4.1 Apply Triangle Sum Properties

Before You classified angles and found their measures.

Now You will classify triangles and find measures of their angles.

Why? So you can place actors on stage, as in Ex. 40.

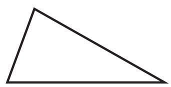
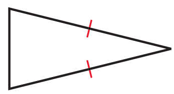
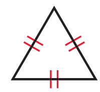
Key Vocabulary
- **triangle**
  scalene, isosceles, equilateral, acute, right, obtuse, equiangular
- **interior angles**
- **exterior angles**
- **corollary to a theorem**

A **triangle** is a polygon with three sides. A triangle with vertices *A*, *B*, and *C* is called "triangle *ABC*" or "$\triangle ABC$."

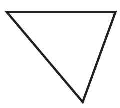
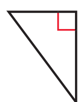
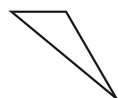
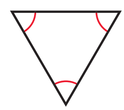
READ VOCABULARY
Notice that an equilateral triangle is also isosceles. An equiangular triangle is also acute.

KEY CONCEPT — *For Your Notebook*

Classifying Triangles by Sides

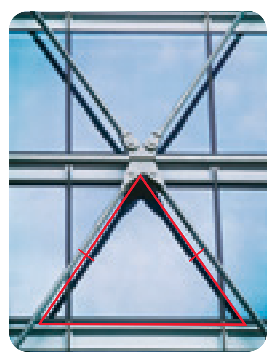
Scalene Triangle	Isosceles Triangle	Equilateral Triangle
		
No congruent sides	At least 2 congruent sides	3 congruent sides

Classifying Triangles by Angles

Acute Triangle	Right Triangle	Obtuse Triangle	Equiangular Triangle
			
3 acute angles	1 right angle	1 obtuse angle	3 congruent angles

EXAMPLE 1 Classify triangles by sides and by angles

SUPPORT BEAMS Classify the triangular shape of the support beams in the diagram by its sides and by measuring its angles.

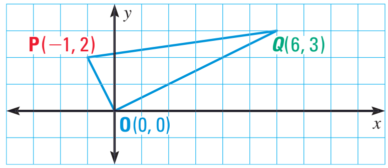


Solution

The triangle has a pair of congruent sides, so it is isosceles. By measuring, the angles are 55°, 55°, and 70°. It is an acute isosceles triangle.

EXAMPLE 2 Classify a triangle in a coordinate plane

Classify $\triangle PQO$ by its sides. Then determine if the triangle is a right triangle.



Solution

STEP 1 **Use** the distance formula to find the side lengths.

$$OP = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{((-1) - 0)^2 + (2 - 0)^2} = \sqrt{5} \approx 2.2$$

$$OQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(6 - 0)^2 + (3 - 0)^2} = \sqrt{45} \approx 6.7$$

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(6 - (-1))^2 + (3 - 2)^2} = \sqrt{50} \approx 7.1$$

STEP 2 **Check** for right angles. The slope of $\overline{OP}$ is $\frac{2 - 0}{-1 - 0} = -2$. The slope of $\overline{OQ}$ is $\frac{3 - 0}{6 - 0} = \frac{1}{2}$. The product of the slopes is $-2\left(\frac{1}{2}\right) = -1$, so $\overline{OP} \perp \overline{OQ}$ and $\angle POQ$ is a right angle.

▸ Therefore, $\triangle PQO$ is a right scalene triangle.

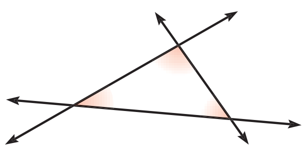
✓ GUIDED PRACTICE for Examples 1 and 2

1. Draw an obtuse isosceles triangle and an acute scalene triangle. **See margin.**

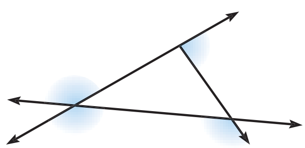
2. Triangle ABC has the vertices $A(0, 0)$, $B(3, 3)$, and $C(-3, 3)$. Classify it by its sides. Then determine if it is a right triangle. **isosceles; right triangle**

ANGLES When the sides of a polygon are extended, other angles are formed. The original angles are the **interior angles**. The angles that form linear pairs with the interior angles are the **exterior angles**.

READ DIAGRAMS
Each vertex has a *pair* of congruent exterior angles. However, it is common to show only *one* exterior angle at each vertex.



interior angles



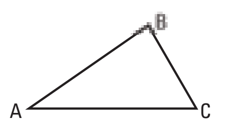
exterior angles

THEOREM *For Your Notebook*

THEOREM 4.1 Triangle Sum Theorem

The sum of the measures of the interior angles of a triangle is 180°.

Proof: p. 219; Ex. 53, p. 224



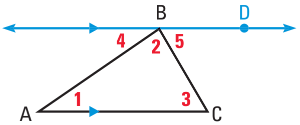
$m\angle A + m\angle B + m\angle C = 180°$

AUXILIARY LINES To prove certain theorems, you may need to add a line, a segment, or a ray to a given diagram. An *auxiliary* line is used in the proof of the Triangle Sum Theorem.

PROOF Triangle Sum Theorem

GIVEN ▶ $\triangle ABC$

PROVE ▶ $m\angle 1 + m\angle 2 + m\angle 3 = 180°$



Plan for Proof

a. Draw an auxiliary line through B and parallel to $\overline{AC}$.

b. Show that $m\angle 4 + m\angle 2 + m\angle 5 = 180°$, $\angle 1 \cong \angle 4$, and $\angle 3 \cong \angle 5$.

c. By substitution, $m\angle 1 + m\angle 2 + m\angle 3 = 180°$.

Plan in Action

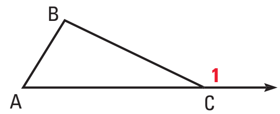
	STATEMENTS	REASONS
a.	1. Draw $\overleftrightarrow{BD}$ parallel to $\overline{AC}$.	1. Parallel Postulate
b.	2. $m\angle 4 + m\angle 2 + m\angle 5 = 180°$	2. Angle Addition Postulate and definition of straight angle
	3. $\angle 1 \cong \angle 4$, $\angle 3 \cong \angle 5$	3. Alternate Interior Angles Theorem
	4. $m\angle 1 = m\angle 4$, $m\angle 3 = m\angle 5$	4. Definition of congruent angles
c.	5. $m\angle 1 + m\angle 2 + m\angle 3 = 180°$	5. Substitution Property of Equality

THEOREM *For Your Notebook*

THEOREM 4.2 Exterior Angle Theorem

The measure of an exterior angle of a triangle is equal to the sum of the measures of the two nonadjacent interior angles.

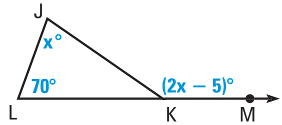
Proof: Ex. 50, p. 223



$m\angle 1 = m\angle A + m\angle B$

EXAMPLE 3 Find an angle measure

ALGEBRA **Find $m\angle JKM$.**



Solution

STEP 1 **Write** and solve an equation to find the value of x.

$(2x - 5)° = 70° + x°$ — **Apply the Exterior Angle Theorem.**

$x = 75$ — **Solve for x.**

STEP 2 **Substitute** 75 for x in $2x - 5$ to find $m\angle JKM$.

$2x - 5 = 2 \cdot 75 - 5 = 145$

▶ The measure of $\angle JKM$ is 145°.

Differentiated Instruction

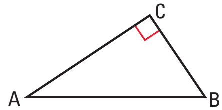
A **corollary to a theorem** is a statement that can be proved easily using the theorem. The corollary below follows from the Triangle Sum Theorem.

COROLLARY *For Your Notebook*

Corollary to the Triangle Sum Theorem

The acute angles of a right triangle are complementary.

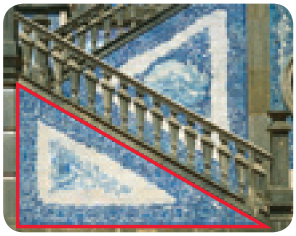
Proof: Ex. 48, p. 223



$m\angle A + m\angle B = 90°$

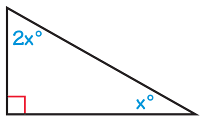
EXAMPLE 4 Find angle measures from a verbal description

ARCHITECTURE The tiled staircase shown forms a right triangle. The measure of one acute angle in the triangle is twice the measure of the other. Find the measure of each acute angle.



Solution

First, sketch a diagram of the situation. Let the measure of the smaller acute angle be $x°$. Then the measure of the larger acute angle is $2x°$. The Corollary to the Triangle Sum Theorem states that the acute angles of a right triangle are complementary.



Use the corollary to set up and solve an equation.

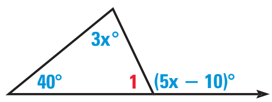
$x° + 2x° = 90°$ **Corollary to the Triangle Sum Theorem**

$x = 30$ **Solve for *x*.**

▶ So, the measures of the acute angles are $30°$ and $2(30°) = 60°$.

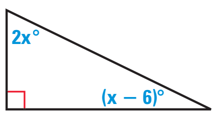
✓ GUIDED PRACTICE for Examples 3 and 4

3. Find the measure of $\angle 1$ in the diagram shown. **65°**



4. Find the measure of each interior angle of $\triangle ABC$, where $m\angle A = x°$, $m\angle B = 2x°$, and $m\angle C = 3x°$. **$m\angle A = 30°, m\angle B = 60°, m\angle C = 90°$**

5. Find the measures of the acute angles of the right triangle in the diagram shown. **26°, 64°**



6. In Example 4, what is the measure of the obtuse angle formed between the staircase and a segment extending from the horizontal leg? **150°**

4.2 Apply Congruence and Triangles

Before You identified congruent angles.

