

2.7 Prove Angle Pair Relationships



Before

You identified relationships between pairs of angles.

Now

You will use properties of special pairs of angles.

Why?

So you can describe angles found in a home, as in Ex. 44.

Key Vocabulary

- **complementary angles**, p. 35
- **supplementary angles**, p. 35
- **linear pair**, p. 37
- **vertical angles**, p. 37

Sometimes, a new theorem describes a relationship that is useful in writing proofs. For example, using the *Right Angles Congruence Theorem* will reduce the number of steps you need to include in a proof involving right angles.

THEOREM

For Your Notebook

THEOREM 2.3 Right Angles Congruence Theorem

All right angles are congruent.

Proof: below

PROOF

Right Angles Congruence Theorem

WRITE PROOFS

When you prove a theorem, write the hypothesis of the theorem as the GIVEN statement. The conclusion is what you must PROVE.

GIVEN ▶ $\angle 1$ and $\angle 2$ are right angles.

PROVE ▶ $\angle 1 \cong \angle 2$



STATEMENTS

1. $\angle 1$ and $\angle 2$ are right angles.
2. $m\angle 1 = 90^\circ$, $m\angle 2 = 90^\circ$
3. $m\angle 1 = m\angle 2$
4. $\angle 1 \cong \angle 2$

REASONS

1. Given
2. Definition of right angle
3. Transitive Property of Equality
4. Definition of congruent angles

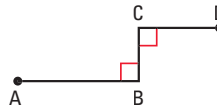
EXAMPLE 1

Use right angle congruence

Write a proof.

GIVEN ▶ $\overline{AB} \perp \overline{BC}$, $\overline{DC} \perp \overline{BC}$

PROVE ▶ $\angle B \cong \angle C$



STATEMENTS

1. $\overline{AB} \perp \overline{BC}$, $\overline{DC} \perp \overline{BC}$
2. $\angle B$ and $\angle C$ are right angles.
3. $\angle B \cong \angle C$

REASONS

1. Given
2. Definition of perpendicular lines
3. Right Angles Congruence Theorem

AVOID ERRORS

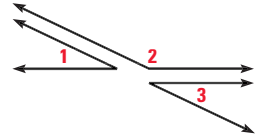
The given information in Example 1 is about perpendicular lines. You must then use deductive reasoning to show the angles are right angles.

THEOREM 2.4 Congruent Supplements Theorem

If two angles are supplementary to the same angle (or to congruent angles), then they are congruent.

If $\angle 1$ and $\angle 2$ are supplementary and $\angle 3$ and $\angle 2$ are supplementary, then $\angle 1 \cong \angle 3$.

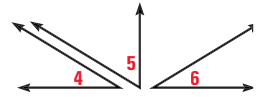
Proof: Example 2, below; Ex. 36, p. 129

**THEOREM 2.5 Congruent Complements Theorem**

If two angles are complementary to the same angle (or to congruent angles), then they are congruent.

If $\angle 4$ and $\angle 5$ are complementary and $\angle 6$ and $\angle 5$ are complementary, then $\angle 4 \cong \angle 6$.

Proof: Ex. 37, p. 129; Ex. 41, p. 130



To prove Theorem 2.4, you must prove two cases: one with angles supplementary to the same angle and one with angles supplementary to congruent angles. The proof of Theorem 2.5 also requires two cases.

EXAMPLE 2 Prove a case of Congruent Supplements Theorem

Prove that two angles supplementary to the same angle are congruent.

GIVEN ▶ $\angle 1$ and $\angle 2$ are supplements.
 $\angle 3$ and $\angle 2$ are supplements.

PROVE ▶ $\angle 1 \cong \angle 3$

**STATEMENTS**

- $\angle 1$ and $\angle 2$ are supplements.
 $\angle 3$ and $\angle 2$ are supplements.
- $m\angle 1 + m\angle 2 = 180^\circ$
 $m\angle 3 + m\angle 2 = 180^\circ$
- $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 2$
- $m\angle 1 = m\angle 3$
- $\angle 1 \cong \angle 3$

REASONS

- Given
- Definition of supplementary angles
- Transitive Property of Equality
- Subtraction Property of Equality
- Definition of congruent angles

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**GUIDED PRACTICE** for Examples 1 and 2

- How many steps do you save in the proof in Example 1 by using the *Right Angles Congruence Theorem*? **2 steps**
- Draw a diagram and write GIVEN and PROVE statements for a proof of each case of the *Congruent Complements Theorem*. **See margin.**

INTERSECTING LINES When two lines intersect, pairs of vertical angles and linear pairs are formed. The relationship that you used in Lesson 1.5 for linear pairs is formally stated below as the *Linear Pair Postulate*. This postulate is used in the proof of the *Vertical Angles Congruence Theorem*.

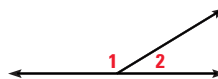
POSTULATE

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POSTULATE 12 Linear Pair Postulate

If two angles form a linear pair, then they are supplementary.

$\angle 1$ and $\angle 2$ form a linear pair, so $\angle 1$ and $\angle 2$ are supplementary and $m\angle 1 + m\angle 2 = 180^\circ$.



