angles of a convex polygon is 1620° . Classify the polygon by the number of sides.

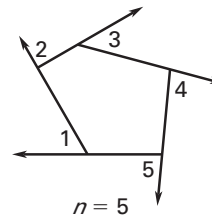
3. Use the diagram at the right.
Find $m\angle K$ and $m\angle L$.



THEOREM 8.2: POLYGON EXTERIOR ANGLES THEOREM

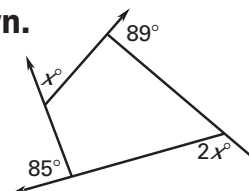
The sum of the measures of the exterior angles of a convex polygon, one angle at each vertex, is _____.

$$m\angle 1 + m\angle 2 + \dots + m\angle n = \underline{\hspace{2cm}}$$



Example 4 Find unknown exterior angle measures

Find the value of x in the diagram shown.



Solution

Use the Polygon Exterior Angles Theorem to write and solve an equation.

$$x^\circ + \underline{\hspace{1cm}} + \underline{\hspace{1cm}} + \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$$

$$\underline{\hspace{1cm}}x + \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$$

$$x = \underline{\hspace{1cm}}$$

Polygon Exterior Angles Theorem.

Combine like terms.

Solve for x .

Your Notes

Example 5 Find angle measures in regular polygons

Lamps The base of a lamp is in the shape of a regular 15-gon. Find (a) the measure of each interior angle and (b) the measure of each exterior angle.

Solution

- a. Use the Polygon Interior Angles Theorem to find the sum of the measures of the interior angles.

$$(n - \underline{\quad}) \cdot \underline{\quad} = (\underline{\quad} - \underline{\quad}) \cdot \underline{\quad}$$
$$= \underline{\quad}$$

Then find the measure of one interior angle. A regular 15-gon has $\underline{\quad}$ congruent interior angles.

Divide $\underline{\quad}$ by $\underline{\quad}$: $\underline{\quad} \div \underline{\quad} = \underline{\quad}$.

The measure of each interior angle in the 15-gon is $\underline{\quad}$.

- b. By the Polygon Exterior Angles Theorem, the sum of the measures of the exterior angles, one angle at each vertex, is $\underline{\quad}$. Divide $\underline{\quad}$ by $\underline{\quad}$:

$$\underline{\quad} \div \underline{\quad} = \underline{\quad}$$

The measure of each exterior angle in the 15-gon is $\underline{\quad}$.

✓ Checkpoint Complete the following exercises.

4. A convex pentagon has exterior angles with measures 66° , 77° , 82° , and 62° . What is the measure of an exterior angle at the fifth vertex?

5. Find the measure of (a) each interior angle and (b) each exterior angle of a regular nonagon.

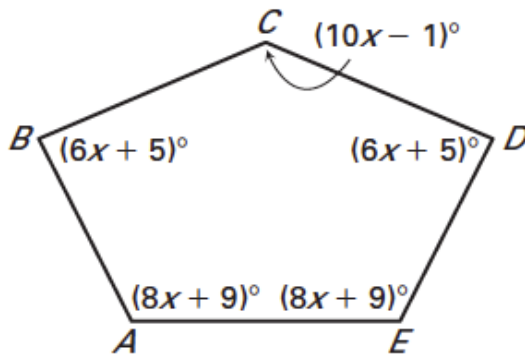
Stop and get the teacher's signature before you move on.

Homework

Complete the following problem. Make sure you justify your answer by either:

- Checking your answer
- Providing a theorem, postulate, or definition
- Showing your work.

Light Fixture The side view of a light fixture is shown below. Find the value of x . Then determine the measure of each angle.



Answer the following questions. Make sure you use full sentences and use the correct vocabulary words when appropriate.

1. Describe how section is connected to something that we learned previously this year.
2. What skills did you use in this section that you learned earlier in this class?
3. What do you think will come next?

8.2

Use Properties of Parallelograms

Goal • Find angle and side measures in parallelograms.

Rewrite the Goal as an "I can" statement!

VOCABULARY

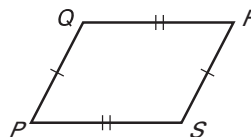
Parallelogram

Complete the vocab. with definitions or pictures that make sense to you.

THEOREM 8.3

If a quadrilateral is a parallelogram, then its opposite sides are congruent.

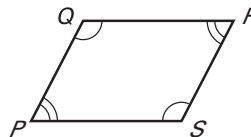
If $PQRS$ is a parallelogram, then $\overline{PQ} \cong \overline{RS}$ and $\overline{QR} \cong \overline{PS}$.



THEOREM 8.4

If a quadrilateral is a parallelogram, then its opposite angles are congruent.