Now You will identify congruent figures.

Why? So you can determine if shapes are identical, as in Example 3.

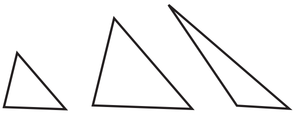
Key Vocabulary
- **congruent figures**
- **corresponding parts**

Two geometric figures are *congruent* if they have exactly the same size and shape. Imagine cutting out one of the congruent figures. You could then position the cut-out figure so that it fits perfectly onto the other figure.

Congruent

Same size and shape

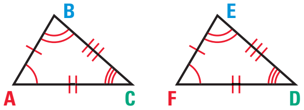
Not congruent



Different sizes or shapes

In two **congruent figures**, all the parts of one figure are congruent to the **corresponding parts** of the other figure. In congruent polygons, this means that the *corresponding sides* and the *corresponding angles* are congruent.

CONGRUENCE STATEMENTS When you write a congruence statement for two polygons, always list the corresponding vertices in the same order. You can write congruence statements in more than one way. Two possible congruence statements for the triangles at the right are $\triangle ABC \cong \triangle FED$ or $\triangle BCA \cong \triangle EDF$.

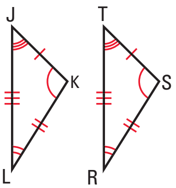


Corresponding angles $\angle A \cong \angle F$ $\angle B \cong \angle E$ $\angle C \cong \angle D$

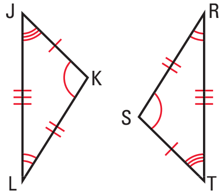
Corresponding sides $\overline{AB} \cong \overline{FE}$ $\overline{BC} \cong \overline{ED}$ $\overline{AC} \cong \overline{FD}$

EXAMPLE 1 Identify congruent parts

VISUAL REASONING
To help you identify corresponding parts, turn $\triangle RST$.



Write a congruence statement for the triangles. Identify all pairs of congruent corresponding parts.



Solution

The diagram indicates that $\triangle JKL \cong \triangle TSR$.

Corresponding angles $\angle J \cong \angle T$, $\angle K \cong \angle S$, $\angle L \cong \angle R$

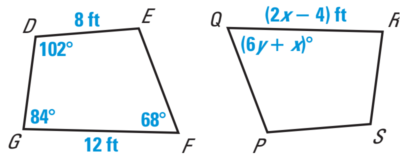
Corresponding sides $\overline{JK} \cong \overline{TS}$, $\overline{KL} \cong \overline{SR}$, $\overline{LJ} \cong \overline{RT}$

EXAMPLE 2 Use properties of congruent figures

In the diagram, *DEFG* ≅ *SPQR*.

a. Find the value of x.

b. Find the value of y.



Solution

a. You know that $\overline{FG} \cong \overline{QR}$.

$FG = QR$

$12 = 2x - 4$

$16 = 2x$

$8 = x$

b. You know that $\angle F \cong \angle Q$.

$m\angle F = m\angle Q$

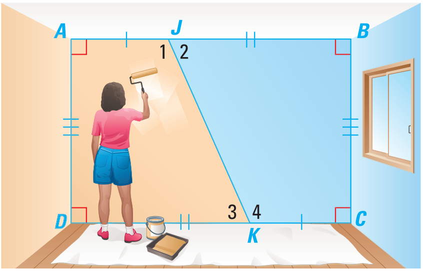
$68° = (6y + x)°$

$68 = 6y + 8$

$10 = y$

EXAMPLE 3 Show that figures are congruent

PAINTING If you divide the wall into orange and blue sections along $\overline{JK}$, will the sections of the wall be the same size and shape? *Explain.*



Solution

From the diagram, $\angle A \cong \angle C$ and $\angle D \cong \angle B$ because all right angles are congruent. Also, by the Lines Perpendicular to a Transversal Theorem, $\overline{AB} \parallel \overline{DC}$. Then, $\angle 1 \cong \angle 4$ and $\angle 2 \cong \angle 3$ by the Alternate Interior Angles Theorem. So, all pairs of corresponding angles are congruent.

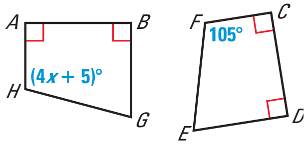
The diagram shows $\overline{AJ} \cong \overline{CK}$, $\overline{KD} \cong \overline{JB}$, and $\overline{DA} \cong \overline{BC}$. By the Reflexive Property, $\overline{JK} \cong \overline{KJ}$. All corresponding parts are congruent, so *AJKD* ≅ *CKJB*.

▶ Yes, the two sections will be the same size and shape.

✓ GUIDED PRACTICE for Examples 1, 2, and 3

1. $\overline{AB} \cong \overline{CD}$, $\overline{BG} \cong \overline{DE}$, $\overline{GH} \cong \overline{EF}$, $\overline{HA} \cong \overline{FC}$, $\angle A \cong \angle C$, $\angle B \cong \angle D$, $\angle G \cong \angle E$, $\angle H \cong \angle F$

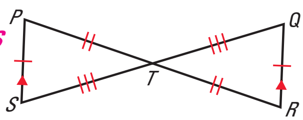
In the diagram at the right, *ABGH* ≅ *CDEF*.



1. Identify all pairs of congruent corresponding parts.

2. Find the value of x and find $m\angle H$. **25, 105°**

3. Show that $\triangle PTS \cong \triangle RTQ$.
All of the corresponding parts of △*PTS* are congruent to those of △*RTQ* by the indicated markings, the Vertical Angles Theorem, and the Alternate Interior Angles Theorem.

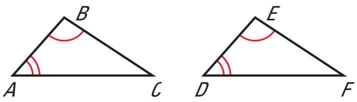


THEOREM — *For Your Notebook*

THEOREM 4.3 Third Angles Theorem

If two angles of one triangle are congruent to two angles of another triangle, then the third angles are also congruent.

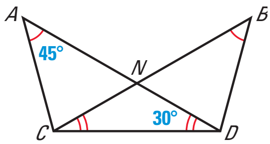
Proof: Ex. 28, p. 230



If $\angle A \cong \angle D$, and $\angle B \cong \angle E$, then $\angle C \cong \angle F$.

EXAMPLE 4 Use the Third Angles Theorem

Find $m\angle BDC$.



Solution

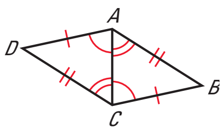
$\angle A \cong \angle B$ and $\angle ADC \cong \angle BCD$, so by the Third Angles Theorem, $\angle ACD \cong \angle BDC$. By the Triangle Sum Theorem, $m\angle ACD = 180° - 45° - 30° = 105°$.

▶ So, $m\angle ACD = m\angle BDC = 105°$ by the definition of congruent angles.

ANOTHER WAY
For an alternative method for solving the problem in Example 4, turn to page 232 for the **Problem Solving Workshop**.

EXAMPLE 5 Prove that triangles are congruent

Write a proof.



GIVEN ▶ $\overline{AD} \cong \overline{CB}$, $\overline{DC} \cong \overline{BA}$, $\angle ACD \cong \angle CAB$, $\angle CAD \cong \angle ACB$

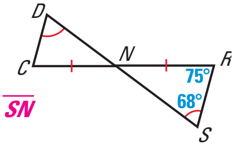
PROVE ▶ $\triangle ACD \cong \triangle CAB$

Plan for Proof
a. Use the Reflexive Property to show that $\overline{AC} \cong \overline{AC}$.
b. Use the Third Angles Theorem to show that $\angle B \cong \angle D$.

Plan in Action

	STATEMENTS	REASONS
	1. $\overline{AD} \cong \overline{CB}$, $\overline{DC} \cong \overline{BA}$	**1.** Given
a.	**2.** $\overline{AC} \cong \overline{AC}$	**2.** Reflexive Property of Congruence
	3. $\angle ACD \cong \angle CAB$, $\angle CAD \cong \angle ACB$	**3.** Given
b.	**4.** $\angle B \cong \angle D$	**4.** Third Angles Theorem
	5. $\triangle ACD \cong \triangle CAB$	**5.** Definition of $\cong$ $\triangle$s

✓ GUIDED PRACTICE for Examples 4 and 5



4. In the diagram, what is $m\angle DCN$? **75°**

5. By the definition of congruence, what additional information is needed to know that $\triangle NDC \cong \triangle NSR$? **$\overline{DC} \cong \overline{SR}$ and $\overline{DN} \cong \overline{SN}$**

Differentiated Instruction

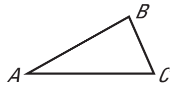
PROPERTIES OF CONGRUENT TRIANGLES The properties of congruence that are true for segments and angles are also true for triangles.

THEOREM *For Your Notebook*

THEOREM 4.4 Properties of Congruent Triangles

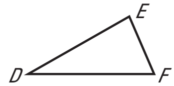
Reflexive Property of Congruent Triangles

For any triangle ABC, $\triangle ABC \cong \triangle ABC$.



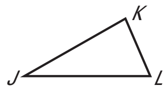
Symmetric Property of Congruent Triangles

If $\triangle ABC \cong \triangle DEF$, then $\triangle DEF \cong \triangle ABC$.



Transitive Property of Congruent Triangles

If $\triangle ABC \cong \triangle DEF$ and $\triangle DEF \cong \triangle JKL$, then $\triangle ABC \cong \triangle JKL$.



