2.3 Apply Deductive Reasoning

Before	You used inductive reasoning to form a conjecture.
Now	You will use deductive reasoning to form a logical argument.
Why	So you can reach logical conclusions about locations, as in Ex. 18.

Key Vocabulary
- **deductive reasoning**

Deductive reasoning uses facts, definitions, accepted properties, and the laws of logic to form a logical argument. This is different from *inductive reasoning*, which uses specific examples and patterns to form a conjecture.

READ VOCABULARY
The Law of Detachment is also called a *direct argument*. The Law of Syllogism is sometimes called the *chain rule*.

KEY CONCEPT *For Your Notebook*

Laws of Logic

Law of Detachment

If the hypothesis of a true conditional statement is true, then the conclusion is also true.

Law of Syllogism

If **hypothesis** p, then **conclusion** q.
If **hypothesis** q, then **conclusion** r. } If these statements are true,
If **hypothesis** p, then **conclusion** r. ← then this statement is true.

EXAMPLE 1 Use the Law of Detachment

Use the Law of Detachment to make a valid conclusion in the true situation.

a. If two segments have the same length, then they are congruent. You know that $BC = XY$.

b. Mary goes to the movies every Friday and Saturday night. Today is Friday.

Solution

a. Because $BC = XY$ satisfies the hypothesis of a true conditional statement, the conclusion is also true. So, $\overline{BC} \cong \overline{XY}$.

b. First, identify the hypothesis and the conclusion of the first statement. The hypothesis is "If it is Friday or Saturday night," and the conclusion is "then Mary goes to the movies."

"Today is Friday" satisfies the hypothesis of the conditional statement, so you can conclude that Mary will go to the movies tonight.

EXAMPLE 2 Use the Law of Syllogism

If possible, use the Law of Syllogism to write a new conditional statement that follows from the pair of true statements.

a. If Rick takes chemistry this year, then Jesse will be Rick's lab partner.

If Jesse is Rick's lab partner, then Rick will get an A in chemistry.

b. If $x^2 > 25$, then $x^2 > 20$.

If $x > 5$, then $x^2 > 25$.

c. If a polygon is regular, then all angles in the interior of the polygon are congruent.

If a polygon is regular, then all of its sides are congruent.

Solution

a. The conclusion of the first statement is the hypothesis of the second statement, so you can write the following new statement.

If Rick takes chemistry this year, then Rick will get an A in chemistry.

AVOID ERRORS The order in which the statements are given does not affect whether you can use the Law of Syllogism.

b. Notice that the conclusion of the second statement is the hypothesis of the first statement, so you can write the following new statement.

If $x > 5$, then $x^2 > 20$.

c. Neither statement's conclusion is the same as the other statement's hypothesis. You cannot use the Law of Syllogism to write a new conditional statement.

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GUIDED PRACTICE for Examples 1 and 2

1. If $90° < m\angle R < 180°$, then $\angle R$ is obtuse. The measure of $\angle R$ is 155°. Using the Law of Detachment, what statement can you make? $\angle R$ is obtuse.

2. If Jenelle gets a job, then she can afford a car. If Jenelle can afford a car, then she will drive to school. Using the Law of Syllogism, what statement can you make? If Janelle gets a job, then she will drive to school.

State the law of logic that is illustrated.

3. If you get an A or better on your math test, then you can go to the movies. If you go to the movies, then you can watch your favorite actor.

If you get an A or better on your math test, then you can watch your favorite actor. Law of Syllogism

4. If $x > 12$, then $x + 9 > 20$. The value of x is 14.

Therefore, $x + 9 > 20$. Law of Detachment

ANALYZING REASONING In Geometry, you will frequently use inductive reasoning to make conjectures. You will also be using deductive reasoning to show that conjectures are true or false. You will need to know which type of reasoning is being used.

EXAMPLE 3 Use inductive and deductive reasoning

ALGEBRA What conclusion can you make about the product of an even integer and any other integer?

Solution

STEP 1 **Look** for a pattern in several examples. Use inductive reasoning to make a conjecture.

$(-2)(2) = -4$, $(-1)(2) = -2$, $2(2) = 4$, $3(2) = 6$,

$(-2)(-4) = 8$, $(-1)(-4) = 4$, $2(-4) = -8$, $3(-4) = -12$

Conjecture Even integer • Any integer = Even integer

STEP 2 **Let** n and m each be any integer. Use deductive reasoning to show the conjecture is true.

$2n$ is an even integer because any integer multiplied by 2 is even.

$2nm$ represents the product of an even integer and any integer m.

$2nm$ is the product of 2 and an integer nm. So, $2nm$ is an even integer.

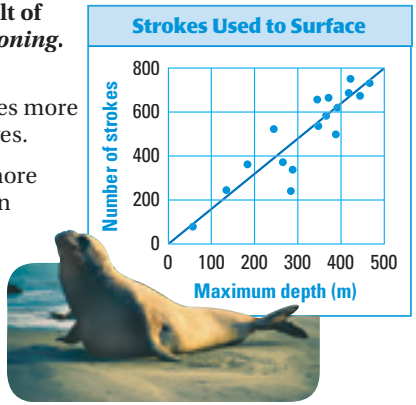
▸ The product of an even integer and any integer is an even integer.

EXAMPLE 4 Reasoning from a graph

Tell whether the statement is the result of *inductive reasoning* or *deductive reasoning*. Explain your choice.

a. The northern elephant seal requires more strokes to surface the deeper it dives.

b. The northern elephant seal uses more strokes to surface from 60 feet than from 250 feet.



Solution

a. Inductive reasoning, because it is based on a pattern in the data

b. Deductive reasoning, because you are comparing values that are given on the graph

5. The sum of a number and itself is twice the number; $n + n = 2n$.

6. Sample answer; The more strokes it takes for the northern elephant to surface, the deeper it dove; the northern elephant seal uses less strokes to surface from 200 feet than from 400 feet.

GUIDED PRACTICE for Examples 3 and 4

5. Use inductive reasoning to make a conjecture about the sum of a number and itself. Then use deductive reasoning to show the conjecture is true. See margin.

6. Use inductive reasoning to write another statement about the graph in Example 4. Then use deductive reasoning to write another statement. See margin.

Differentiated Instruction