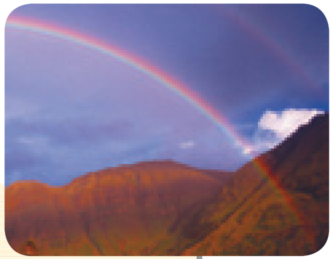
3.2 Use Parallel Lines and Transversals



Before You identified angle pairs formed by a transversal.

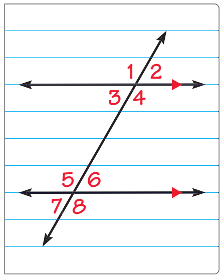
Now You will use angles formed by parallel lines and transversals.

Why? So you can understand angles formed by light, as in Example 4.

Key Vocabulary
- **corresponding angles,** *p. 149*
- **alternate interior angles,** *p. 149*
- **alternate exterior angles,** *p. 149*
- **consecutive interior angles,** *p. 149*

ACTIVITY EXPLORE PARALLEL LINES

Materials: lined paper, tracing paper, straightedge



STEP 1 **Draw** a pair of parallel lines cut by a nonperpendicular transversal on lined paper. Label the angles as shown.

STEP 2 **Trace** your drawing onto tracing paper.

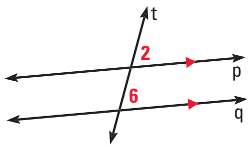
STEP 3 **Move** the tracing paper to position $\angle 1$ of the traced figure over $\angle 5$ of the original figure. Compare the angles. Are they congruent?

STEP 4 **Compare** the eight angles and list all the congruent pairs. What do you notice about the special angle pairs formed by the transversal?

POSTULATE — *For Your Notebook*

POSTULATE 15 Corresponding Angles Postulate

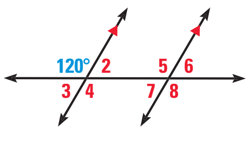
If two parallel lines are cut by a transversal, then the pairs of corresponding angles are congruent.



$\angle 2 \cong \angle 6$

EXAMPLE 1 Identify congruent angles

The measure of three of the numbered angles is 120°. Identify the angles. Explain your reasoning.



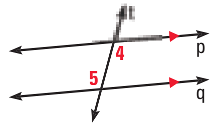
Solution

By the Corresponding Angles Postulate, $m\angle 5 = 120°$.
Using the Vertical Angles Congruence Theorem, $m\angle 4 = 120°$.
Because $\angle 4$ and $\angle 8$ are corresponding angles, by the Corresponding Angles Postulate, you know that $m\angle 8 = 120°$.

THEOREMS — *For Your Notebook*

THEOREM 3.1 Alternate Interior Angles Theorem

If two parallel lines are cut by a transversal, then the pairs of alternate interior angles are congruent.

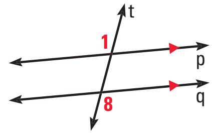


$\angle 4 \cong \angle 5$

Proof: Example 3, p. 156

THEOREM 3.2 Alternate Exterior Angles Theorem

If two parallel lines are cut by a transversal, then the pairs of alternate exterior angles are congruent.

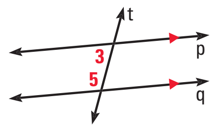


$\angle 1 \cong \angle 8$

Proof: Ex. 37, p. 159

THEOREM 3.3 Consecutive Interior Angles Theorem

If two parallel lines are cut by a transversal, then the pairs of consecutive interior angles are supplementary.

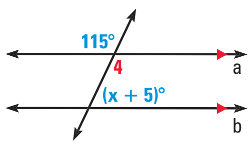


$\angle 3$ and $\angle 5$ are supplementary.

Proof: Ex. 41, p. 159

EXAMPLE 2 Use properties of parallel lines

ALGEBRA **Find the value of x.**



Solution

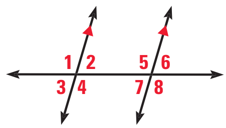
By the Vertical Angles Congruence Theorem, $m\angle 4 = 115°$. Lines a and b are parallel, so you can use the theorems about parallel lines.

$m\angle 4 + (x + 5)° = 180°$	Consecutive Interior Angles Theorem
$115° + (x + 5)° = 180°$	Substitute 115° for $m\angle 4$.
$x + 120 = 180$	Combine like terms.
$x = 60$	Subtract 120 from each side.