If $PQRS$ is a parallelogram, then $\angle P \cong \angle R$ and $\angle Q \cong \angle S$.

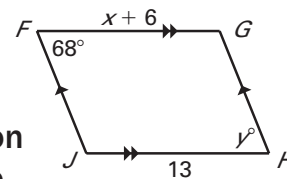


Example 1 Use properties of parallelograms

Find the values of x and y .

Solution

$FGHJ$ is a parallelogram by the definition of a parallelogram. Use Theorem 8.3 to find the value of x .



$FG = \underline{\hspace{2cm}}$ Opposite sides of a \square are \cong .

$x + 6 = \underline{\hspace{2cm}}$ Substitute $x + 6$ for FG and $\underline{\hspace{2cm}}$ for $\underline{\hspace{2cm}}$.

$x = \underline{\hspace{2cm}}$ Subtract 6 from each side.

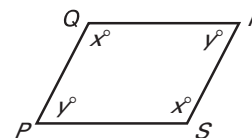
By Theorem 8.4, $\angle F \cong \angle H$, or $m\angle F = \underline{\hspace{2cm}}$. So, $y^\circ = \underline{\hspace{2cm}}$.

In $\square FGHJ$, $x = \underline{\hspace{2cm}}$ and $y = \underline{\hspace{2cm}}$.

Your Notes

THEOREM 8.5

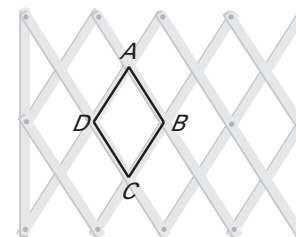
If a quadrilateral is a parallelogram, then its consecutive angles are _____.



If $PQRS$ is a parallelogram, then $x^\circ + y^\circ =$ _____.

Example 2 Use properties of a parallelogram

Gates As shown, a gate contains several parallelograms. Find $m\angle ADC$ when $m\angle DAB = 65^\circ$.

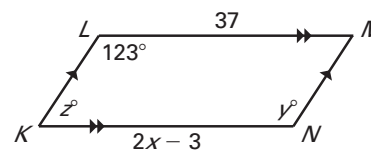


Solution

By Theorem 8.5, the consecutive angle pairs in $\square ABCD$ are _____. So, $m\angle ADC + m\angle DAB =$ _____. Because $m\angle DAB = 65^\circ$, $m\angle ADC =$ _____ - _____ = _____.

Stop and get the teacher's signature before you move on.

✔ **Checkpoint** Find the indicated measure in $\square KLMN$ shown at the right.

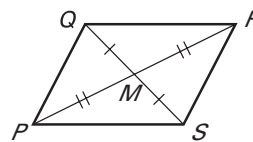


<p>1. x</p>	<p>2. y</p>
<p>3. z</p>	

Your Notes

THEOREM 8.6

If a quadrilateral is a parallelogram, then its diagonals _____ each other.

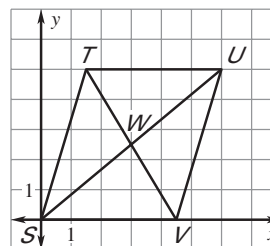


$$\overline{QM} \cong \underline{\hspace{2cm}} \text{ and}$$

$$\overline{PM} \cong \underline{\hspace{2cm}}$$

Example 3 Use properties of a parallelogram

The diagonals of $\square STUV$ intersect at point W . Find the coordinates of W .



Solution

By Theorem 8.6, the diagonals of a parallelogram _____ each other.

So, W is the _____ of the diagonals \overline{TV} and \overline{SU} .

Use the _____.

Coordinates of midpoint W of

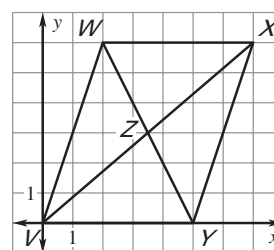
$$\overline{SU} = \left(\underline{\hspace{2cm}} \right) = \left(\underline{\hspace{2cm}} \right)$$

In Example 3, you can use either diagonal to find the coordinates of W . Using \overline{SU} simplifies calculations because one endpoint is $(0, 0)$.

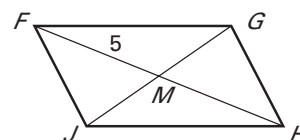
Stop and get the teacher's signature before you move on.

Checkpoint Complete the following exercises.

4. The diagonals of $\square VWXY$ intersect at point Z . Find the coordinates of Z .



5. Given that $\square FGHJ$ is a parallelogram, find MH and FH .



Homework

Complete the following problem. Make sure you justify your answer by either:

- **Checking your answer**
- **Providing a theorem, postulate, or definition**
- **Showing your work.**

The measure of one interior angle of a parallelogram is 2.6 times the measure of another angle. Find the measure of each angle.

