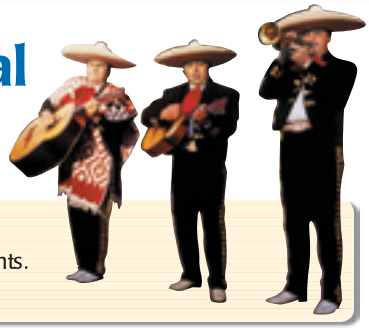
2.2 Analyze Conditional Statements



Before You used definitions.
Now You will write definitions as conditional statements.
Why? So you can verify statements, as in Example 2.

Key Vocabulary
- **conditional statement** converse, inverse, contrapositive
- **if-then form** hypothesis, conclusion
- **negation**
- **equivalent statements**
- **perpendicular lines**
- **biconditional statement**

A **conditional statement** is a logical statement that has two parts, a *hypothesis* and a *conclusion*. When a conditional statement is written in **if-then form**, the "if" part contains the **hypothesis** and the "then" part contains the **conclusion**. Here is an example:

If **it is raining**, then **there are clouds in the sky**.
Hypothesis — Conclusion

EXAMPLE 1 Rewrite a statement in if-then form

Rewrite the conditional statement in if-then form.

a. All birds have feathers.

b. Two angles are supplementary if they are a linear pair.

Solution

First, identify the **hypothesis** and the **conclusion**. When you rewrite the statement in if-then form, you may need to reword the hypothesis or conclusion.

a. **All birds** have **feathers**.
If **an animal is a bird**, then **it has feathers**.

b. **Two angles are supplementary** if **they are a linear pair**.
If **two angles are a linear pair**, then **they are supplementary**.

GUIDED PRACTICE for Example 1

Rewrite the conditional statement in if-then form.

1. All 90° angles are right angles.
If the measure of an angle is 90°, then it is a right angle.

2. $2x + 7 = 1$, because $x = -3$.
If $2x + 7 = 1$, then $x = -3$.

3. When $n = 9$, $n^2 = 81$.
If $n = 9$, then $n^2 = 81$.

4. Tourists at the Alamo are in Texas.
If tourists are at the Alamo, then they are in Texas.

NEGATION The **negation** of a statement is the *opposite* of the original statement. Notice that Statement 2 is already negative, so its negation is positive.

Statement 1 The ball is red.
Negation 1 The ball is *not* red.

Statement 2 The cat is *not* black.
Negation 2 The cat is black.

VERIFYING STATEMENTS Conditional statements can be true or false. To show that a conditional statement is true, you must prove that the conclusion is true every time the hypothesis is true. To show that a conditional statement is false, you need to give *only one* counterexample.

RELATED CONDITIONALS To write the **converse** of a conditional statement, exchange the **hypothesis** and **conclusion**.

READ VOCABULARY
To *negate* part of a conditional statement, you write its negation.

To write the **inverse** of a conditional statement, negate both the hypothesis and the conclusion. To write the **contrapositive**, first write the converse and then negate both the hypothesis and the conclusion.

Conditional statement If $m\angle A = 99°$, then $\angle A$ is obtuse.	both true
Converse If $\angle A$ is obtuse, then $m\angle A = 99°$.	both false
Inverse If $m\angle A \neq 99°$, then $\angle A$ is not obtuse.	both false
Contrapositive If $\angle A$ is not obtuse, then $m\angle A \neq 99°$.	both true

EXAMPLE 2 Write four related conditional statements

Write the if-then form, the converse, the inverse, and the contrapositive of the conditional statement "Guitar players are musicians." Decide whether each statement is *true* or *false*.

Solution

If-then form If you are a guitar player, then you are a musician.
True, guitars players are musicians.

Converse If you are a musician, then you are a guitar player.
False, not all musicians play the guitar.

Inverse If you are not a guitar player, then you are not a musician.
False, even if you don't play a guitar, you can still be a musician.

Contrapositive If you are not a musician, then you are not a guitar player. *True*, a person who is not a musician cannot be a guitar player.

GUIDED PRACTICE for Example 2

Write the converse, the inverse, and the contrapositive of the conditional statement. Tell whether each statement is *true* or *false*.

5. If a dog is a Great Dane, then it is large.

6. If a polygon is equilateral, then the polygon is regular.

5. If a dog is large, then it is a Great Dane, false; if a dog is not a Great Dane, then it is not large, false; if a dog is not large, then it is not a Great Dane, true.

6. If a polygon is regular, then it is equilateral, true; if a polygon is not equilateral, then it is not regular, true; if a polygon is not regular, then it is not equilateral, false.

EQUIVALENT STATEMENTS A conditional statement and its contrapositive are either both true or both false. Similarly, the converse and inverse of a conditional statement are either both true or both false. Pairs of statements such as these are called *equivalent statements*. In general, when two statements are both true or both false, they are called **equivalent statements**.

