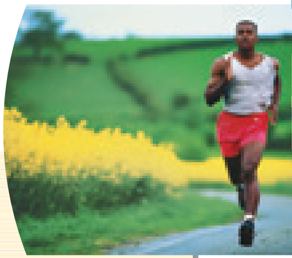
2.5 Reason Using Properties from Algebra



Before You used deductive reasoning to form logical arguments.

Now You will use algebraic properties in logical arguments too.

Why So you can apply a heart rate formula, as in Example 3.

Key Vocabulary
- **equation,** *p. 875*
- **solve an equation,** *p. 875*

When you *solve an equation*, you use properties of real numbers. Segment lengths and angle measures are real numbers, so you can also use these properties to write logical arguments about geometric figures.

KEY CONCEPT *For Your Notebook*

Algebraic Properties of Equality

Let a, b, and c be real numbers.

Addition Property	If $a = b$, then $a + c = b + c$.
Subtraction Property	If $a = b$, then $a - c = b - c$.
Multiplication Property	If $a = b$, then $ac = bc$.
Division Property	If $a = b$ and $c \neq 0$, then $\frac{a}{c} = \frac{b}{c}$.
Substitution Property	If $a = b$, then a can be substituted for b in any equation or expression.

EXAMPLE 1 Write reasons for each step

Solve $2x + 5 = 20 - 3x$. Write a reason for each step.

Equation	Explanation	Reason
$2x + 5 = 20 - 3x$	Write original equation.	Given
$2x + 5 + 3x = 20 - 3x + 3x$	Add $3x$ to each side.	Addition Property of Equality
$5x + 5 = 20$	Combine like terms.	Simplify.
$5x = 15$	Subtract 5 from each side.	Subtraction Property of Equality
$x = 3$	Divide each side by 5.	Division Property of Equality

▶ The value of x is 3.

KEY CONCEPT — For Your Notebook

Distributive Property

$a(b + c) = ab + ac$, where a, b, and c are real numbers.

EXAMPLE 2 Use the Distributive Property

Solve $-4(11x + 2) = 80$. Write a reason for each step.

Solution

Equation	Explanation	Reason
$-4(11x + 2) = 80$	Write original equation.	Given
$-44x - 8 = 80$	Multiply.	Distributive Property
$-44x = 88$	Add 8 to each side.	Addition Property of Equality
$x = -2$	Divide each side by -44.	Division Property of Equality

Animated Geometry at classzone.com

EXAMPLE 3 Use properties in the real world

HEART RATE When you exercise, your target heart rate should be between 50% to 70% of your maximum heart rate. Your target heart rate r at 70% can be determined by the formula $r = 0.70(220 - a)$ where a represents your age in years. Solve the formula for a.

Solution

Equation	Explanation	Reason
$r = 0.70(220 - a)$	Write original equation.	Given
$r = 154 - 0.70a$	Multiply.	Distributive Property
$r - 154 = -0.70a$	Subtract 154 from each side.	Subtraction Property of Equality
$\frac{r - 154}{-0.70} = a$	Divide each side by -0.70.	Division Property of Equality

✓ GUIDED PRACTICE for Examples 1, 2, and 3

In Exercises 1 and 2, solve the equation and write a reason for each step.

1, 2. See margin.

1. $4x + 9 = -3x + 2$
2. $14x + 3(7 - x) = -1$
3. Solve the formula $A = \frac{1}{2}bh$ for b. $b = \frac{2A}{h}$

Differentiated Instruction

PROPERTIES The following properties of equality are true for all real numbers. Segment lengths and angle measures are real numbers, so these properties of equality are true for segment lengths and angle measures.

KEY CONCEPT *For Your Notebook*

Reflexive Property of Equality

Real Numbers	For any real number a, $a = a$.
Segment Length	For any segment $\overline{AB}$, $AB = AB$.
Angle Measure	For any angle $\angle A$, $m\angle A = m\angle A$.