4.2 EXERCISES

HOMEWORK KEY

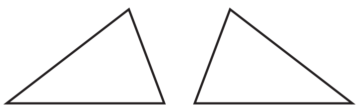
○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 9, 15, and 25

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 18, 21, 24, 27, and 30

SKILL PRACTICE

A

1. **VOCABULARY** Copy the congruent triangles shown. Then label the vertices of the triangles so that $\triangle JKL \cong \triangle RST$. Identify all pairs of congruent *corresponding angles* and *corresponding sides*.
   $\overline{JK} \cong \overline{RS}$, $\overline{KL} \cong \overline{ST}$, $\overline{JL} \cong \overline{RT}$, $\angle J \cong \angle R$, $\angle K \cong \angle S$, $\angle L \cong \angle T$; see margin for art.

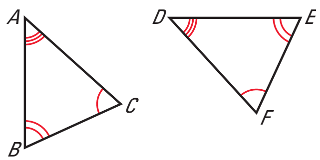


2. ★ **WRITING** Based on this lesson, what information do you need to prove that two triangles are congruent? *Explain*. **See margin.**

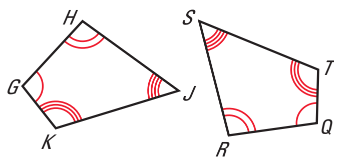
EXAMPLE 1 on p. 225 for Exs. 3–4

USING CONGRUENCE Identify all pairs of congruent corresponding parts. Then write another congruence statement for the figures. **3, 4. See margin.**

3. $\triangle ABC \cong \triangle DEF$



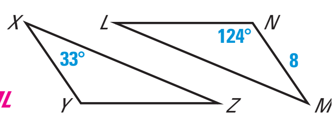
4. $GHJK \cong QRST$



EXAMPLE 2 on p. 226 for Exs. 5–10

READING A DIAGRAM In the diagram, $\triangle XYZ \cong \triangle MNL$. Copy and complete the statement.

5. $m\angle Y =$ _?_ **124°**
6. $m\angle M =$ _?_ **33°**
7. $YX =$ _?_ **8**
8. $\overline{YZ} \cong$ _?_ **$\overline{NL}$**
9. $\triangle LNM \cong$ _?_ **$\triangle ZYX$**
10. $\triangle YXZ \cong$ _?_ **$\triangle NML$**



4.3 Prove Triangles Congruent by SSS

Before	You used the definition of congruent figures.
Now	You will use the side lengths to prove triangles are congruent.
Why	So you can determine if triangles in a tile floor are congruent, as in Ex. 22.

Key Vocabulary
- **congruent figures,** *p. 225*
- **corresponding parts,** *p. 225*

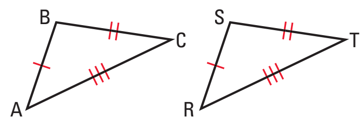
In the Activity on page 233, you saw that there is only one way to form a triangle given three side lengths. In general, any two triangles with the same three side lengths must be congruent.

POSTULATE *For Your Notebook*

POSTULATE 19 Side-Side-Side (SSS) Congruence Postulate

If three sides of one triangle are congruent to three sides of a second triangle, then the two triangles are congruent.

If Side $\overline{AB} \cong \overline{RS}$,
Side $\overline{BC} \cong \overline{ST}$, and
Side $\overline{CA} \cong \overline{TR}$,
then $\triangle ABC \cong \triangle RST$.

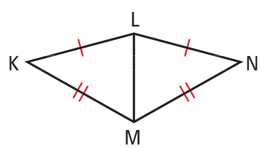


EXAMPLE 1 Use the SSS Congruence Postulate

Write a proof.

GIVEN ▶ $\overline{KL} \cong \overline{NL}$, $\overline{KM} \cong \overline{NM}$

PROVE ▶ $\triangle KLM \cong \triangle NLM$



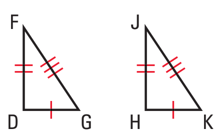
Proof It is given that $\overline{KL} \cong \overline{NL}$ and $\overline{KM} \cong \overline{NM}$. By the Reflexive Property, $\overline{LM} \cong \overline{LM}$. So, by the SSS Congruence Postulate, $\triangle KLM \cong \triangle NLM$.

AnimatedGeometry at classzone.com

✓ GUIDED PRACTICE for Example 1

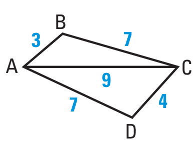
Decide whether the congruence statement is true. *Explain* your reasoning.

1. $\triangle DFG \cong \triangle HJK$



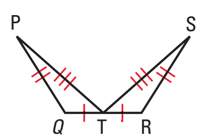
yes; SSS

2. $\triangle ACB \cong \triangle CAD$



2. No; corresponding sides are not congruent.

3. $\triangle QPT \cong \triangle RST$



yes; SSS

EXAMPLE 2 Standardized Test Practice

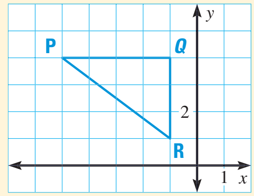
Which are the coordinates of the vertices of a triangle congruent to $\triangle PQR$?

Ⓐ $(-1, 1), (-1, 5), (-4, 5)$

Ⓑ $(-2, 4), (-7, 4), (-4, 6)$

Ⓒ $(-3, 2), (-1, 3), (-3, 1)$

Ⓓ $(-7, 7), (-7, 9), (-3, 7)$



Solution

ELIMINATE CHOICES
Once you know the side lengths of $\triangle PQR$, look for pairs of coordinates with the same x-coordinates or the same y-coordinates. In Choice C, $(-3, 2)$ and $(-3, 1)$ are only 1 unit apart. You can eliminate D in the same way.

By counting, $PQ = 4$ and $QR = 3$. Use the Distance Formula to find PR.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$PR = \sqrt{(-1 - (-5))^2 + (1 - 4)^2} = \sqrt{4^2 + (-3)^2} = \sqrt{25} = 5$$

By the SSS Congruence Postulate, any triangle with side lengths 3, 4, and 5 will be congruent to $\triangle PQR$. The distance from $(-1, 1)$ to $(-1, 5)$ is 4. The distance from $(-1, 5)$ to $(-4, 5)$ is 3. The distance from $(-1, 1)$ to $(-4, 5)$ is $\sqrt{(5 - 1)^2 + ((-4) - (-1))^2} = \sqrt{4^2 + (-3)^2} = \sqrt{25} = 5$.

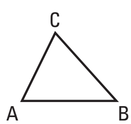
▸ The correct answer is A. Ⓐ Ⓑ Ⓒ Ⓓ

✓ GUIDED PRACTICE for Example 2

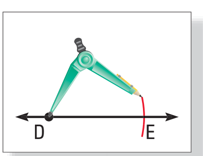
4. $\triangle JKL$ has vertices $J(-3, -2)$, $K(0, -2)$, and $L(-3, -8)$. $\triangle RST$ has vertices $R(10, 0)$, $S(10, -3)$, and $T(4, 0)$. Graph the triangles in the same coordinate plane and show that they are congruent. **$KJ = SR = 3$, $JL = RT = 6$, $LK = TS = 3\sqrt{5}$; see margin for art.**

ACTIVITY COPY A TRIANGLE

Follow the steps below to construct a triangle that is congruent to $\triangle ABC$.

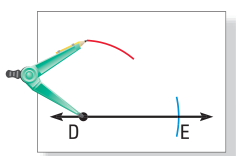


STEP 1



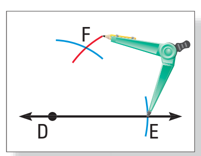
Construct $\overline{DE}$ so that it is congruent to $\overline{AB}$.

STEP 2



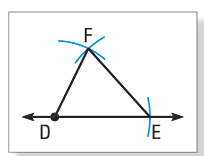
Open your compass to the length AC. Use this length to draw an arc with the compass point at D.

STEP 3



Draw an arc with radius BC and center E that intersects the arc from Step 2. Label the intersection point F.

STEP 4



Draw $\triangle DEF$. By the SSS Congruence Postulate, $\triangle ABC \cong \triangle DEF$.

EXAMPLE 3 Solve a real-world problem

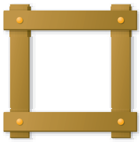
STRUCTURAL SUPPORT Explain why the bench with the diagonal support is stable, while the one without the support can collapse.

Solution

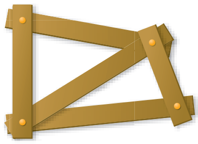
The bench with a diagonal support forms triangles with fixed side lengths. By the SSS Congruence Postulate, these triangles cannot change shape, so the bench is stable. The bench without a diagonal support is not stable because there are many possible quadrilaterals with the given side lengths.

✓ GUIDED PRACTICE for Example 3

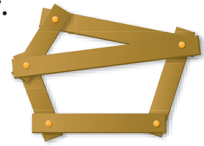
Determine whether the figure is stable. *Explain* your reasoning.

5. 

Not stable; a figure without diagonal support is not stable.

6. 

Stable; the figure has diagonal support with fixed side lengths.

7. 

Not stable; the lower half of the figure does not have diagonal support.

