

3.3 Prove Lines are Parallel



Before

You used properties of parallel lines to determine angle relationships.

Now

You will use angle relationships to prove that lines are parallel.

Why?

So you can describe how sports equipment is arranged, as in Ex. 32.

Key Vocabulary

- **paragraph proof**
- **converse**, p. 80
- **two-column proof**, p. 112

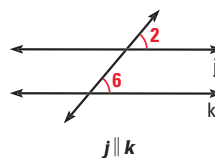
Postulate 16 below is the converse of Postulate 15 in Lesson 3.2. Similarly, the theorems in Lesson 3.2 have true converses. Remember that the converse of a true conditional statement is not necessarily true, so each converse of a theorem must be proved, as in Example 3.

POSTULATE

For Your Notebook

POSTULATE 16 Corresponding Angles Converse

If two lines are cut by a transversal so the corresponding angles are congruent, then the lines are parallel.



EXAMPLE 1 Apply the Corresponding Angles Converse

xy ALGEBRA Find the value of x that makes $m \parallel n$.

Solution

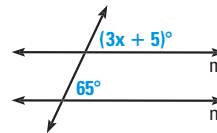
Lines m and n are parallel if the marked corresponding angles are congruent.

$$(3x + 5)^\circ = 65^\circ \quad \text{Use Postulate 16 to write an equation.}$$

$$3x = 60 \quad \text{Subtract 5 from each side.}$$

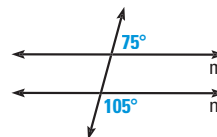
$$x = 20 \quad \text{Divide each side by 3.}$$

► The lines m and n are parallel when $x = 20$.



GUIDED PRACTICE for Example 1

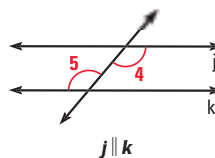
1. Is there enough information in the diagram to conclude that $m \parallel n$? *Explain.* **See margin.**
2. *Explain* why Postulate 16 is the converse of Postulate 15. **Postulate 16 switches the hypothesis and conclusion of Postulate 15.**



THEOREM 3.4 Alternate Interior Angles Converse

If two lines are cut by a transversal so the alternate interior angles are congruent, then the lines are parallel.

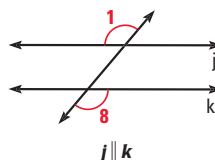
Proof: Example 3, p. 163



THEOREM 3.5 Alternate Exterior Angles Converse

If two lines are cut by a transversal so the alternate exterior angles are congruent, then the lines are parallel.

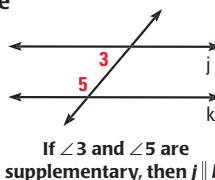
Proof: Ex. 36, p. 168



THEOREM 3.6 Consecutive Interior Angles Converse

If two lines are cut by a transversal so the consecutive interior angles are supplementary, then the lines are parallel.

Proof: Ex. 37, p. 168



EXAMPLE 2 Solve a real-world problem

SNAKE PATTERNS How can you tell whether the sides of the pattern are parallel in the photo of a diamond-back snake?



Solution

Because the alternate interior angles are congruent, you know that the sides of the pattern are parallel.

3. yes; Alternate Exterior Angles Converse

4. yes; Corresponding Angles Converse

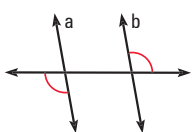
5. No. Sample answer: Supplementary angles do not have to be congruent.



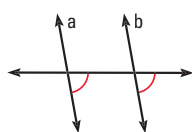
GUIDED PRACTICE for Example 2

Can you prove that lines a and b are parallel? Explain why or why not.

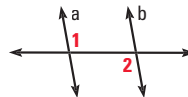
3.



4.



5. $m\angle 1 + m\angle 2 = 180^\circ$



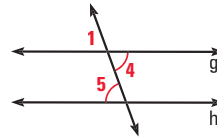
EXAMPLE 3 Prove the Alternate Interior Angles Converse

Prove that if two lines are cut by a transversal so the alternate interior angles are congruent, then the lines are parallel.

Solution

GIVEN $\angle 4 \cong \angle 5$

PROVE $g \parallel h$



STATEMENTS	REASONS
1. $\angle 4 \cong \angle 5$	1. Given
2. $\angle 1 \cong \angle 4$	2. Vertical Angles Congruence Theorem
3. $\angle 1 \cong \angle 5$	3. Transitive Property of Congruence
4. $g \parallel h$	4. Corresponding Angles Converse

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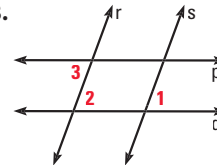
AVOID ERRORS

Before you write a proof, identify the GIVEN and PROVE statements for the situation described or for any diagram you draw.

