3.6 Prove Theorems About Perpendicular Lines

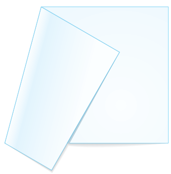
Before You found the distance between points in the coordinate plane.
Now You will find the distance between a point and a line.
Why? So you can determine lengths in art, as in Example 4.

Key Vocabulary
- **distance from a point to a line**

ACTIVITY FOLD PERPENDICULAR LINES

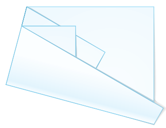
Materials: paper, protractor

STEP 1



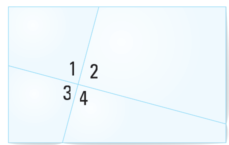
Fold a piece of paper.

STEP 2



Fold the paper again, so that the original fold lines up on itself.

STEP 3



Unfold the paper.

DRAW CONCLUSIONS

1. What type of angles appear to be formed where the fold lines intersect?
2. Measure the angles with a protractor. Which angles are congruent? Which angles are right angles?

The activity above suggests several properties of perpendicular lines.

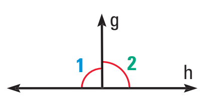
THEOREMS *For Your Notebook*

THEOREM 3.8

If two lines intersect to form a linear pair of congruent angles, then the lines are perpendicular.

If $\angle 1 \cong \angle 2$, then $g \perp h$.

Proof: Ex. 31, p. 196

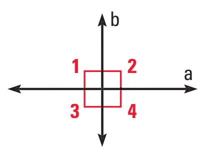


THEOREM 3.9

If two lines are perpendicular, then they intersect to form four right angles.

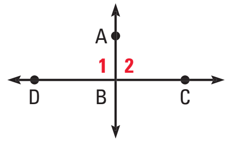
If $a \perp b$, then $\angle 1$, $\angle 2$, $\angle 3$, $\angle 4$ are right angles.

Proof: Ex. 32, p. 196



EXAMPLE 1 Draw conclusions

In the diagram at the right, $\overleftrightarrow{AB} \perp \overleftrightarrow{BC}$. What can you conclude about $\angle 1$ and $\angle 2$?



Solution

$\overleftrightarrow{AB}$ and $\overleftrightarrow{BC}$ are perpendicular, so by Theorem 3.9, they form four right angles. You can conclude that $\angle 1$ and $\angle 2$ are right angles, so $\angle 1 \cong \angle 2$.

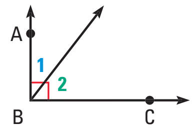
THEOREM *For Your Notebook*

THEOREM 3.10

If two sides of two adjacent acute angles are perpendicular, then the angles are complementary.

If $\overleftrightarrow{BA} \perp \overleftrightarrow{BC}$, then $\angle 1$ and $\angle 2$ are complementary.

Proof: Example 2, below

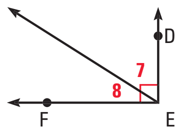


EXAMPLE 2 Prove Theorem 3.10

Prove that if two sides of two adjacent acute angles are perpendicular, then the angles are complementary.

GIVEN ▶ $\overrightarrow{ED} \perp \overrightarrow{EF}$

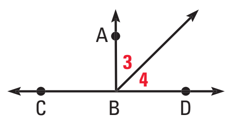
PROVE ▶ $\angle 7$ and $\angle 8$ are complementary.



STATEMENTS	REASONS
1. $\overrightarrow{ED} \perp \overrightarrow{EF}$	**1.** Given
2. $\angle DEF$ is a right angle.	**2.** $\perp$ lines intersect to form 4 rt. $\angle$s. (Theorem 3.9)
3. $m\angle DEF = 90°$	**3.** Definition of a right angle
4. $m\angle 7 + m\angle 8 = m\angle DEF$	**4.** Angle Addition Postulate
5. $m\angle 7 + m\angle 8 = 90°$	**5.** Substitution Property of Equality
6. $\angle 7$ and $\angle 8$ are complementary.	**6.** Definition of complementary angles

✓ GUIDED PRACTICE for Examples 1 and 2

1. Given that $\angle ABC \cong \angle ABD$, what can you conclude about $\angle 3$ and $\angle 4$? *Explain* how you know.



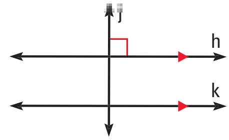
1. They are complementary. Sample answer: $\angle ABD$ is a right angle since 2 lines intersect to form a linear pair of congruent angles (Theorem 3.8), $\angle 3$ and $\angle 4$ are complementary.

2. Write a plan for proof for Theorem 3.9, that if two lines are perpendicular, then they intersect to form four right angles. Sample answer: The definition of perpendicular lines implies that angles formed by the intersecting lines are right angles.

THEOREMS *For Your Notebook*

THEOREM 3.11 Perpendicular Transversal Theorem

If a transversal is perpendicular to one of two parallel lines, then it is perpendicular to the other.

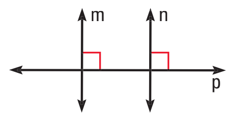


If $h \parallel k$ and $j \perp h$, then $j \perp k$.

Proof: Ex. 42, p. 160; Ex. 33, p. 196

THEOREM 3.12 Lines Perpendicular to a Transversal Theorem

In a plane, if two lines are perpendicular to the same line, then they are parallel to each other.

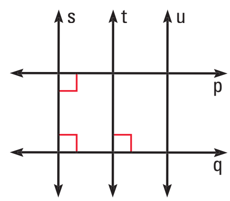


If $m \perp p$ and $n \perp p$, then $m \parallel n$.

Proof: Ex. 34, p. 196

EXAMPLE 3 Draw conclusions

Determine which lines, if any, must be parallel in the diagram. Explain your reasoning.



Solution

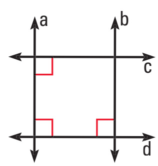
Lines p and q are both perpendicular to s, so by Theorem 3.12, $p \parallel q$. Also, lines s and t are both perpendicular to q, so by Theorem 3.12, $s \parallel t$.

✓ GUIDED PRACTICE for Example 3

Use the diagram at the right.

3. Is $b \parallel a$? *Explain* your reasoning.

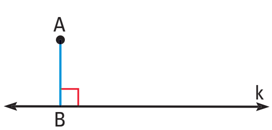
4. Is $b \perp c$? *Explain* your reasoning.



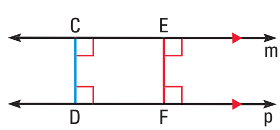
3. yes; Lines Perpendicular to a Transversal Theorem

4. yes; c ∥ d by the Lines Perpendicular to a Transversal Theorem, therefore b ⊥ c by the Perpendicular Transversal Theorem

DISTANCE FROM A LINE The **distance from a point to a line** is the length of the perpendicular segment from the point to the line. This perpendicular segment is the shortest distance between the point and the line. For example, the distance between point A and line k is AB. You will prove this in Chapter 5.



Distance from a point to a line



Distance between two parallel lines

The *distance between two parallel lines* is the length of any perpendicular segment joining the two lines. For example, the distance between line p and line m above is CD or EF.

Differentiated Instruction