Answer the following questions. Make sure you use full sentences and use the correct vocabulary words when appropriate.

1. Describe how section is connected to something that we learned previously this year.
2. What skills did you use in this section that you learned earlier in this class?
3. What do you think will come next?

8.3

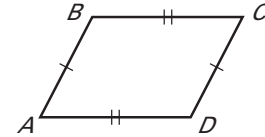
Show that a Quadrilateral is a Parallelogram

Goal • Use properties to identify parallelograms.

Rewrite the Goal as an "I can" statement!

THEOREM 8.7

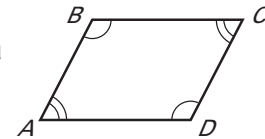
If both pairs of opposite _____ of a quadrilateral are congruent, then the quadrilateral is a parallelogram.



If $\overline{AB} \cong \overline{CD}$ and $\overline{BC} \cong \overline{AD}$, then $ABCD$ is a parallelogram.

THEOREM 8.8

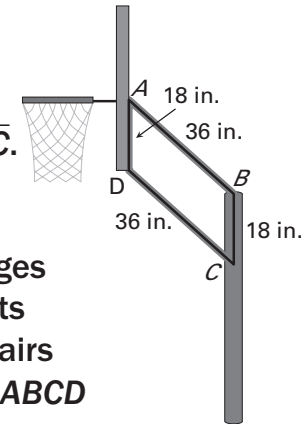
If both pairs of opposite _____ of a quadrilateral are congruent, then the quadrilateral is a parallelogram.



If $\angle A \cong \angle C$ and $\angle B \cong \angle D$, then $ABCD$ is a parallelogram.

Example 1 Solve a real-world problem

Basketball In the diagram at the right, \overline{AB} and \overline{DC} represent adjustable supports of a basketball hoop. Explain why \overline{AD} is always parallel to \overline{BC} .



Solution

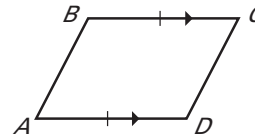
The shape of quadrilateral $ABCD$ changes as the adjustable supports move, but its _____ do not change. Both pairs of opposite _____ are congruent, so $ABCD$ is a parallelogram by _____.

By the definition of a parallelogram, $\overline{AD} \parallel \underline{\hspace{1cm}}$.

Your Notes

THEOREM 8.9

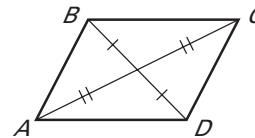
If one pair of opposite sides of a quadrilateral are _____ and _____, then the quadrilateral is a parallelogram.



If $\overline{BC} \parallel \overline{AD}$ and $\overline{BC} \cong \overline{AD}$, then $ABCD$ is a parallelogram.

THEOREM 8.10

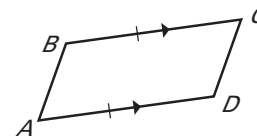
If the diagonals of a quadrilateral _____ each other, then the quadrilateral is a parallelogram.



If \overline{BD} and \overline{AC} _____ each other, then $ABCD$ is a parallelogram.

Example 2 Identify a parallelogram

Lights The headlights of a car have the shape shown at the right. Explain how you know that $\angle B \cong \angle D$.



Solution

In the diagram, $\overline{BC} \parallel \overline{AD}$ and $\overline{BC} \cong \overline{AD}$. By _____, quadrilateral $ABCD$ is a parallelogram.

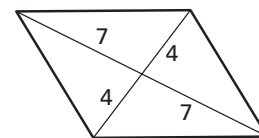
By _____, you know that opposite angles of a parallelogram are congruent. So, $\angle B \cong \angle D$.

Stop and get the teacher's signature before you move on.

Checkpoint Complete the following exercises.

1. In quadrilateral $GHJK$, $m\angle G = 55^\circ$, $m\angle H = 125^\circ$, and $m\angle J = 55^\circ$. Find $m\angle K$. What theorem can you use to show that $GHJK$ is a parallelogram?

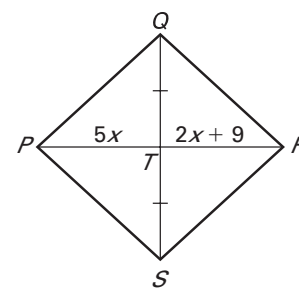
2. What theorem can you use to show that the quadrilateral is a parallelogram?



Your Notes

Example 3 Use algebra with parallelograms

For what value of x is quadrilateral $PQRS$ a parallelogram?



Solution

By Theorem 8.10, if the diagonals of $PQRS$ _____ each other, then it is a parallelogram. You are given that $\overline{QT} \cong$ _____. Find x so that $\overline{PT} \cong$ _____.