4.3 EXERCISES

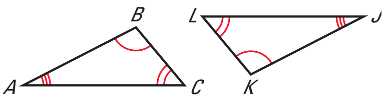
HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 7, 9, and 25

★ = **STANDARDIZED TEST PRACTICE** Exs. 16, 17, and 28

SKILL PRACTICE

A **VOCABULARY** **Tell whether the angles or sides are *corresponding angles*, *corresponding sides*, or *neither*.**

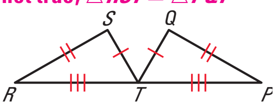


1. $\angle C$ and $\angle L$
   corresponding angles
2. $\overline{AC}$ and $\overline{JK}$ neither
3. $\overline{BC}$ and $\overline{KL}$
   corresponding sides
4. $\angle B$ and $\angle L$
   neither

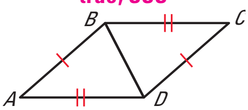
EXAMPLE 1 on p. 234 for Exs. 5–7

DETERMINING CONGRUENCE **Decide whether the congruence statement is true. *Explain* your reasoning.**

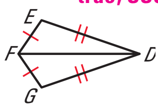
5. $\triangle RST \cong \triangle TQP$
   not true; $\triangle RST \cong \triangle PQT$



6. $\triangle ABD \cong \triangle CDB$
   true; SSS



7. $\triangle DEF \cong \triangle DGF$
   true; SSS



4.4 Prove Triangles Congruent by SAS and HL

Before You used the SSS Congruence Postulate.

Now You will use sides and angles to prove congruence.

Why? So you can show triangles are congruent, as in Ex. 33.

Key Vocabulary
- **leg of a right triangle**
- **hypotenuse**

Consider a relationship involving two sides and the angle they form, their *included* angle. To picture the relationship, form an angle using two pencils.

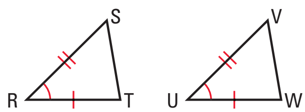
Any time you form an angle of the same measure with the pencils, the side formed by connecting the pencil points will have the same length. In fact, any two triangles formed in this way are congruent.

POSTULATE — *For Your Notebook*

POSTULATE 20 Side-Angle-Side (SAS) Congruence Postulate

If two sides and the included angle of one triangle are congruent to two sides and the included angle of a second triangle, then the two triangles are congruent.

If **S**ide $\overline{RS} \cong \overline{UV}$,
Angle $\angle R \cong \angle U$, and
Side $\overline{RT} \cong \overline{UW}$,
then $\triangle RST \cong \triangle UVW$.

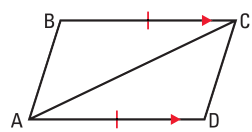


EXAMPLE 1 Use the SAS Congruence Postulate

Write a proof.

GIVEN ▶ $\overline{BC} \cong \overline{DA}$, $\overline{BC} \parallel \overline{AD}$

PROVE ▶ $\triangle ABC \cong \triangle CDA$

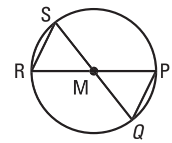


WRITE PROOFS
Make your proof easier to read by identifying the steps where you show congruent sides (S) and angles (A).

	STATEMENTS	REASONS
S	1. $\overline{BC} \cong \overline{DA}$	1. Given
	2. $\overline{BC} \parallel \overline{AD}$	2. Given
A	3. $\angle BCA \cong \angle DAC$	3. Alternate Interior Angles Theorem
S	4. $\overline{AC} \cong \overline{CA}$	4. Reflexive Property of Congruence
	5. $\triangle ABC \cong \triangle CDA$	5. SAS Congruence Postulate

EXAMPLE 2 Use SAS and properties of shapes

In the diagram, $\overline{QS}$ and $\overline{RP}$ pass through the center M of the circle. What can you conclude about $\triangle MRS$ and $\triangle MPQ$?



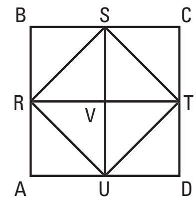
Solution

Because they are vertical angles, $\angle PMQ \cong \angle RMS$. All points on a circle are the same distance from the center, so MP, MQ, MR, and MS are all equal.

▶ $\triangle MRS$ and $\triangle MPQ$ are congruent by the SAS Congruence Postulate.

✓ GUIDED PRACTICE for Examples 1 and 2

In the diagram, $ABCD$ is a square with four congruent sides and four right angles. R, S, T, and U are the midpoints of the sides of $ABCD$. Also, $\overline{RT} \perp \overline{SU}$ and $\overline{SV} \cong \overline{VU}$.



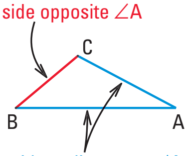
1. Prove that $\triangle SVR \cong \triangle UVR$. **See margin.**
2. Prove that $\triangle BSR \cong \triangle DUT$. **See margin.**

In general, if you know the lengths of two sides and the measure of an angle that is *not included* between them, you can create two different triangles.

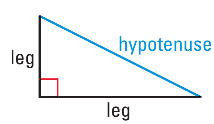
Therefore, SSA is *not* a valid method for proving that triangles are congruent, although there is a special case for right triangles.

READ VOCABULARY

The two sides of a triangle that form an angle are *adjacent* to the angle. The side not adjacent to the angle is *opposite* the angle.



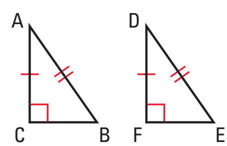
RIGHT TRIANGLES In a right triangle, the sides adjacent to the right angle are called the **legs**. The side opposite the right angle is called the **hypotenuse** of the right triangle.



THEOREM *For Your Notebook*

THEOREM 4.5 Hypotenuse-Leg (HL) Congruence Theorem

If the hypotenuse and a leg of a right triangle are congruent to the hypotenuse and a leg of a second right triangle, then the two triangles are congruent.



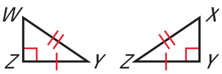
$\triangle ABC \cong \triangle DEF$

Proofs: Ex. 37, p. 439; p. 932

Differentiated Instruction

EXAMPLE 3 Use the Hypotenuse-Leg Congruence Theorem

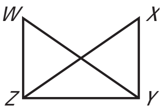
USE DIAGRAMS
If you have trouble matching vertices to letters when you separate the overlapping triangles, leave the triangles in their original orientations.



Write a proof.

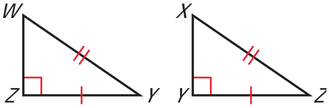
GIVEN ▶ $\overline{WY} \cong \overline{XZ}$, $\overline{WZ} \perp \overline{ZY}$, $\overline{XY} \perp \overline{ZY}$

PROVE ▶ $\triangle WYZ \cong \triangle XZY$



Solution

Redraw the triangles so they are side by side with corresponding parts in the same position. Mark the given information in the diagram.



STATEMENTS	REASONS
H **1.** $\overline{WY} \cong \overline{XZ}$	**1.** Given
2. $\overline{WZ} \perp \overline{ZY}$, $\overline{XY} \perp \overline{ZY}$	**2.** Given
3. $\angle Z$ and $\angle Y$ are right angles.	**3.** Definition of $\perp$ lines
4. $\triangle WYZ$ and $\triangle XZY$ are right triangles.	**4.** Definition of a right triangle
L **5.** $\overline{ZY} \cong \overline{YZ}$	**5.** Reflexive Property of Congruence
6. $\triangle WYZ \cong \triangle XZY$	**6.** HL Congruence Theorem

Animated Geometry at classzone.com

EXAMPLE 4 Choose a postulate or theorem

SIGN MAKING You are making a canvas sign to hang on the triangular wall over the door to the barn shown in the picture. You think you can use two identical triangular sheets of canvas. You know that $\overline{RP} \perp \overline{QS}$ and $\overline{PQ} \cong \overline{PS}$. What postulate or theorem can you use to conclude that $\triangle PQR \cong \triangle PSR$?

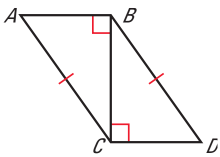
Solution

You are given that $\overline{PQ} \cong \overline{PS}$. By the Reflexive Property, $\overline{RP} \cong \overline{RP}$. By the definition of perpendicular lines, both $\angle RPQ$ and $\angle RPS$ are right angles, so they are congruent. So, two sides and their included angle are congruent.

▶ You can use the SAS Congruence Postulate to conclude that $\triangle PQR \cong \triangle PSR$.

✓ GUIDED PRACTICE for Examples 3 and 4

Use the diagram at the right.



3. Redraw $\triangle ACB$ and $\triangle DBC$ side by side with corresponding parts in the same position. **See margin.**
4. Use the information in the diagram to prove that $\triangle ACB \cong \triangle DBC$. **See margin.**

Differentiated Instruction

4.5 Prove Triangles Congruent by ASA and AAS

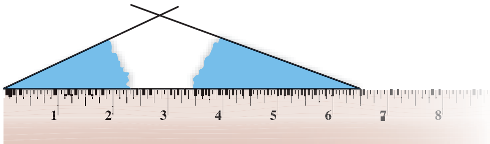
Before You used the SSS, SAS, and HL congruence methods.

Now You will use two more methods to prove congruences.

Why? So you can recognize congruent triangles in bikes, as in Exs. 23–24.

Key Vocabulary
- flow proof

Suppose you tear two angles out of a piece of paper and place them at a fixed distance on a ruler. Can you form more than one triangle with a given length and two given angle measures as shown below?



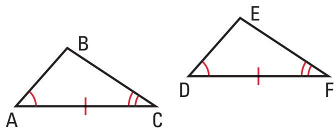
In a polygon, the side connecting the vertices of two angles is the *included* side. Given two angle measures and the length of the included side, you can make only one triangle. So, all triangles with those measurements are congruent.

THEOREMS *For Your Notebook*

POSTULATE 21 Angle-Side-Angle (ASA) Congruence Postulate

If two angles and the included side of one triangle are congruent to two angles and the included side of a second triangle, then the two triangles are congruent.