Symmetric Property of Equality

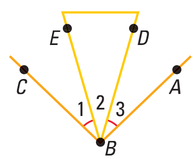
Real Numbers	For any real numbers a and b, if $a = b$, then $b = a$.
Segment Length	For any segments $\overline{AB}$ and $\overline{CD}$, if $AB = CD$, then $CD = AB$.
Angle Measure	For any angles $\angle A$ and $\angle B$, if $m\angle A = m\angle B$, then $m\angle B = m\angle A$.

Transitive Property of Equality

Real Numbers	For any real numbers a, b, and c, if $a = b$ and $b = c$, then $a = c$.
Segment Length	For any segments $\overline{AB}$, $\overline{CD}$, and $\overline{EF}$, if $AB = CD$ and $CD = EF$, then $AB = EF$.
Angle Measure	For any angles $\angle A$, $\angle B$, and $\angle C$, if $m\angle A = m\angle B$ and $m\angle B = m\angle C$, then $m\angle A = m\angle C$.

EXAMPLE 4 Use properties of equality

LOGO You are designing a logo to sell daffodils. Use the information given. Determine whether $m\angle EBA = m\angle DBC$.

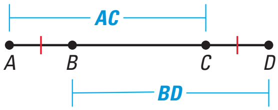


Solution

Equation	Explanation	Reason
$m\angle 1 = m\angle 3$	Marked in diagram.	**Given**
$m\angle EBA = m\angle 3 + m\angle 2$	Add measures of adjacent angles.	**Angle Addition Postulate**
$m\angle EBA = m\angle 1 + m\angle 2$	Substitute $m\angle 1$ for $m\angle 3$.	**Substitution Property of Equality**
$m\angle 1 + m\angle 2 = m\angle DBC$	Add measures of adjacent angles.	**Angle Addition Postulate**
$m\angle EBA = m\angle DBC$	Both measures are equal to the sum of $m\angle 1 + m\angle 2$.	**Transitive Property of Equality**

EXAMPLE 5 Use properties of equality

In the diagram, $AB = CD$. Show that $AC = BD$.



Solution

Equation	Explanation	Reason
$AB = CD$	Marked in diagram.	**Given**
$AC = AB + BC$	Add lengths of adjacent segments.	**Segment Addition Postulate**
$BD = BC + CD$	Add lengths of adjacent segments.	**Segment Addition Postulate**
$AB + BC = CD + BC$	Add BC to each side of $AB = CD$.	**Addition Property of Equality**
$AC = BD$	Substitute AC for $AB + BC$ and BD for $BC + CD$.	**Substitution Property of Equality**

✓ GUIDED PRACTICE for Examples 4 and 5

Name the property of equality the statement illustrates.

4. If $m\angle 6 = m\angle 7$, then $m\angle 7 = m\angle 6$. **Symmetric Property of Equality**
5. If $JK = KL$ and $KL = 12$, then $JK = 12$. **Transitive Property of Equality**
6. $m\angle W = m\angle W$ **Reflexive Property of Equality**

2.5 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 9, 21, and 31

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 5, 27, and 35

◆ = **MULTIPLE REPRESENTATIONS** Ex. 36

SKILL PRACTICE

A

1. **VOCABULARY** The following statement is true because of what property? The measure of an angle is equal to itself. **Reflexive Property of Equality for Angle Measure**

2. ★ **WRITING** *Explain* how to check the answer to Example 3 on page 106. **Substitute the value of *a* into the original equation to see if it is a solution.**

EXAMPLES 1 and 2 on pp. 105–106 for Exs. 3–14

WRITING REASONS **Copy the logical argument. Write a reason for each step.** **3, 4. See margin.**

3. $3x - 12 = 7x + 8$ **Given**
   $-4x - 12 = 8$?
   $-4x = 20$?
   $x = -5$?

4. $5(x - 1) = 4x + 13$ **Given**
   $5x - 5 = 4x + 13$?
   $x - 5 = 13$?
   $x = 18$?