$PT =$ _____ Set the segment lengths equal.

$5x =$ _____ Substitute $5x$ for PT and _____ for _____.

_____ $x =$ _____ Subtract _____ from each side.

$x =$ _____ Divide each side by _____.

When $x =$ _____, $PT = 5(\text{_____}) =$ _____ and $RT = 2(\text{_____}) + 9 =$ _____.

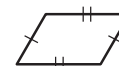
Quadrilateral $PQRS$ is a parallelogram when $x =$ _____.

CONCEPT SUMMARY: WAYS TO PROVE A QUADRILATERAL IS A PARALLELOGRAM

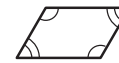
1. Show both pairs of opposite sides are parallel. (Definition)



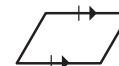
2. Show both pairs of opposite sides are congruent. (Theorem 8.7)



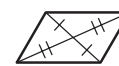
3. Show both pairs of opposite angles are congruent. (Theorem 8.8)



4. Show one pair of opposite sides are congruent and parallel. (Theorem 8.9)



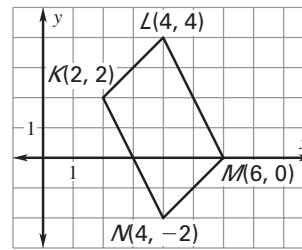
5. Show the diagonals bisect each other. (Theorem 8.10)



Your Notes

Example 4 Use coordinate geometry

Show that quadrilateral $KLMN$ is a parallelogram.



Solution

One way is to show that a pair of sides are congruent and parallel. Then apply _____.

First use the Distance Formula to show that \overline{KL} and \overline{MN} are _____.

$$KL = \sqrt{\quad} = \sqrt{\quad}$$

$$MN = \sqrt{\quad} = \sqrt{\quad}$$

Because $KL = MN = \sqrt{\quad}$, $\overline{KL} \cong \overline{MN}$.

Then use the slope formula to show that $\overline{KL} \parallel \overline{MN}$.

$$\text{Slope of } \overline{KL} = \frac{\boxed{\quad}}{\boxed{\quad}} = \underline{\quad}$$

$$\text{Slope of } \overline{MN} = \frac{\boxed{\quad}}{\boxed{\quad}} = \underline{\quad}$$

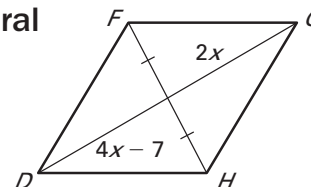
\overline{KL} and \overline{MN} have the same slope, so they are _____.

\overline{KL} and \overline{MN} are congruent and parallel. So, $KLMN$ is a parallelogram by _____.

Stop and get the teacher's signature before you move on.

Checkpoint Complete the following exercises.

3. For what value of x is quadrilateral $DFGH$ a parallelogram?



Homework

4. Explain another method that can be used to show that quadrilateral $KLMN$ in Example 4 is a parallelogram.

Complete the following problem. Make sure you justify your answer by either:

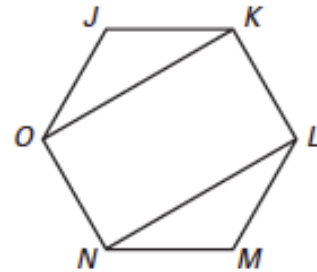
- Checking your answer
- Providing a theorem, postulate, or definition
- Showing your work.

Complete the proof.

19. GIVEN: Regular hexagon $JKLMNO$

PROVE: $OKLN$ is a parallelogram.

Statements	Reasons
1. <u>?</u>	1. Given
2. $\overline{JO} \cong \overline{NM}$ $\overline{JK} \cong \overline{ML}$ $\angle J \cong \angle M$	2. <u>?</u>
3. <u>?</u>	3. SAS Congruence Postulate
4. $\overline{OK} \cong \overline{NL}$	4. <u>?</u>
5. <u>?</u>	5. Definition of regular polygon
6. $OKLN$ is a \square .	6. <u>?</u>



Answer the following questions. Make sure you use full sentences and use the correct vocabulary words when appropriate.

1. Describe how section is connected to something that we learned previously this year.
2. What skills did you use in this section that you learned earlier in this class?
3. What do you think will come next?

8.4

Properties of Rhombuses, Rectangles, and Squares

- Goal** • Use properties of rhombuses, rectangles, and squares.

Rewrite the Goal as an "I can" statement!

Complete the vocab. with definitions or pictures that make sense to you.

VOCABULARY

Rhombus

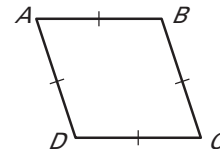
Rectangle

Square

RHOMBUS COROLLARY

A quadrilateral is a rhombus if and only if it has four congruent _____.