Animated Geometry at classzone.com

GUIDED PRACTICE for Examples 1 and 2

Use the diagram at the right.



1. If $m\angle 1 = 105°$, find $m\angle 4$, $m\angle 5$, and $m\angle 8$. Tell which postulate or theorem you use in each case.

2. If $m\angle 3 = 68°$ and $m\angle 8 = (2x + 4)°$, what is the value of x? Show your steps. **54; $m\angle 7 + m\angle 8 = 180$, $m\angle 3 = m\angle 7$, $68 + 2x + 4 = 180$, $2x + 72 = 180$, $2x = 108$, $x = 54$**

1. $m\angle 4 = 105°$, Vertical Angles Congruence Theorem; $m\angle 5 = 105°$, Corresponding Angles Postulate; $m\angle 8 = 105°$, Alternate Exterior Angles Theorem

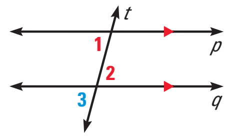
EXAMPLE 3 Prove the Alternate Interior Angles Theorem

Prove that if two parallel lines are cut by a transversal, then the pairs of alternate interior angles are congruent.

Solution

WRITE PROOFS
You can use the information from the diagram in your proof. Find any special angle pairs. Then decide what you know about those pairs.

Draw a diagram. Label a pair of alternate interior angles as $\angle 1$ and $\angle 2$. You are looking for an angle that is related to both $\angle 1$ and $\angle 2$. Notice that one angle is a vertical angle with $\angle 2$ and a corresponding angle with $\angle 1$. Label it $\angle 3$.



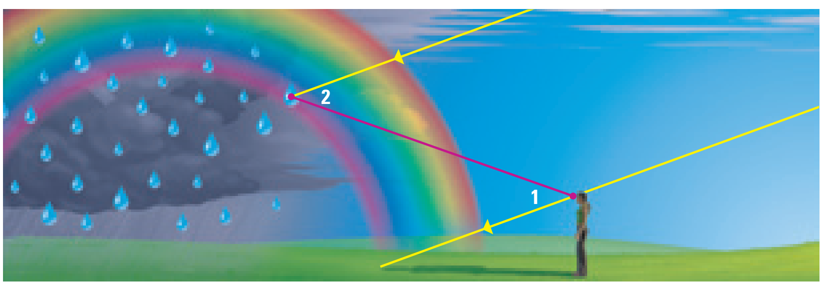
GIVEN ▶ $p \parallel q$

PROVE ▶ $\angle 1 \cong \angle 2$

STATEMENTS	REASONS
1. $p \parallel q$	**1.** Given
2. $\angle 1 \cong \angle 3$	**2.** Corresponding Angles Postulate
3. $\angle 3 \cong \angle 2$	**3.** Vertical Angles Congruence Theorem
4. $\angle 1 \cong \angle 2$	**4.** Transitive Property of Congruence

EXAMPLE 4 Solve a real-world problem

SCIENCE When sunlight enters a drop of rain, different colors of light leave the drop at different angles. This process is what makes a rainbow. For violet light, $m\angle 2 = 40°$. What is $m\angle 1$? How do you know?



Solution

Because the sun's rays are parallel, $\angle 1$ and $\angle 2$ are alternate interior angles. By the Alternate Interior Angles Theorem, $\angle 1 \cong \angle 2$. By the definition of congruent angles, $m\angle 1 = m\angle 2 = 40°$.

✓ GUIDED PRACTICE for Examples 3 and 4

3. In the proof in Example 3, if you use the third statement before the second statement, could you still prove the theorem? *Explain.* **Yes. *Sample answer:* $\angle 3$ and $\angle 2$ congruence is not dependent on the congruence of $\angle 1$ and $\angle 3$.**

4. **WHAT IF?** Suppose the diagram in Example 4 shows yellow light leaving a drop of rain. Yellow light leaves the drop at an angle of 41°. What is $m\angle 1$ in this case? How do you know?

4. 41°; $\angle 1$ and $\angle 2$ are alternate interior angles. By the Alternate Interior Angles Theorem $\angle 1 \cong \angle 2$, and by definition of congruent angles $m\angle 1 = m\angle 2 = 41°$.