

3.5 Write and Graph Equations of Lines



Before

You found slopes of lines.

Now

You will find equations of lines.

Why?

So you can find monthly gym costs, as in Example 4.

Key Vocabulary

- slope-intercept form
- standard form
- x-intercept, p. 879
- y-intercept, p. 879

Linear equations may be written in different forms. The general form of a linear equation in **slope-intercept form** is $y = mx + b$, where m is the slope and b is the y-intercept.

EXAMPLE 1 Write an equation of a line from a graph

Write an equation of the line in slope-intercept form.

Solution

STEP 1 Find the slope. Choose two points on the graph of the line, $(0, 4)$ and $(3, -2)$.

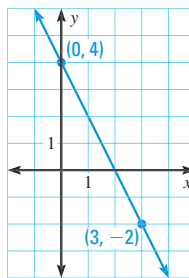
$$m = \frac{4 - (-2)}{0 - 3} = \frac{6}{-3} = -2$$

STEP 2 Find the y-intercept. The line intersects the y-axis at the point $(0, 4)$, so the y-intercept is 4.

STEP 3 Write the equation.

$$y = mx + b \quad \text{Use slope-intercept form.}$$

$$y = -2x + 4 \quad \text{Substitute } -2 \text{ for } m \text{ and } 4 \text{ for } b.$$



EXAMPLE 2 Write an equation of a parallel line

Write an equation of the line passing through the point $(-1, 1)$ that is parallel to the line with the equation $y = 2x - 3$.

Solution

STEP 1 Find the slope m . The slope of a line parallel to $y = 2x - 3$ is the same as the given line, so the slope is 2.

STEP 2 Find the y-intercept b by using $m = 2$ and $(x, y) = (-1, 1)$.

$$y = mx + b \quad \text{Use slope-intercept form.}$$

$$1 = 2(-1) + b \quad \text{Substitute for } x, y, \text{ and } m.$$

$$3 = b \quad \text{Solve for } b.$$

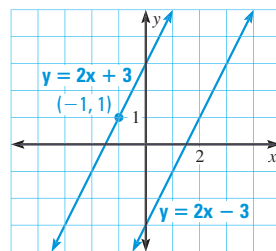
► Because $m = 2$ and $b = 3$, an equation of the line is $y = 2x + 3$.

LINEAR EQUATIONS

The graph of a linear equation represents all the solutions of the equation. So, the given point must be a solution of the equation.

CHECKING BY GRAPHING You can check that equations are correct by graphing. In Example 2, you can use a graph to check that $y = 2x - 3$ is parallel to $y = 2x + 3$.

 **Animated Geometry** at classzone.com



EXAMPLE 3 Write an equation of a perpendicular line

Write an equation of the line j passing through the point $(2, 3)$ that is perpendicular to the line k with the equation $y = -2x + 2$.

Solution

STEP 1 Find the slope m of line j . Line k has a slope of -2 .

$$-2 \cdot m = -1 \quad \text{The product of the slopes of } \perp \text{ lines is } -1.$$

$$m = \frac{1}{2} \quad \text{Divide each side by } -2.$$

STEP 2 Find the y -intercept b by using $m = \frac{1}{2}$ and $(x, y) = (2, 3)$.

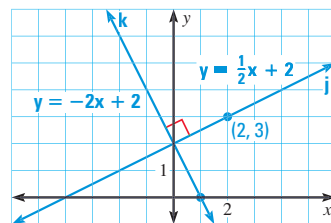
$$y = mx + b \quad \text{Use slope-intercept form.}$$

$$3 = \frac{1}{2}(2) + b \quad \text{Substitute for } x, y, \text{ and } m.$$

$$2 = b \quad \text{Solve for } b.$$

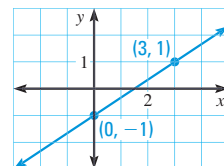
► Because $m = \frac{1}{2}$ and $b = 2$, an equation

of line j is $y = \frac{1}{2}x + 2$. You can check that the lines j and k are perpendicular by graphing, then using a protractor to measure one of the angles formed by the lines.



✓ GUIDED PRACTICE for Examples 1, 2, and 3

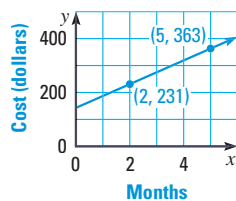
- Write an equation of the line in the graph at the right. $y = \frac{2}{3}x - 1$
- Write an equation of the line that passes through $(-2, 5)$ and $(1, 2)$. $y = -x + 3$
- Write an equation of the line that passes through the point $(1, 5)$ and is parallel to the line with the equation $y = 3x - 5$. Graph the lines to check that they are parallel. $y = 3x + 2$; see margin for art.
- How do you know the lines $x = 4$ and $y = 2$ are perpendicular?
Sample answer: $x = 4$ is a horizontal line while $y = 2$ is a vertical line.