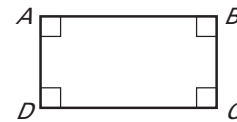
$ABCD$ is a rhombus if and only if $\overline{AB} \cong \overline{BC} \cong \overline{CD} \cong \overline{AD}$.



RECTANGLE COROLLARY

A quadrilateral is a rectangle if and only if it has four _____.

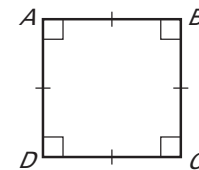
$ABCD$ is a rectangle if and only if $\angle A$, $\angle B$, $\angle C$, and $\angle D$ are right angles.



SQUARE COROLLARY

A quadrilateral is a square if and only if it is a _____ and a _____.

$ABCD$ is a square if and only if $\overline{AB} \cong \overline{BC} \cong \overline{CD} \cong \overline{AD}$ and $\angle A$, $\angle B$, $\angle C$, and $\angle D$ are right angles.



Example 1 Use properties of special quadrilaterals

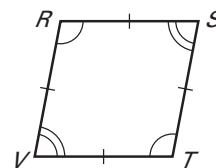
For any rhombus $RSTV$, decide whether the statement is always or sometimes true. Draw a sketch and explain your reasoning.

a. $\angle S \cong \angle V$

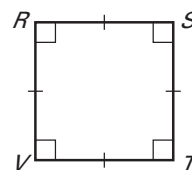
b. $\angle T \cong \angle V$

Solution

a. By definition, a rhombus is a parallelogram with four congruent _____. By Theorem 8.4, opposite angles of a parallelogram are _____. So, $\angle S \cong \angle V$. The statement is _____ true.

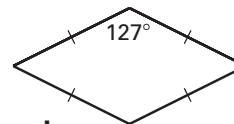


b. If rhombus $RSTV$ is a _____, then all four angles are congruent right angles. So $\angle T \cong \angle V$ if $RSTV$ is a _____. Because not all rhombuses are also _____, the statement is _____ true.



Example 2 Classify special quadrilaterals

Classify the special quadrilateral. Explain your reasoning.



The quadrilateral has four congruent _____. One of the angles is not a _____, so the rhombus is not also a _____. By the Rhombus Corollary, the quadrilateral is a _____.

Stop and get the teacher's signature before you move on.

Checkpoint Complete the following exercises.

1. For any square $CDEF$, is it *always* or *sometimes* true that $\overline{CD} \cong \overline{DE}$? Explain your reasoning.

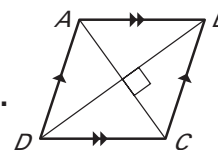
2. A quadrilateral has four congruent sides and four congruent angles. Classify the quadrilateral.

Your Notes

THEOREM 8.11

A parallelogram is a rhombus if and only if its diagonals are _____.

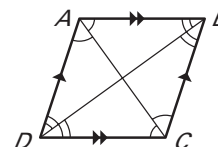
$\square ABCD$ is a rhombus if and only if _____ \perp _____.



THEOREM 8.12

A parallelogram is a rhombus if and only if each diagonal bisects a pair of opposite angles.

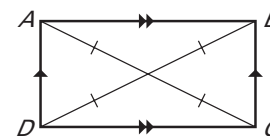
$\square ABCD$ is a rhombus if and only if \overline{AC} bisects \angle _____ and \angle _____ and \overline{BD} bisects \angle _____ and \angle _____.



THEOREM 8.13

A parallelogram is a rectangle if and only if its diagonals are _____.

$\square ABCD$ is a rectangle if and only if _____ \cong _____.



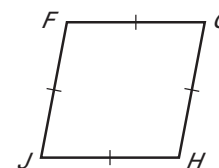
Example 3 List properties of special parallelograms

Sketch rhombus $FGHJ$. List everything you know about it.

Solution

By definition, you need to draw a figure with the following properties:

- The figure is a _____.
- The figure has four congruent _____.



Because $FGHJ$ is a parallelogram, it has these properties:

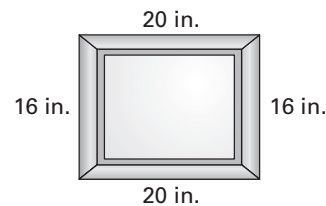
- Opposite sides are _____ and _____.
- Opposite angles are _____. Consecutive angles are _____.
- Diagonals _____ each other.

By Theorem 8.11, the diagonals of $FGHJ$ are _____. By Theorem 8.12, each diagonal bisects a pair of _____.

Your Notes

Example 4 Solve a real-world problem

Framing You are building a frame for a painting. The measurements of the frame are shown at the right.