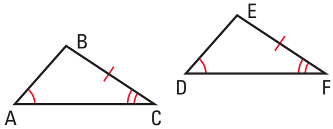
If **A**ngle $\angle A \cong \angle D$,
Side $\overline{AC} \cong \overline{DF}$, and
Angle $\angle C \cong \angle F$,
then $\triangle ABC \cong \triangle DEF$.



THEOREM 4.6 Angle-Angle-Side (AAS) Congruence Theorem

If two angles and a non-included side of one triangle are congruent to two angles and the corresponding non-included side of a second triangle, then the two triangles are congruent.

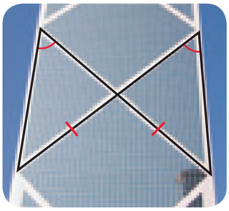
If **A**ngle $\angle A \cong \angle D$,
Angle $\angle C \cong \angle F$, and
Side $\overline{BC} \cong \overline{EF}$,
then $\triangle ABC \cong \triangle DEF$.

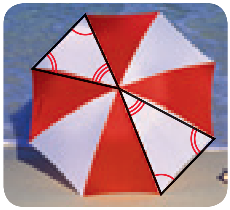


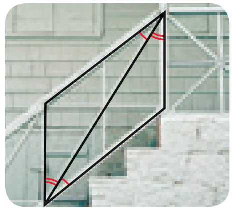
Proof: Example 2, p. 250

EXAMPLE 1 Identify congruent triangles

Can the triangles be proven congruent with the information given in the diagram? If so, state the postulate or theorem you would use.

a. 

b. 

c. 

Solution

a. The vertical angles are congruent, so two pairs of angles and a pair of non-included sides are congruent. The triangles are congruent by the AAS Congruence Theorem.

b. There is not enough information to prove the triangles are congruent, because no sides are known to be congruent.

c. Two pairs of angles and their included sides are congruent. The triangles are congruent by the ASA Congruence Postulate.

AVOID ERRORS
You need at least one pair of congruent corresponding sides to prove two triangles congruent.

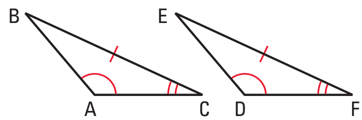
FLOW PROOFS You have written two-column proofs and paragraph proofs. A **flow proof** uses arrows to show the flow of a logical argument. Each reason is written below the statement it justifies.

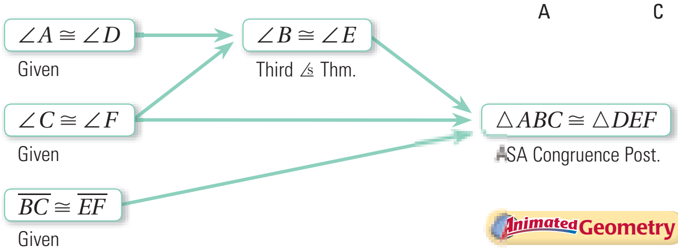
EXAMPLE 2 Prove the AAS Congruence Theorem

Prove the Angle-Angle-Side Congruence Theorem.

GIVEN ▶ $\angle A \cong \angle D$, $\angle C \cong \angle F$, $\overline{BC} \cong \overline{EF}$

PROVE ▶ $\triangle ABC \cong \triangle DEF$

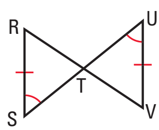




AnimatedGeometry at classzone.com

✓ GUIDED PRACTICE for Examples 1 and 2

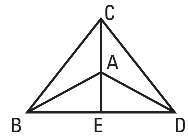
1. In the diagram at the right, what postulate or theorem can you use to prove that $\triangle RST \cong \triangle VUT$? *Explain.*
   AAS; ∠RTS and ∠VTU are congruent.
2. Rewrite the proof of the Triangle Sum Theorem on page 219 as a flow proof. **See margin.**



Differentiated Instruction

EXAMPLE 3 Write a flow proof

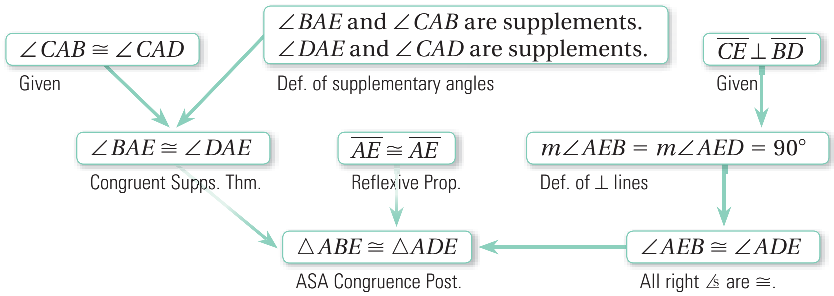
In the diagram, $\overline{CE} \perp \overline{BD}$ and $\angle CAB \cong \angle CAD$. Write a flow proof to show $\triangle ABE \cong \triangle ADE$.



Solution

GIVEN ▶ $\overline{CE} \perp \overline{BD}$, $\angle CAB \cong \angle CAD$

PROVE ▶ $\triangle ABE \cong \triangle ADE$

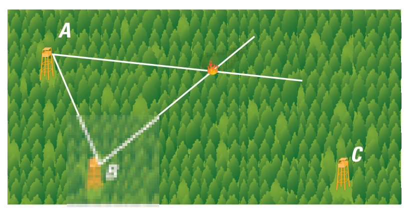


★ EXAMPLE 4 Standardized Test Practice

FIRE TOWERS The forestry service uses fire tower lookouts to watch for forest fires. When the lookouts spot a fire, they measure the angle of their view and radio a dispatcher. The dispatcher then uses the angles to locate the fire. How many lookouts are needed to locate a fire?

Ⓐ 1 Ⓑ 2 Ⓒ 3 Ⓓ Not enough information

The locations of tower A, tower B, and the fire form a triangle. The dispatcher knows the distance from tower A to tower B and the measures of $\angle A$ and $\angle B$. So, he knows the measures of two angles and an included side of the triangle.



By the ASA Congruence Postulate, all triangles with these measures are congruent. So, the triangle formed is unique and the fire location is given by the third vertex. Two lookouts are needed to locate the fire.

▶ The correct answer is B. Ⓐ Ⓑ Ⓒ Ⓓ

✓ GUIDED PRACTICE for Examples 3 and 4

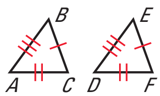
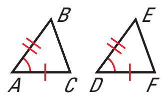
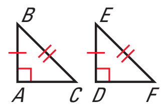
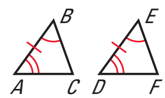
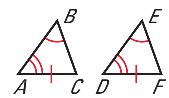
3. In Example 3, suppose $\angle ABE \cong \angle ADE$ is also given. What theorem or postulate besides ASA can you use to prove that $\triangle ABE \cong \triangle ADE$?
   AAS Congruence Theorem

4. **WHAT IF?** In Example 4, suppose a fire occurs directly between tower B and tower C. Could towers B and C be used to locate the fire? *Explain.*
   No; no triangle is formed by the location of the fire and the towers, so the fire could be anywhere between towers B and C.

CONCEPT SUMMARY *For Your Notebook*

Triangle Congruence Postulates and Theorems

You have learned five methods for proving that triangles are congruent.

SSS	SAS	HL (right △ only)	ASA	AAS
				
All three sides are congruent.	Two sides and the included angle are congruent.	The hypotenuse and one of the legs are congruent.	Two angles and the included side are congruent.	Two angles and a (non-included) side are congruent.

In the Exercises, you will prove three additional theorems about the congruence of right triangles: **A**ngle-**L**eg, **L**eg-**L**eg, and **H**ypotenuse-**A**ngle.

4.5 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 5, 9, and 27

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 7, 21, and 26

SKILL PRACTICE

A

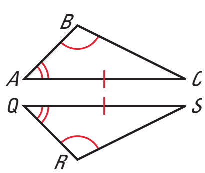
1. **VOCABULARY** Name one advantage of using a flow proof rather than a two-column proof. ***Sample answer:*** **A flow proof shows the flow of a logical argument.**

2. ★ **WRITING** You know that a pair of triangles has two pairs of congruent corresponding angles. What other information do you need to show that the triangles are congruent? **an included side or a non-included side**

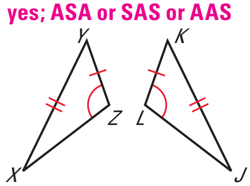
EXAMPLE 1 on p. 250 for Exs. 3–7

IDENTIFY CONGRUENT TRIANGLES **Is it possible to prove that the triangles are congruent? If so, state the postulate or theorem you would use.**

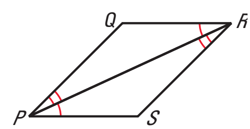
3. $\triangle ABC$, $\triangle QRS$ **yes; AAS**



4. $\triangle XYZ$, $\triangle JKL$ **yes; ASA or SAS or AAS**

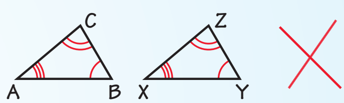


5. $\triangle PQR$, $\triangle RSP$ **yes; ASA**

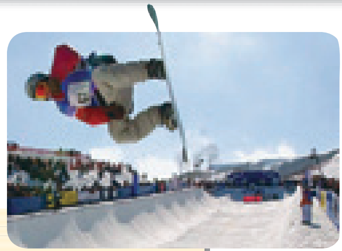


6. **ERROR ANALYSIS** *Describe* the error in concluding that $\triangle ABC \cong \triangle XYZ$. **There is no AAA postulate or theorem.**

By AAA,
△ABC ≅ △XYZ.



4.6 Use Congruent Triangles



Before You used corresponding parts to prove triangles congruent.

Now You will use congruent triangles to prove corresponding parts congruent.