PARAGRAPH PROOFS A proof can also be written in paragraph form, called a **paragraph proof**. The statements and reasons in a paragraph proof are written in sentences, using words to explain the logical flow of the argument.

EXAMPLE 4 Write a paragraph proof

In the figure, $r \parallel s$ and $\angle 1$ is congruent to $\angle 3$.
Prove $p \parallel q$.

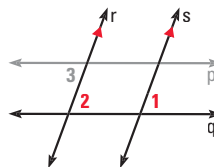


Solution

Look at the diagram to make a plan. The diagram suggests that you look at angles 1, 2, and 3. Also, you may find it helpful to focus on one pair of lines and one transversal at a time.

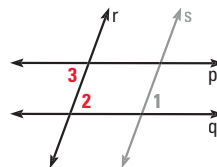
Plan for Proof

a. Look at $\angle 1$ and $\angle 2$.



$\angle 1 \cong \angle 2$ because $r \parallel s$.

b. Look at $\angle 2$ and $\angle 3$.



If $\angle 2 \cong \angle 3$, then $p \parallel q$.

TRANSITIONAL WORDS

In paragraph proofs, **transitional words** such as *so*, *then*, and *therefore* help to make the logic clear.

Plan in Action

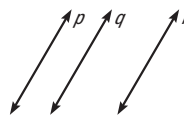
- a. It is given that $r \parallel s$, **so** by the Corresponding Angles Postulate, $\angle 1 \cong \angle 2$.
- b. It is also given that $\angle 1 \cong \angle 3$. **Then** $\angle 2 \cong \angle 3$ by the Transitive Property of Congruence for angles. **Therefore**, by the Alternate Interior Angles Converse, $p \parallel q$.

THEOREM

For Your Notebook

THEOREM 3.7 Transitive Property of Parallel Lines

If two lines are parallel to the same line, then they are parallel to each other.

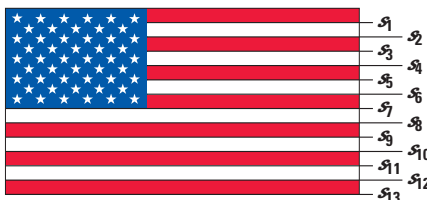


If $p \parallel q$ and $q \parallel r$, then $p \parallel r$.

Proofs: Ex. 38, p. 168; Ex. 38, p. 177

EXAMPLE 5 Use the Transitive Property of Parallel Lines

U.S. FLAG The flag of the United States has 13 alternating red and white stripes. Each stripe is parallel to the stripe immediately below it. Explain why the top stripe is parallel to the bottom stripe.



Solution

The stripes from top to bottom can be named $s_1, s_2, s_3, \dots, s_{13}$. Each stripe is parallel to the one below it, so $s_1 \parallel s_2, s_2 \parallel s_3$, and so on. Then $s_1 \parallel s_3$ by the Transitive Property of Parallel Lines. Similarly, because $s_3 \parallel s_4$, it follows that $s_1 \parallel s_4$. By continuing this reasoning, $s_1 \parallel s_{13}$. So, the top stripe is parallel to the bottom stripe.

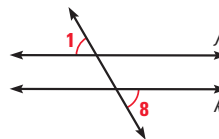
USE SUBSCRIPTS

When you name several similar items, you can use one variable with subscripts to keep track of the items.

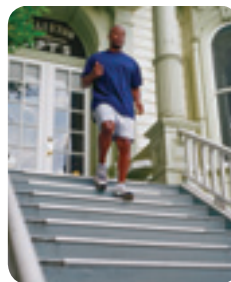


GUIDED PRACTICE for Examples 3, 4, and 5

6. If you use the diagram at the right to prove the Alternate Exterior Angles Converse, what GIVEN and PROVE statements would you use? **Given:** $\angle 1 \cong \angle 8$, **Prove:** $j \parallel k$



7. Copy and complete the following paragraph proof of the Alternate Interior Angles Converse using the diagram in Example 3.
It is given that $\angle 4 \cong \angle 5$. By the ?, $\angle 1 \cong \angle 4$. Then by the Transitive Property of Congruence, ?. So, by the ?, $g \parallel h$.
8. Each step is parallel to the step immediately above it. The bottom step is parallel to the ground. *Explain* why the top step is parallel to the ground.



7. Vertical Angles Congruence Theorem; $\angle 1 \cong \angle 5$;
Corresponding Angles Converse Postulate

8. All of the steps are parallel. Since the bottom step is parallel to the ground, the Transitive Property of Parallel Lines applies, and the top step is parallel to the ground.