- The frame must be a rectangle. Given the measurements in the diagram, can you assume that it is? *Explain.*
- You measure the diagonals of the frame. The diagonals are about 25.6 inches. What can you conclude about the shape of the frame?

Solution

- No, you cannot. The boards on opposite sides are the same length, so they form a _____. But you do not know whether the angles are _____.
- By Theorem 8.13, the diagonals of a rectangle are _____. The diagonals of the frame are _____, so the frame forms a _____.

Stop and get the teacher's signature before you move on.

✓ Checkpoint Complete the following exercises.

- Sketch rectangle WXYZ. List everything that you know about it.

- Suppose the diagonals of the frame in Example 4 are not congruent.

Could the frame still be a rectangle? *Explain.*

Homework

Complete the following problem. Make sure you justify your answer by either:

- Checking your answer
- Providing a theorem, postulate, or definition
- Showing your work.

Find the length or angle measure.

$WXYZ$ is a square.

$$WX = 1 - 10x$$

$$YZ = 14 + 3x$$

$$XY = \underline{\quad?}$$

Answer the following questions. Make sure you use full sentences and use the correct vocabulary words when appropriate.

1. Describe how section is connected to something that we learned previously this year.
2. What skills did you use in this section that you learned earlier in this class?
3. What do you think will come next?

8.5

Use Properties of Trapezoids and Kites

Goal • Use properties of trapezoids and kites.

Rewrite the Goal as an "I can" statement!

VOCABULARY

Trapezoid

Bases of a trapezoid

Base angles of a trapezoid

Legs of a trapezoid

Isosceles trapezoid

Midsegment of a trapezoid

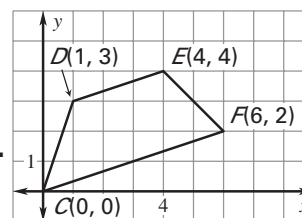
Kite

Complete the vocab. with definitions or pictures that make sense to you.

Your Notes

Example 1 Use a coordinate plane

Show that $CDEF$ is a trapezoid.



Solution

Compare the slopes of opposite sides.

Slope of \overline{DE} = _____ = _____

Slope of \overline{CF} = _____ = _____ = _____

The slopes of \overline{DE} and \overline{CF} are the same, so $\overline{DE} \parallel \overline{CF}$.

Slope of \overline{EF} = _____ = _____ = _____

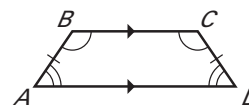
Slope of \overline{CD} = _____ = _____ = _____

The slopes of \overline{EF} and \overline{CD} are not the same, so \overline{EF} is _____ to \overline{CD} .

Because quadrilateral $CDEF$ has exactly one pair of _____, it is a trapezoid.

THEOREM 8.14

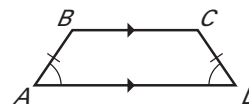
If a trapezoid is isosceles, then each pair of base angles is _____.



If trapezoid $ABCD$ is isosceles, then $\angle A \cong \angle$ _____ and \angle _____ $\cong \angle C$.

THEOREM 8.15

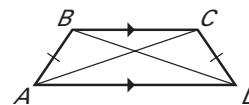
If a trapezoid has a pair of congruent _____, then it is an isosceles trapezoid.



If $\angle A \cong \angle D$ (or if $\angle B \cong \angle C$), then trapezoid $ABCD$ is isosceles.

THEOREM 8.16

A trapezoid is isosceles if and only if its diagonals are _____.

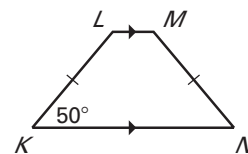


Trapezoid $ABCD$ is isosceles if and only if _____ \cong _____.

Your Notes

Example 2 Use properties of isosceles trapezoids

Kitchen A shelf fitting into a cupboard in the corner of a kitchen is an isosceles trapezoid. Find $m\angle N$, $m\angle L$, and $m\angle M$.



Solution

Step 1 Find $m\angle N$. $KLMN$ is an _____, so $\angle N$ and \angle _____ are congruent base angles, and $m\angle N = m\angle$ _____ = _____.

Step 2 Find $m\angle L$. Because $\angle K$ and $\angle L$ are consecutive interior angles formed by \overleftrightarrow{KL} intersecting two parallel lines, they are _____. So, $m\angle L =$ _____ - _____ = _____.

Step 3 Find $m\angle M$. Because $\angle M$ and \angle _____ are a pair of base angles, they are congruent, and $m\angle M = m\angle$ _____ = _____.

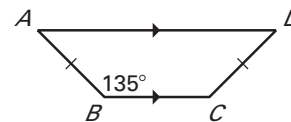
So, $m\angle N =$ _____, $m\angle L =$ _____, and $m\angle M =$ _____.