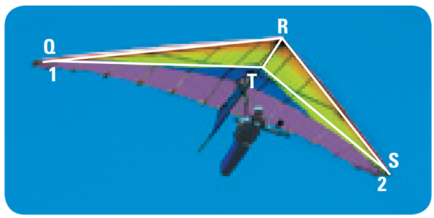
Why? So you can find the distance across a half pipe, as in Ex. 30.

Key Vocabulary
- **corresponding parts,** *p. 225*

By definition, congruent triangles have congruent corresponding parts. So, if you can prove that two triangles are congruent, you know that their corresponding parts must be congruent as well.

EXAMPLE 1 Use congruent triangles

Explain how you can use the given information to prove that the hanglider parts are congruent.



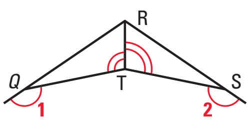
GIVEN ▶ $\angle 1 \cong \angle 2, \angle RTQ \cong \angle RTS$

PROVE ▶ $\overline{QT} \cong \overline{ST}$

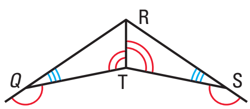
Solution

If you can show that $\triangle QRT \cong \triangle SRT$, you will know that $\overline{QT} \cong \overline{ST}$. First, copy the diagram and mark the given information. Then add the information that you can deduce. In this case, $\angle RQT$ and $\angle RST$ are supplementary to congruent angles, so $\angle RQT \cong \angle RST$. Also, $\overline{RT} \cong \overline{RT}$.

Mark given information.



Add deduced information.



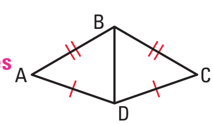
Two angle pairs and a non-included side are congruent, so by the AAS Congruence Theorem, $\triangle QRT \cong \triangle SRT$. Because corresponding parts of congruent triangles are congruent, $\overline{QT} \cong \overline{ST}$.

Animated Geometry at classzone.com

✓ GUIDED PRACTICE for Example 1

1. *Explain* how you can prove that $\angle A \cong \angle C$.

Since $\overline{BD} \cong \overline{BD}$ by the Reflexive Property, the triangles are congruent by SSS.

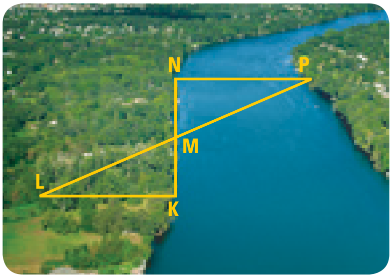


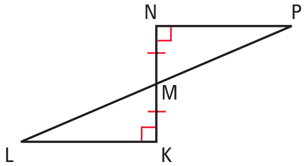
EXAMPLE 2 Use congruent triangles for measurement

INDIRECT MEASUREMENT
When you cannot easily measure a length directly, you can make conclusions about the length *indirectly,* usually by calculations based on known lengths.

SURVEYING Use the following method to find the distance across a river, from point N to point P.

- Place a stake at K on the near side so that $\overline{NK} \perp \overline{NP}$.
- Find M, the midpoint of $\overline{NK}$.
- Locate the point L so that $\overline{NK} \perp \overline{KL}$ and L, P, and M are collinear.
- Explain how this plan allows you to find the distance.





Solution

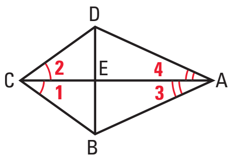
Because $\overline{NK} \perp \overline{NP}$ and $\overline{NK} \perp \overline{KL}$, $\angle N$ and $\angle K$ are congruent right angles. Because M is the midpoint of $\overline{NK}$, $\overline{NM} \cong \overline{KM}$. The vertical angles $\angle KML$ and $\angle NMP$ are congruent. So, $\triangle MLK \cong \triangle MPN$ by the ASA Congruence Postulate. Then, because corresponding parts of congruent triangles are congruent, $\overline{KL} \cong \overline{NP}$. So, you can find the distance NP across the river by measuring $\overline{KL}$.

EXAMPLE 3 Plan a proof involving pairs of triangles

Use the given information to write a plan for proof.

GIVEN ▶ $\angle 1 \cong \angle 2$, $\angle 3 \cong \angle 4$

PROVE ▶ $\triangle BCE \cong \triangle DCE$



Solution

In $\triangle BCE$ and $\triangle DCE$, you know $\angle 1 \cong \angle 2$ and $\overline{CE} \cong \overline{CE}$. If you can show that $\overline{CB} \cong \overline{CD}$, you can use the SAS Congruence Postulate.

To prove that $\overline{CB} \cong \overline{CD}$, you can first prove that $\triangle CBA \cong \triangle CDA$. You are given $\angle 1 \cong \angle 2$ and $\angle 3 \cong \angle 4$. $\overline{CA} \cong \overline{CA}$ by the Reflexive Property. You can use the ASA Congruence Postulate to prove that $\triangle CBA \cong \triangle CDA$.

▶ **Plan for Proof** Use the ASA Congruence Postulate to prove that $\triangle CBA \cong \triangle CDA$. Then state that $\overline{CB} \cong \overline{CD}$. Use the SAS Congruence Postulate to prove that $\triangle BCE \cong \triangle DCE$.

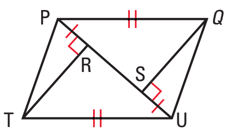
Animated Geometry at classzone.com

2. No; since M is the midpoint of $\overline{NK}$, $\overline{NM} \cong \overline{MK}$. No matter how far apart the stakes at K and M are placed, the triangles will be congruent by ASA.

3. Since you already know that $\overline{TU} \cong \overline{QP}$ and $\overline{UP} \cong \overline{PU}$ you need only show $\overline{PT} \cong \overline{UQ}$ to prove the triangles are congruent by SSS. This can be done by showing right triangles QSP and TRU are congruent by LL leading to right triangles USQ and PRT being congruent by LL which gives you $\overline{PT} \cong \overline{UQ}$.

✓ GUIDED PRACTICE for Examples 2 and 3

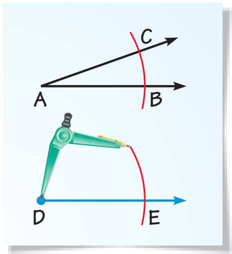
2. In Example 2, does it matter how far from point N you place a stake at point K? *Explain.* **See margin.**
3. Using the information in the diagram at the right, write a plan to prove that $\triangle PTU \cong \triangle UQP$. **See margin.**



Differentiated Instruction

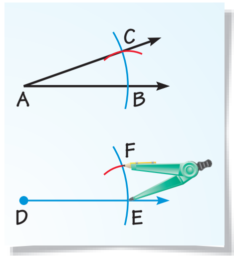
PROVING CONSTRUCTIONS On page 34, you learned how to use a compass and a straightedge to copy an angle. The construction is shown below. You can use congruent triangles to prove that this construction is valid.

STEP 1



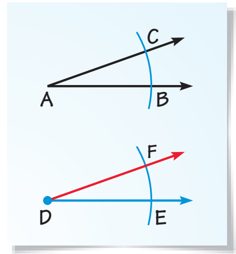
To copy $\angle A$, draw a segment with initial point D. Draw an arc with center A. Using the same radius, draw an arc with center D. Label points B, C, and E.

STEP 2



Draw an arc with radius BC and center E. Label the intersection F.

STEP 3



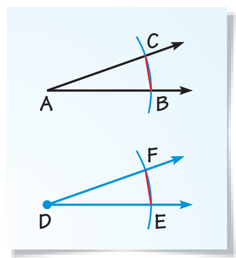
Draw $\overrightarrow{DF}$. In Example 4, you will prove that $\angle D \cong \angle A$.

EXAMPLE 4 Prove a construction

Write a proof to verify that the construction for copying an angle is valid.

Solution

Add $\overline{BC}$ and $\overline{EF}$ to the diagram. In the construction, $\overline{AB}$, $\overline{DE}$, $\overline{AC}$, and $\overline{DF}$ are all determined by the same compass setting, as are $\overline{BC}$ and $\overline{EF}$. So, you can assume the following as given statements.



GIVEN ▶ $\overline{AB} \cong \overline{DE}$, $\overline{AC} \cong \overline{DF}$, $\overline{BC} \cong \overline{EF}$

PROVE ▶ $\angle D \cong \angle A$

Plan for Proof Show that $\triangle CAB \cong \triangle FDE$, so you can conclude that the corresponding parts $\angle A$ and $\angle D$ are congruent.

Plan in Action

STATEMENTS	REASONS
1. $\overline{AB} \cong \overline{DE}$, $\overline{AC} \cong \overline{DF}$, $\overline{BC} \cong \overline{EF}$	**1.** Given
2. $\triangle FDE \cong \triangle CAB$	**2.** SSS Congruence Postulate
3. $\angle D \cong \angle A$	**3.** Corresp. parts of $\cong$ △ are $\cong$.

✓ GUIDED PRACTICE for Example 4

4. Look back at the construction of an angle bisector in Explore 4 on page 34. What segments can you assume are congruent? $\overline{CA}$ and $\overline{CB}$, $\overline{AD}$ and $\overline{BD}$

4.7 Use Isosceles and Equilateral Triangles

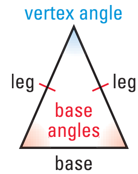
Before You learned about isosceles and equilateral triangles.

Now You will use theorems about isosceles and equilateral triangles.

Why? So you can solve a problem about architecture, as in Ex. 40.

