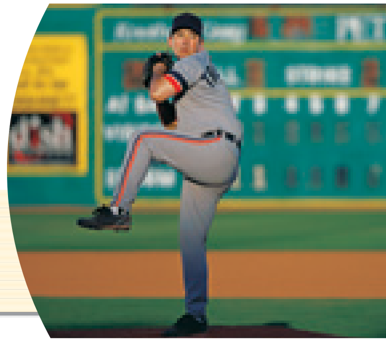
2.1 Use Inductive Reasoning



Before You classified polygons by the number of sides.

Now You will describe patterns and use inductive reasoning.

Why? So you can make predictions about baseball, as in Ex. 32.

Key Vocabulary
- **conjecture**
- **inductive reasoning**
- **counterexample**

Geometry, like much of science and mathematics, was developed partly as a result of people recognizing and describing patterns. In this lesson, you will discover patterns yourself and use them to make predictions.

EXAMPLE 1 Describe a visual pattern

Describe how to sketch the fourth figure in the pattern. Then sketch the fourth figure.

Figure 1

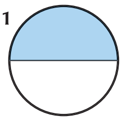



Figure 2

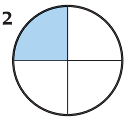
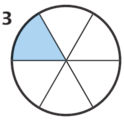



Figure 3


Solution

Each circle is divided into twice as many equal regions as the figure number. Sketch the fourth figure by dividing a circle into eighths. Shade the section just above the horizontal segment at the left.

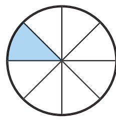


Figure 4

EXAMPLE 2 Describe a number pattern

READ SYMBOLS
The three dots (. . .) tell you that the pattern continues.

Describe the pattern in the numbers −7, −21, −63, −189, . . . and write the next three numbers in the pattern.

Notice that each number in the pattern is three times the previous number.

$$-7, \xrightarrow{\times 3} -21, \xrightarrow{\times 3} -63, \xrightarrow{\times 3} -189, \ldots \xrightarrow{\times 3}$$

▶ Continue the pattern. The next three numbers are −567, −1701, and −5103.

Animated Geometry at classzone.com

✓ GUIDED PRACTICE for Examples 1 and 2

1. Sketch the fifth figure in the pattern in Example 1. **See margin.**

2. *Describe* the pattern in the numbers 5.01, 5.03, 5.05, 5.07, ... Write the next three numbers in the pattern. **The numbers are increasing by 0.02; 5.09, 5.11, 5.13.**

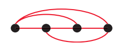
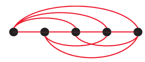
INDUCTIVE REASONING A **conjecture** is an unproven statement that is based on observations. You use **inductive reasoning** when you find a pattern in specific cases and then write a conjecture for the general case.

EXAMPLE 3 Make a conjecture

Given five collinear points, make a conjecture about the number of ways to connect different pairs of the points.

Solution

Make a table and look for a pattern. Notice the pattern in how the number of connections increases. You can use the pattern to make a conjecture.

Number of points	1	2	3	4	5
Picture					
Number of connections	0	1	3	6	?

+ 1   + 2   + 3   + ?

▶ **Conjecture** You can connect five collinear points 6 + **4**, or 10 different ways.

EXAMPLE 4 Make and test a conjecture

Numbers such as 3, 4, and 5 are called *consecutive numbers*. Make and test a conjecture about the sum of any three consecutive numbers.

Solution

STEP 1 **Find** a pattern using a few groups of small numbers.

$3 + 4 + 5 = 12 = 4 \cdot 3$ $7 + 8 + 9 = 24 = 8 \cdot 3$

$10 + 11 + 12 = 33 = 11 \cdot 3$ $16 + 17 + 18 = 51 = 17 \cdot 3$

▶ **Conjecture** The sum of any three consecutive integers is three times the second number.

STEP 2 **Test** your conjecture using other numbers. For example, test that it works with the groups −1, 0, 1 and 100, 101, 102.

$-1 + 0 + 1 = 0 = 0 \cdot 3$ ✓ $100 + 101 + 102 = 303 = 101 \cdot 3$ ✓

✓ GUIDED PRACTICE for Examples 3 and 4

3. Suppose you are given seven collinear points. Make a conjecture about the number of ways to connect different pairs of the points. **You can connect seven collinear points 15 + 6 or 21 different ways.**
4. Make and test a conjecture about the sign of the product of any three negative integers.

4. The result of the product of three negative numbers is a negative number. Sample answer: $-2 \cdot -5 \cdot -4 = -40$.

Differentiated Instruction

DISPROVING CONJECTURES To show that a conjecture is true, you must show that it is true for all cases. You can show that a conjecture is false, however, by simply finding one *counterexample*. A **counterexample** is a specific case for which the conjecture is false.

EXAMPLE 5 Find a counterexample

A student makes the following conjecture about the sum of two numbers. Find a counterexample to disprove the student's conjecture.

Conjecture The sum of two numbers is always greater than the larger number.

Solution

To find a counterexample, you need to find a sum that is less than the larger number.

$-2 + -3 = -5$

$-5 \ngtr -3$

▶ Because a counterexample exists, the conjecture is false.

★ EXAMPLE 6 Standardized Test Practice

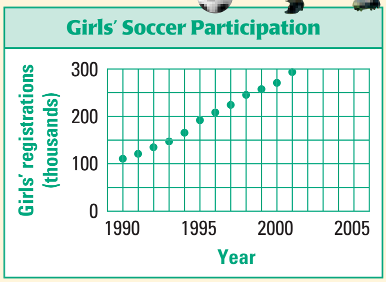
Which conjecture could a high school athletic director make based on the graph at the right?

Ⓐ More boys play soccer than girls.

Ⓑ More girls are playing soccer today than in 1995.

Ⓒ More people are playing soccer today than in the past because the 1994 World Cup games were held in the United States.

Ⓓ The number of girls playing soccer was more in 1995 than in 2001.



ELIMINATE CHOICES
Because the graph does not show data about boys or the World Cup games, you can eliminate choices A and C.

Solution

Choices A and C can be eliminated because they refer to facts not presented by the graph. Choice B is a reasonable conjecture because the graph shows an increase from 1990–2001, but does not give any reasons for that increase.

▶ The correct answer is B. Ⓐ Ⓑ Ⓒ Ⓓ

6. The number of girls playing soccer in the year 2001 will increase over previous years; the number of girls participating in soccer has increased for the past 11 years.

✓ GUIDED PRACTICE for Examples 5 and 6

5. Find a counterexample to show that the following conjecture is false.
Conjecture The value of x^2 is always greater than the value of x.

Sample answer: $x = \frac{1}{2}, x^2 = \frac{1}{4}$

6. Use the graph in Example 6 to make a conjecture that *could* be true. Give an explanation that supports your reasoning.

Differentiated Instruction