DEFINITIONS You can write a definition as a conditional statement in if-then form or as its converse. Both the conditional statement and its converse are true. For example, consider the definition of *perpendicular lines*.

READ DIAGRAMS
In a diagram, a red square may be used to indicate a right angle or that two intersecting lines are perpendicular.

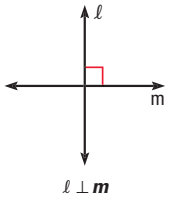
KEY CONCEPT *For Your Notebook*

Perpendicular Lines

Definition If two lines intersect to form a right angle, then they are **perpendicular lines**.

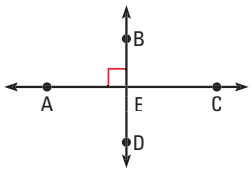
The definition can also be written using the converse: If two lines are perpendicular lines, then they intersect to form a right angle.

You can write "line ℓ is perpendicular to line m" as $\ell \perp m$.



EXAMPLE 3 Use definitions

Decide whether each statement about the diagram is true. Explain your answer using the definitions you have learned.



a. $\overleftrightarrow{AC} \perp \overleftrightarrow{BD}$

b. $\angle AEB$ and $\angle CEB$ are a linear pair.

c. $\overrightarrow{EA}$ and $\overrightarrow{EB}$ are opposite rays.

Solution

a. This statement is *true*. The right angle symbol in the diagram indicates that the lines intersect to form a right angle. So you can say the lines are perpendicular.

b. This statement is *true*. By definition, if the noncommon sides of adjacent angles are opposite rays, then the angles are a linear pair. Because $\overrightarrow{EA}$ and $\overrightarrow{EC}$ are opposite rays, $\angle AEB$ and $\angle CEB$ are a linear pair.

c. This statement is *false*. Point E does not lie on the same line as A and B, so the rays are not opposite rays.

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GUIDED PRACTICE for Example 3

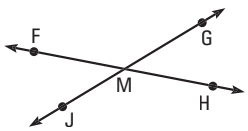
Use the diagram shown. Decide whether each statement is true. *Explain* your answer using the definitions you have learned.

7. $\angle JMF$ and $\angle FMG$ are supplementary.

8. Point M is the midpoint of $\overline{FH}$.

9. $\angle JMF$ and $\angle HMG$ are vertical angles.

10. $\overleftrightarrow{FH} \perp \overleftrightarrow{JG}$



7. True; linear pairs of angles are supplementary.

8. False; it is not known that $\overline{FM} \cong \overline{MH}$.

9. True; two intersecting lines form 2 pair of vertical angles.

10. False; it is not known that the lines intersect at right angles.

READ DEFINITIONS
All definitions can be interpreted forward and backward in this way.

BICONDITIONAL STATEMENTS When a conditional statement and its converse are both true, you can write them as a single *biconditional statement*. A **biconditional statement** is a statement that contains the phrase "if and only if."

Any valid definition can be written as a biconditional statement.

EXAMPLE 4 Write a biconditional

Write the definition of perpendicular lines as a biconditional.

Solution

Definition If two lines intersect to form a right angle, then they are perpendicular.
Converse If two lines are perpendicular, then they intersect to form a right angle.
Biconditional Two lines are perpendicular if and only if they intersect to form a right angle.

GUIDED PRACTICE for Example 4

11. Rewrite the definition of *right angle* as a biconditional statement.

12. Rewrite the statements as a biconditional.

If Mary is in theater class, she will be in the fall play. If Mary is in the fall play, she must be taking theater class.

11. An angle is a right angle if and only if the measure of the angle is 90°.

12. Mary is in the theater class if and only if she will be in the fall play.

2.2 EXERCISES

HOMEWORK KEY
- = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 11, 17, and 33
- ★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 25, 29, 33, 34, and 35

SKILL PRACTICE

A

1. VOCABULARY Copy and complete: The ? of a conditional statement is found by switching the hypothesis and the conclusion. converse

2. ★ WRITING Write a definition for the term *collinear points*, and show how the definition can be interpreted as a biconditional. Points are collinear if one line contains them; points are collinear if and only if one line contains the points.

EXAMPLE 1 on p. 79 for Exs. 3–6

REWRITING STATEMENTS **Rewrite the conditional statement in if-then form.**

3. When $x = 6$, $x^2 = 36$. If $x = 6$, then $x^2 = 36$.

4. The measure of a straight angle is 180°.
If an angle is a straight angle, then its measure is 180°.

5. Only people who are registered are allowed to vote.
If a person is registered to vote, then they are allowed to vote.

6. ERROR ANALYSIS *Describe* and correct the error in writing the if-then statement.

Given statement: All high school students take four English courses.

If-then statement: If a high school student takes four courses, then all four are English courses.

6. The hypothesis and conclusion are not written correctly; if a student is a high school student, then they take four English courses.