Key Vocabulary
- **legs**
- **vertex angle**
- **base**
- **base angles**

In Lesson 4.1, you learned that a triangle is isosceles if it has at least two congruent sides. When an isosceles triangle has exactly two congruent sides, these two sides are the **legs**. The angle formed by the legs is the **vertex angle**. The third side is the **base** of the isosceles triangle. The two angles adjacent to the base are called **base angles**.



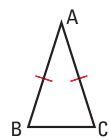
THEOREMS *For Your Notebook*

THEOREM 4.7 Base Angles Theorem

If two sides of a triangle are congruent, then the angles opposite them are congruent.

If $\overline{AB} \cong \overline{AC}$, then $\angle B \cong \angle C$.

Proof: p. 265

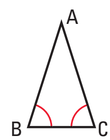


THEOREM 4.8 Converse of Base Angles Theorem

If two angles of a triangle are congruent, then the sides opposite them are congruent.

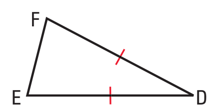
If $\angle B \cong \angle C$, then $\overline{AB} \cong \overline{AC}$.

Proof: Ex. 45, p. 269



EXAMPLE 1 Apply the Base Angles Theorem

In $\triangle DEF$, $\overline{DE} \cong \overline{DF}$. Name two congruent angles.



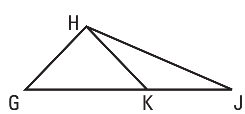
Solution

▶ $\overline{DE} \cong \overline{DF}$, so by the Base Angles Theorem, $\angle E \cong \angle F$.

✓ GUIDED PRACTICE for Example 1

Copy and complete the statement.

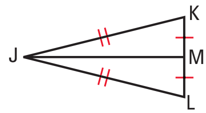
1. If $\overline{HG} \cong \overline{HK}$, then $\angle$ _?_ $\cong \angle$ _?_. **HGK, HKG**
2. If $\angle KHJ \cong \angle KJH$, then _?_ $\cong$ _?_. **KH, KJ**



PROOF Base Angles Theorem

GIVEN ▶ $\overline{JK} \cong \overline{JL}$

PROVE ▶ $\angle K \cong \angle L$



Plan for Proof

a. Draw $\overline{JM}$ so that it bisects $\overline{KL}$.

b. Use SSS to show that $\triangle JMK \cong \triangle JML$.

c. Use properties of congruent triangles to show that $\angle K \cong \angle L$.

Plan in Action

	STATEMENTS	REASONS
	1. M is the midpoint of $\overline{KL}$.	**1.** Definition of midpoint
a.	**2.** Draw $\overline{JM}$.	**2.** Two points determine a line.
	3. $\overline{MK} \cong \overline{ML}$	**3.** Definition of midpoint
	4. $\overline{JK} \cong \overline{JL}$	**4.** Given
	5. $\overline{JM} \cong \overline{JM}$	**5.** Reflexive Property of Congruence
b.	**6.** $\triangle JMK \cong \triangle JML$	**6.** SSS Congruence Postulate
c.	**7.** $\angle K \cong \angle L$	**7.** Corresp. parts of $\cong$ $\triangle$s are $\cong$.

Recall that an equilateral triangle has three congruent sides.

WRITE A BICONDITIONAL

The corollaries state that a triangle is *equilateral* if and only if it is *equiangular*.

COROLLARIES

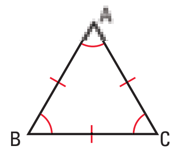
For Your Notebook

Corollary to the Base Angles Theorem

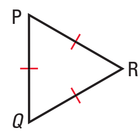
If a triangle is equilateral, then it is equiangular.

Corollary to the Converse of Base Angles Theorem

If a triangle is equiangular, then it is equilateral.



EXAMPLE 2 Find measures in a triangle



Find the measures of $\angle P$, $\angle Q$, and $\angle R$.

The diagram shows that $\triangle PQR$ is equilateral. Therefore, by the Corollary to the Base Angles Theorem, $\triangle PQR$ is equiangular. So, $m\angle P = m\angle Q = m\angle R$.

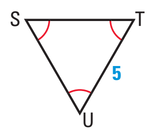
$3(m\angle P) = 180°$ **Triangle Sum Theorem**

$m\angle P = 60°$ **Divide each side by 3.**

▶ The measures of $\angle P$, $\angle Q$, and $\angle R$ are all 60°.

✓ GUIDED PRACTICE for Example 2

3. Find ST in the triangle at the right. **5**

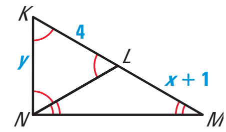


4. Is it possible for an equilateral triangle to have an angle measure other than 60°? *Explain.*

No. Sample answer: The Triangle Sum Theorem and the fact that the triangle is equilateral guarantees the angles measure 60° because all pairs of angles could be considered base angles of an isosceles triangle.

EXAMPLE 3 Use isosceles and equilateral triangles

ALGEBRA Find the values of x and y in the diagram.



Solution

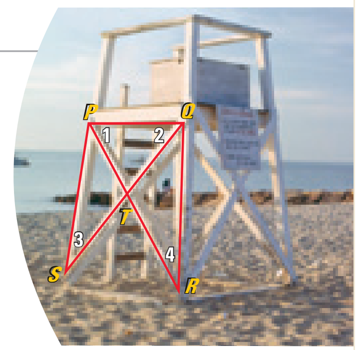
STEP 1 **Find** the value of y. Because $\triangle KLN$ is equiangular, it is also equilateral and $\overline{KN} \cong \overline{KL}$. Therefore, $y = 4$.

AVOID ERRORS
You cannot use $\angle N$ to refer to $\angle LNM$ because three angles have N as their vertex.

STEP 2 **Find** the value of x. Because $\angle LNM \cong \angle LMN$, $\overline{LN} \cong \overline{LM}$ and $\triangle LMN$ is isosceles. You also know that $LN = 4$ because $\triangle KLN$ is equilateral.

$LN = LM$	Definition of congruent segments
$4 = x + 1$	Substitute 4 for LN and $x + 1$ for LM.
$3 = x$	Subtract 1 from each side.

EXAMPLE 4 Solve a multi-step problem



LIFEGUARD TOWER In the lifeguard tower, $\overline{PS} \cong \overline{QR}$ and $\angle QPS \cong \angle PQR$.

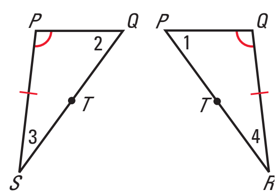
a. What congruence postulate can you use to prove that $\triangle QPS \cong \triangle PQR$?

b. Explain why $\triangle PQT$ is isosceles.

c. Show that $\triangle PTS \cong \triangle QTR$.

Solution

AVOID ERRORS
When you redraw the triangles so that they do not overlap, be careful to copy all given information and labels correctly.

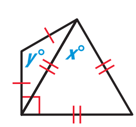


a. Draw and label $\triangle QPS$ and $\triangle PQR$ so that they do not overlap. You can see that $\overline{PQ} \cong \overline{QP}$, $\overline{PS} \cong \overline{QR}$, and $\angle QPS \cong \angle PQR$. So, by the SAS Congruence Postulate, $\triangle QPS \cong \triangle PQR$.

b. From part (a), you know that $\angle 1 \cong \angle 2$ because corresp. parts of $\cong$ △s are $\cong$. By the Converse of the Base Angles Theorem, $\overline{PT} \cong \overline{QT}$, and $\triangle PQT$ is isosceles.

c. You know that $\overline{PS} \cong \overline{QR}$, and $\angle 3 \cong \angle 4$ because corresp. parts of $\cong$ △s are $\cong$. Also, $\angle PTS \cong \angle QTR$ by the Vertical Angles Congruence Theorem. So, $\triangle PTS \cong \triangle QTR$ by the AAS Congruence Theorem.

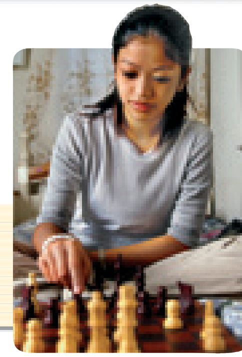
✓ GUIDED PRACTICE for Examples 3 and 4



5. Find the values of x and y in the diagram. **60, 120**

6. **REASONING** Use parts (b) and (c) in Example 4 and the SSS Congruence Postulate to give a different proof that $\triangle QPS \cong \triangle PQR$. **See margin.**

4.8 Perform Congruence Transformations



Before You determined whether two triangles are congruent.

Now You will create an image congruent to a given triangle.

Why So you can describe chess moves, as in Ex. 41.

Key Vocabulary
- **transformation**
- **image**
- **translation**
- **reflection**
- **rotation**
- **congruence transformation**

A **transformation** is an operation that moves or changes a geometric figure in some way to produce a new figure. The new figure is called the **image.** A transformation can be shown using an arrow.

The order of the vertices in the transformation statement tells you that P is the image of A, Q is the image of B, and R is the image of C.

$\triangle ABC \rightarrow \triangle PQR$

Original figure   Image

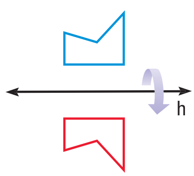
There are three main types of transformations. A **translation** moves every point of a figure the same distance in the same direction. A **reflection** uses a *line of reflection* to create a mirror image of the original figure. A **rotation** turns a figure about a fixed point, called the *center of rotation*.

EXAMPLE 1 Identify transformations

TRANSFORMATIONS You will learn more about transformations in Lesson 6.7 and in Chapter 9.

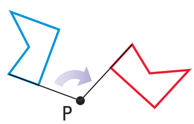
Name the type of transformation demonstrated in each picture.

a.