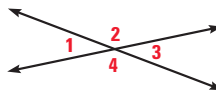
THEOREM

For Your Notebook

THEOREM 2.6 Vertical Angles Congruence Theorem

Vertical angles are congruent.

Proof: Example 3, below



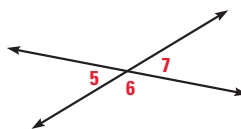
$\angle 1 \cong \angle 3, \angle 2 \cong \angle 4$

EXAMPLE 3 Prove the Vertical Angles Congruence Theorem

Prove vertical angles are congruent.

GIVEN $\angle 5$ and $\angle 7$ are vertical angles.

PROVE $\angle 5 \cong \angle 7$



STATEMENTS

1. $\angle 5$ and $\angle 7$ are vertical angles.
2. $\angle 5$ and $\angle 6$ are a linear pair.
 $\angle 6$ and $\angle 7$ are a linear pair.
3. $\angle 5$ and $\angle 6$ are supplementary.
 $\angle 6$ and $\angle 7$ are supplementary.
4. $\angle 5 \cong \angle 7$

REASONS

1. Given
2. Definition of linear pair, as shown in the diagram
3. Linear Pair Postulate
4. Congruent Supplements Theorem

USE A DIAGRAM

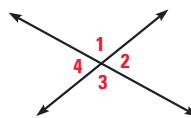
You can use information labeled in a diagram in your proof.



GUIDED PRACTICE for Example 3

In Exercises 3–5, use the diagram.

3. If $m\angle 1 = 112^\circ$, find $m\angle 2$, $m\angle 3$, and $m\angle 4$.
 $68^\circ, 112^\circ, 68^\circ$
4. If $m\angle 2 = 67^\circ$, find $m\angle 1$, $m\angle 3$, and $m\angle 4$.
 $113^\circ, 113^\circ, 67^\circ$
5. If $m\angle 4 = 71^\circ$, find $m\angle 1$, $m\angle 2$, and $m\angle 3$.
 $109^\circ, 71^\circ, 109^\circ$



6. Which previously proven theorem is used in Example 3 as a reason?

Congruent Supplements Theorem



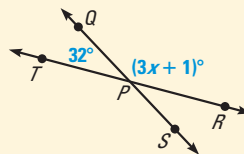
EXAMPLE 4 Standardized Test Practice

ELIMINATE CHOICES

Look for angle pair relationships in the diagram. The angles in the diagram are supplementary, not complementary or congruent, so eliminate choices A and C.

Which equation can be used to find x ?

- (A) $32 + (3x + 1) = 90$
- (B) $32 + (3x + 1) = 180$
- (C) $32 = 3x + 1$
- (D) $3x + 1 = 212$



Solution

Because $\angle TPQ$ and $\angle QPR$ form a linear pair, the sum of their measures is 180° .

► The correct answer is B. (A) (B) (C) (D)



GUIDED PRACTICE for Example 4

Use the diagram in Example 4.

- 7. Solve for x . 49
- 8. Find $m\angle TPS$. 148°

2.7 EXERCISES

HOMEWORK KEY

○ = WORKED-OUT SOLUTIONS on p. WS1 for Exs. 5, 13, and 39

★ = STANDARDIZED TEST PRACTICE Exs. 2, 7, 16, 30, and 45

SKILL PRACTICE

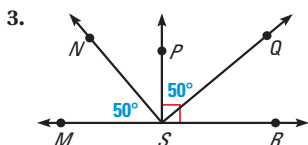
- A** 1. **VOCABULARY** Copy and complete: If two lines intersect at a point, then the ? angles formed by the intersecting lines are congruent. **vertical**
2. **★ WRITING** Describe the relationship between the angle measures of complementary angles, supplementary angles, vertical angles, and linear pairs. **The sum is 90° , the sum is 180° , same, the sum is 180° .**

EXAMPLES 1 and 2

on pp. 124–125 for Exs. 3–7

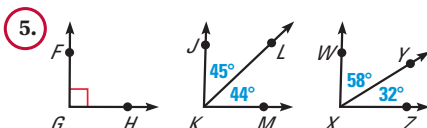
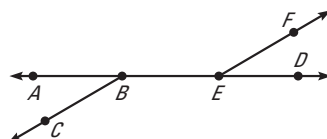
4. $\angle ABC$ and $\angle DEF$, $\angle CBD$ and $\angle FEB$; Congruent Supplements Theorem

IDENTIFY ANGLES Identify the pair(s) of congruent angles in the figures below. Explain how you know they are congruent.

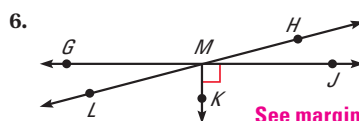


$\angle MSN$ and $\angle PSQ$, $\angle NSP$ and $\angle QSR$; indicated in diagram, Congruent Complements Theorem

4. $\angle ABC$ is supplementary to $\angle CBD$. $\angle CBD$ is supplementary to $\angle DEF$.



$\angle FGH$ and $\angle WXZ$; Right Angles Congruence Theorem



See margin.