Stop and get the teacher's signature before you move on.

✓ Checkpoint Complete the following exercises.

1. In Example 1, suppose the coordinates of point E are $(7, 5)$. What type of quadrilateral is $CDEF$? Explain.

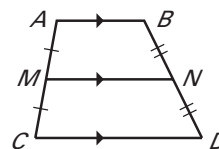
2. Find $m\angle C$, $m\angle A$, and $m\angle D$ in the trapezoid shown.



Your Notes

THEOREM 8.17: MIDSEGMENT THEOREM FOR TRAPEZOIDS

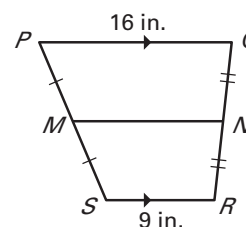
The midsegment of a trapezoid is parallel to each base and its length is one half the sum of the lengths of the bases.



If \overline{MN} is the midsegment of trapezoid $ABCD$, then $\overline{MN} \parallel \underline{\hspace{2cm}}$, $\overline{MN} \parallel \underline{\hspace{2cm}}$, and $MN = \underline{\hspace{1cm}} (\underline{\hspace{1cm}} + \underline{\hspace{1cm}})$.

Example 3 Use the midsegment of a trapezoid

In the diagram, \overline{MN} is the midsegment of trapezoid $PQRS$. Find MN .



Solution

Use Theorem 8.17 to find MN .

$MN = \underline{\hspace{1cm}} (\underline{\hspace{1cm}} + \underline{\hspace{1cm}})$ Apply Theorem 8.17.

$= \underline{\hspace{1cm}} (\underline{\hspace{1cm}} + \underline{\hspace{1cm}})$ Substitute $\underline{\hspace{1cm}}$ for PQ and $\underline{\hspace{1cm}}$ for SR .

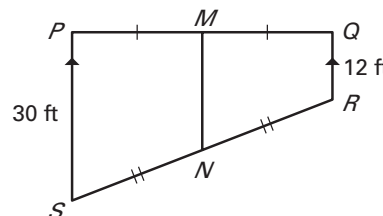
$= \underline{\hspace{1cm}}$ Simplify.

The length MN is $\underline{\hspace{1cm}}$ inches.

Stop and get the teacher's signature before you move on.

Checkpoint Complete the following exercise.

3. Find MN in the trapezoid at the right.

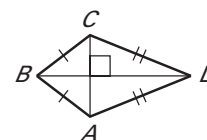


Your Notes

THEOREM 8.18

If a quadrilateral is a kite, then its diagonals are _____.

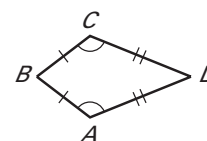
If quadrilateral $ABCD$ is a kite, then _____ \perp _____.



THEOREM 8.19

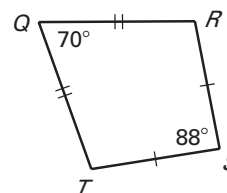
If a quadrilateral is a kite, then exactly one pair of opposite angles are congruent.

If quadrilateral $ABCD$ is a kite and $\overline{BC} \cong \overline{BA}$, then $\angle A$ _____ $\angle C$ and $\angle B$ _____ $\angle D$.



Example 4 Apply Theorem 8.19

Find $m\angle T$ in the kite shown at the right.



Solution

By Theorem 8.19, $QRST$ has exactly one pair of _____ opposite angles.

Because $\angle Q \neq \angle S$, \angle _____ and $\angle T$ must be congruent. So, $m\angle$ _____ = $m\angle T$. Write and solve an equation to find $m\angle T$.

$$m\angle T + m\angle R + \underline{\hspace{1cm}} + \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$$

Corollary to Theorem 8.1

$$m\angle T + m\angle T + \underline{\hspace{1cm}} + \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$$

Substitute $m\angle T$ for $m\angle R$.

$$\underline{\hspace{1cm}}(m\angle T) + \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$$

Combine like terms.

$$m\angle T = \underline{\hspace{1cm}}$$

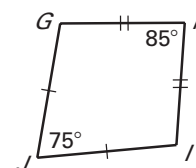
Solve for $m\angle T$.

Stop and get the teacher's signature before you move on.

Homework

Checkpoint Complete the following exercise.

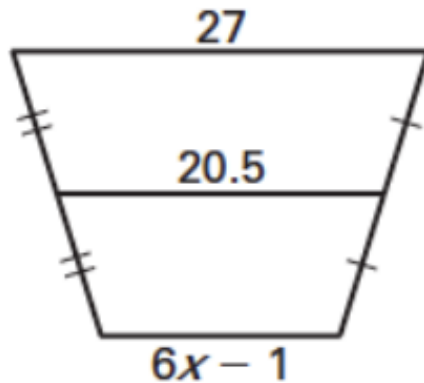
4. Find $m\angle G$ in the kite shown at the right.