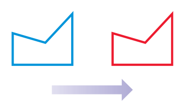
Reflection in a horizontal line

b.



Rotation about a point

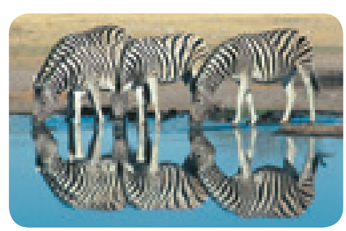
c.



Translation in a straight path

✓ GUIDED PRACTICE for Example 1

1. Name the type of transformation shown. reflection



CONGRUENCE Translations, reflections, and rotations are three types of *congruence transformations*. A **congruence transformation** changes the position of the figure without changing its size or shape.

TRANSLATIONS In a coordinate plane, a translation moves an object a given distance right or left and up or down. You can use coordinate notation to describe a translation.

READ DIAGRAMS
In this book, the original figure is blue and the transformation of the figure is red, unless otherwise stated.

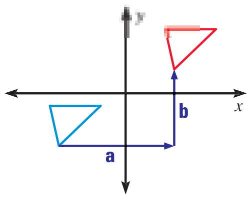
KEY CONCEPT — For Your Notebook

Coordinate Notation for a Translation

You can describe a translation by the notation

$$(x, y) \rightarrow (x + a, y + b)$$

which shows that each point (x, y) of the blue figure is translated horizontally a units and vertically b units.



EXAMPLE 2 Translate a figure in the coordinate plane

Figure *ABCD* has the vertices *A*(−4, 3), *B*(−2, 4), *C*(−1, 1), and *D*(−3, 1). Sketch *ABCD* and its image after the translation $(x, y) \rightarrow (x + 5, y - 2)$.

Solution

First draw *ABCD*. Find the translation of each vertex by adding 5 to its *x*-coordinate and subtracting 2 from its *y*-coordinate. Then draw *ABCD* and its image.

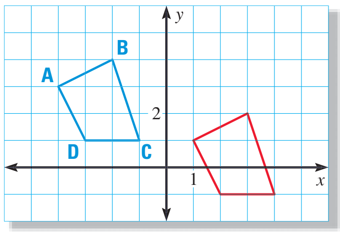
$(x, y) \rightarrow (x + 5, y - 2)$

$A(-4, 3) \rightarrow (1, 1)$

$B(-2, 4) \rightarrow (3, 2)$

$C(-1, 1) \rightarrow (4, -1)$

$D(-3, 1) \rightarrow (2, -1)$

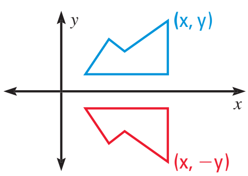


REFLECTIONS In this lesson, when a reflection is shown in a coordinate plane, the line of reflection is always the *x*-axis or the *y*-axis.

KEY CONCEPT — For Your Notebook

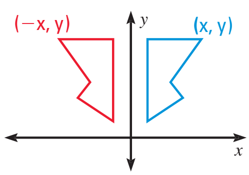
Coordinate Notation for a Reflection

Reflection in the *x*-axis



Multiply the *y*-coordinate by −1.
$(x, y) \rightarrow (x, -y)$

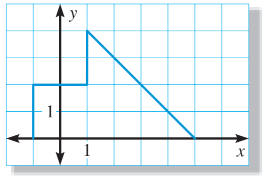
Reflection in the *y*-axis



Multiply the *x*-coordinate by −1.
$(x, y) \rightarrow (-x, y)$

EXAMPLE 3 Reflect a figure in the *y*-axis

WOODWORK **You are drawing a pattern for a wooden sign. Use a reflection in the *x*-axis to draw the other half of the pattern.**



Solution

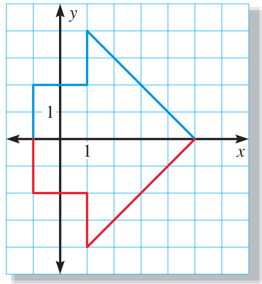
Multiply the *y*-coordinate of each vertex by −1 to find the corresponding vertex in the image.

$$(x, y) \rightarrow (x, -y)$$

$(-1, 0) \rightarrow (-1, 0)$ $\quad$ $(-1, 2) \rightarrow (-1, -2)$

$(1, 2) \rightarrow (1, -2)$ $\quad$ $(1, 4) \rightarrow (1, -4)$

$(5, 0) \rightarrow (5, 0)$



Use the vertices to draw the image. You can check your results by looking to see if each original point and its image are the same distance from the *x*-axis.

AnimatedGeometry at classzone.com

✓ GUIDED PRACTICE for Examples 2 and 3

2. The vertices of $\triangle ABC$ are $A(1, 2)$, $B(0, 0)$, and $C(4, 0)$. A translation of $\triangle ABC$ results in the image $\triangle DEF$ with vertices $D(2, 1)$, $E(1, -1)$, and $F(5, -1)$. *Describe* the translation in words and in coordinate notation.

   2. Add one to each *x* coordinate and subtract one from each *y* coordinate, $(x, y) \rightarrow (x + 1, y - 1)$.

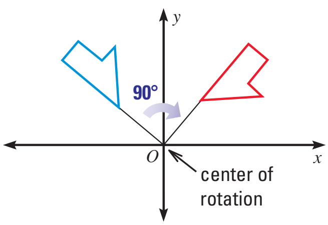
3. The endpoints of $\overline{RS}$ are $R(4, 5)$ and $S(1, -3)$. A reflection of $\overline{RS}$ results in the image $\overline{TU}$, with coordinates $T(4, -5)$ and $U(1, 3)$. Tell which axis $\overline{RS}$ was reflected in and write the coordinate rule for the reflection.

   ***x*-axis, $(x, y) \rightarrow (x, -y)$**

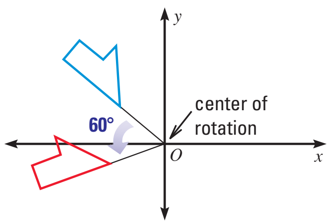
ROTATIONS In this lesson, if a rotation is shown in a coordinate plane, the center of rotation is the origin.

The direction of rotation can be either *clockwise* or *counterclockwise*. The *angle of rotation* is formed by rays drawn from the center of rotation through corresponding points on the original figure and its image.

90° clockwise rotation



60° counterclockwise rotation



Notice that rotations preserve distances from the center of rotation. So, segments drawn from the center of rotation to corresponding points on the figures are congruent.

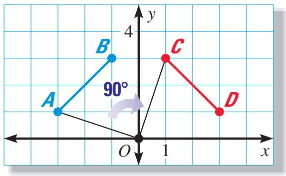
EXAMPLE 4 Identify a rotation

Graph $\overline{AB}$ and $\overline{CD}$. Tell whether $\overline{CD}$ is a rotation of $\overline{AB}$ about the origin. If so, give the angle and direction of rotation.

a. $A(-3, 1)$, $B(-1, 3)$, $C(1, 3)$, $D(3, 1)$ **b.** $A(0, 1)$, $B(1, 3)$, $C(-1, 1)$, $D(-3, 2)$

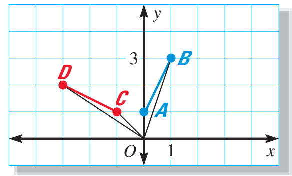
Solution

a.



$m\angle AOC = m\angle BOD = 90°$
This is a 90° clockwise rotation.

b.

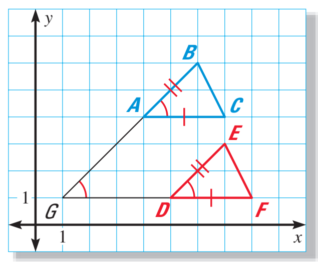


$m\angle AOC < m\angle BOD$
This is not a rotation.

EXAMPLE 5 Verify congruence

The vertices of $\triangle ABC$ are $A(4, 4)$, $B(6, 6)$, and $C(7, 4)$. The notation $(x, y) \rightarrow (x + 1, y - 3)$ describes the translation of $\triangle ABC$ to $\triangle DEF$. Show that $\triangle ABC \cong \triangle DEF$ to verify that the translation is a congruence transformation.

Solution



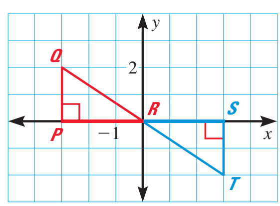
S You can see that $AC = DF = 3$, so $\overline{AC} \cong \overline{DF}$.

A Using the slopes, $\overline{AB} \parallel \overline{DE}$ and $\overline{AC} \parallel \overline{DF}$. If you extend $\overline{AB}$ and $\overline{DF}$ to form $\angle G$, the Corresponding Angles Postulate gives you $\angle BAC \cong \angle G$ and $\angle G \cong \angle EDF$. Then, $\angle BAC \cong \angle EDF$ by the Transitive Property of Congruence.

S Using the Distance Formula, $AB = DE = 2\sqrt{2}$ so $\overline{AB} \cong \overline{DE}$. So, $\triangle ABC \cong \triangle DEF$ by the SAS Congruence Postulate.

▶ Because $\triangle ABC \cong \triangle DEF$, the translation is a congruence transformation.

✓ GUIDED PRACTICE for Examples 4 and 5



4. Tell whether $\triangle PQR$ is a rotation of $\triangle STR$. If so, give the angle and direction of rotation. **no**

5. Show that $\triangle PQR \cong \triangle STR$ to verify that the transformation is a congruence transformation.
$\overline{PQ} \cong \overline{ST}$, $\overline{PR} \cong \overline{SR}$, by LL $\triangle PQR \cong \triangle STR$ so it is a congruence transformation.