EXAMPLE 4 Write an equation of a line from a graph

GYM MEMBERSHIP The graph models the total cost of joining a gym. Write an equation of the line. Explain the meaning of the slope and the y -intercept of the line.

Gym Membership Cost



Solution

STEP 1 Find the slope.

$$m = \frac{363 - 231}{5 - 2} = \frac{132}{3} = 44$$

STEP 2 Find the y -intercept. Use the slope and one of the points on the graph.

$$y = mx + b \quad \text{Use slope-intercept form.}$$

$$231 = 44 \cdot 2 + b \quad \text{Substitute for } x, y, \text{ and } m.$$

$$143 = b \quad \text{Simplify.}$$

STEP 3 Write the equation. Because $m = 44$ and $b = 143$, an equation of the line is $y = 44x + 143$.

► The equation $y = 44x + 143$ models the cost. The slope is the monthly fee, \$44, and the y -intercept is the initial cost to join the gym, \$143.

STANDARD FORM Another form of a linear equation is *standard form*. In **standard form**, the equation is written as $Ax + By = C$, where A and B are not both zero.

EXAMPLE 5 Graph a line with equation in standard form

Graph $3x + 4y = 12$.

Solution

The equation is in standard form, so you can use the intercepts.

STEP 1 Find the intercepts.

To find the x -intercept, let $y = 0$. To find the y -intercept, let $x = 0$.

$$3x + 4y = 12$$

$$3x + 4y = 12$$

$$3x + 4(0) = 12$$

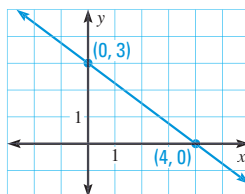
$$3(0) + 4y = 12$$

$$x = 4$$

$$y = 3$$

STEP 2 Graph the line.

The intercepts are $(4, 0)$ and $(0, 3)$. Graph these points, then draw a line through the points.



CHOOSE A METHOD

Another way you could graph the equation is to solve the equation for y . Then the equation will be in slope-intercept form. Use rise and run from the point where the line crosses the y -axis to find a second point. Then graph the line.

**GUIDED PRACTICE** for Examples 4 and 5

5. The equation $y = 50x + 125$ models the total cost of joining a climbing gym. What are the meaning of the slope and the y -intercept of the line?

Slope: monthly fee, **y -intercept:** initial cost to join gym

Graph the equation. 6–8. See margin.

6. $2x - 3y = 6$

7. $y = 4$

8. $x = -3$

WRITING EQUATIONS You can write linear equations to model real-world situations, such as comparing costs to find a better buy.

EXAMPLE 6 Solve a real-world problem

DVD RENTAL You can rent DVDs at a local store for \$4.00 each. An Internet company offers a flat fee of \$15.00 per month for as many rentals as you want. How many DVDs do you need to rent to make the online rental a better buy?

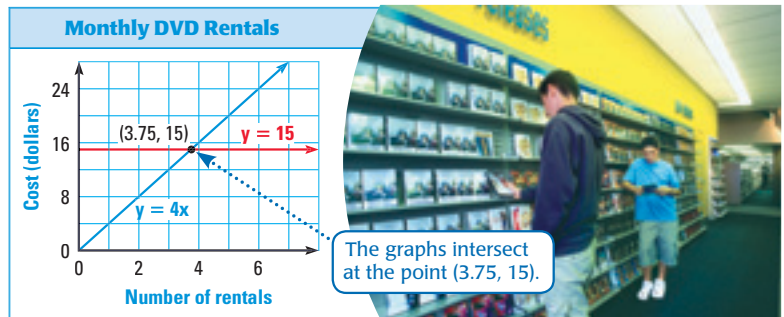
Solution

STEP 1 **Model** each rental with an equation.

Cost of one month's rental online: $y = 15$

Cost of one month's rental locally: $y = 4x$, where x represents the number of DVDs rented

STEP 2 **Graph** each equation.

**ANOTHER WAY**

For alternative methods for solving the problem in Example 6, turn to page 188 for the **Problem Solving Workshop**.

READ VOCABULARY

The point at which the costs are the same is sometimes called the *break-even point*.

- ▶ The point of intersection is $(3.75, 15)$. Using the graph, you can see that it is cheaper to rent locally if you rent 3 or fewer DVDs per month. If you rent 4 or more DVDs per month, it is cheaper to rent online.

**GUIDED PRACTICE** for Example 6

9. **WHAT IF?** In Example 6, suppose the online rental is \$16.50 per month and the local rental is \$4 each. How many DVDs do you need to rent to make the online rental a better buy? **5 DVDs**
10. How would your answer to Exercise 9 change if you had a 2-for-1 coupon that you could use once at the local store?

10. Online rental would have to increase to 6 DVDs to be a better buy.