Complete the following problem. Make sure you justify your answer by either:

- Checking your answer
- Providing a theorem, postulate, or definition
- Showing your work.

Find the value of x .



Answer the following questions. Make sure you use full sentences and use the correct vocabulary words when appropriate.

1. Describe how section is connected to something that we learned previously this year.
2. What skills did you use in this section that you learned earlier in this class?
3. What do you think will come next?

8.6

Identify Special Quadrilaterals

Goal • Identify special quadrilaterals.

Rewrite the Goal as an "I can" statement!

Stop and get the teacher's signature before you move on.

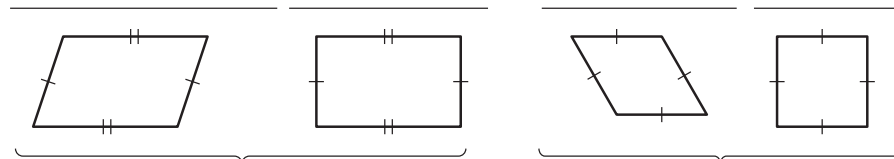
Complete the vocab. with definitions or pictures that make sense to you.

Example 1 Identify quadrilaterals

Quadrilateral $ABCD$ has both pairs of opposite sides congruent. What types of quadrilaterals meet this condition?

Solution

There are many possibilities.



Opposite sides are congruent.

All sides are congruent.

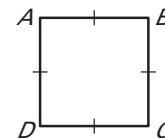
✓ **Checkpoint** Complete the following exercise.

1. Quadrilateral $JKLM$ has both pairs of opposite angles congruent. What types of quadrilaterals meet this condition?

In Example 2, $ABCD$ is shaped like a square. But you must rely only on marked information when you interpret a diagram.

Example 2 Identify a quadrilateral

What is the most specific name for quadrilateral $ABCD$?



Solution

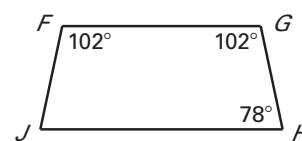
The diagram shows that both pairs of opposite sides are congruent. By Theorem 8.7, $ABCD$ is a _____ . All sides are congruent, so $ABCD$ is a _____ by definition.

_____ are also rhombuses. However, there is no information given about the angle measures of $ABCD$. So, you cannot determine whether it is a _____ .

Your Notes

Example 3 Identify a quadrilateral

Is enough information given in the diagram to show that quadrilateral $FGHJ$ is an isosceles trapezoid? Explain.



Solution

Step 1 Show that $FGHJ$ is a _____. $\angle G$ and $\angle H$ are _____ but $\angle F$ and $\angle G$ are not.

So, _____ \parallel _____, but \overline{FJ} is not _____ to \overline{GH} .

By definition, $FGHJ$ is a _____.

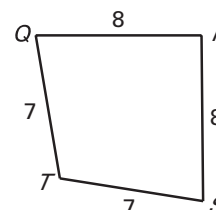
Step 2 Show that trapezoid $FGHJ$ is _____. $\angle F$ and $\angle G$ are a pair of congruent _____. So, $FGHJ$ is an _____ by Theorem 8.15.

Yes, the diagram is sufficient to show that $FGHJ$ is an isosceles trapezoid.

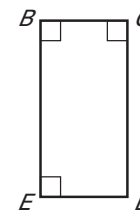
Stop and get the teacher's signature before you move on.

✔ **Checkpoint** Complete the following exercises.

2. What is the most specific name for quadrilateral $QRST$? Explain your reasoning.



3. Is enough information given in the diagram to show that quadrilateral $BCDE$ is a rectangle? Explain.



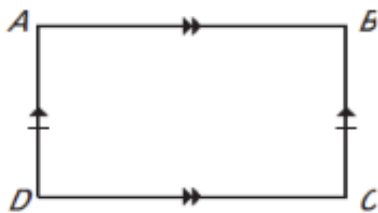
Homework

Complete the following problem. Make sure you justify your answer by either:

- Checking your answer
- Providing a theorem, postulate, or definition
- Showing your work.

Which pairs of segments or angles must be congruent so that you can prove that $ABCD$ is the indicated quadrilateral? *Explain*. There may be more than one right answer.

Rectangle



Answer the following questions. Make sure you use full sentences and use the correct vocabulary words when appropriate.

1. Describe how section is connected to something that we learned previously this year.
2. What skills did you use in this section that you learned earlier in this class?
3. What do you think will come next?