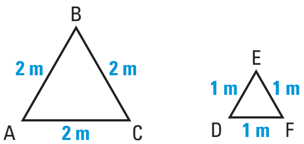
6.1 Ratios, Proportions, and the Geometric Mean

Before You solved problems by writing and solving equations.
Now You will solve problems by writing and solving proportions.
Why? So you can estimate bird populations, as in Ex. 62.

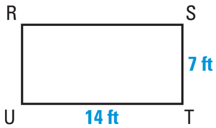
Key Vocabulary
- **ratio**
- **proportion**
  means, extremes
- **geometric mean**

If a and b are two numbers or quantities and $b \neq 0$, then the **ratio of a to b** is $\frac{a}{b}$. The ratio of a to b can also be written as $a:b$.

For example, the ratio of a side length in $\triangle ABC$ to a side length in $\triangle DEF$ can be written as $\frac{2}{1}$ or $2:1$.



Ratios are usually expressed in simplest form. Two ratios that have the same simplified form are called *equivalent ratios*. The ratios $7:14$ and $1:2$ in the example below are *equivalent*.



$$\frac{\text{width of } RSTU}{\text{length of } RSTU} = \frac{7 \text{ ft}}{14 \text{ ft}} = \frac{1}{2}$$

EXAMPLE 1 Simplify ratios

Simplify the ratio.

a. $64 \text{ m} : 6 \text{ m}$

b. $\frac{5 \text{ ft}}{20 \text{ in.}}$

Solution

a. Write $64 \text{ m} : 6 \text{ m}$ as $\frac{64 \text{ m}}{6 \text{ m}}$. Then divide out the units and simplify.

$$\frac{64 \not{\text{m}}}{6 \not{\text{m}}} = \frac{32}{3} = 32:3$$

b. To simplify a ratio with unlike units, multiply by a conversion factor.

$$\frac{5 \text{ ft}}{20 \text{ in.}} = \frac{5 \not{\text{ft}}}{20 \not{\text{in.}}} \cdot \frac{12 \not{\text{in.}}}{1 \not{\text{ft}}} = \frac{60}{20} = \frac{3}{1}$$

REVIEW UNIT ANALYSIS
For help with measures and conversion factors, see p. 886 and the Table of Measures on p. 921.

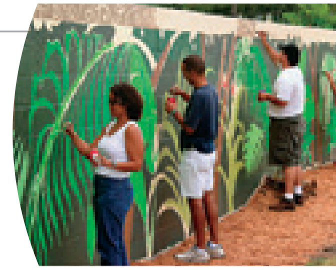
✓ GUIDED PRACTICE for Example 1

Simplify the ratio.

1. 24 yards to 3 yards **8 to 1**

2. $150 \text{ cm} : 6 \text{ m}$ **1 : 4**

EXAMPLE 2 Use a ratio to find a dimension



PAINTING You are planning to paint a mural on a rectangular wall. You know that the perimeter of the wall is 484 feet and that the ratio of its length to its width is 9 : 2. Find the area of the wall.

WRITE EXPRESSIONS

Because the ratio in Example 2 is 9 : 2, you can write an equivalent ratio to find expressions for the length and width.

$$\frac{\text{length}}{\text{width}} = \frac{9}{2} = \frac{9}{2} \cdot \frac{x}{x} = \frac{9x}{2x}$$

Solution

STEP 1 **Write** expressions for the length and width. Because the ratio of length to width is 9 : 2, you can represent the length by $9x$ and the width by $2x$.

STEP 2 **Solve** an equation to find x.

$2\ell + 2w = P$	Formula for perimeter of rectangle
$2(9x) + 2(2x) = 484$	Substitute for ℓ, w, and P.
$22x = 484$	Multiply and combine like terms.
$x = 22$	Divide each side by 22.

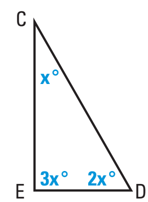
STEP 3 **Evaluate** the expressions for the length and width. Substitute the value of x into each expression.

Length $= 9x = 9(22) = 198$ Width $= 2x = 2(22) = 44$

▶ The wall is 198 feet long and 44 feet wide, so its area is $198 \text{ ft} \cdot 44 \text{ ft} = 8712 \text{ ft}^2$.

EXAMPLE 3 Use extended ratios

ALGEBRA **The measures of the angles in △*CDE* are in the *extended ratio* of 1 : 2 : 3. Find the measures of the angles.**



Solution

Begin by sketching the triangle. Then use the extended ratio of 1 : 2 : 3 to label the measures as $x°$, $2x°$, and $3x°$.

$x° + 2x° + 3x° = 180°$	Triangle Sum Theorem
$6x = 180$	Combine like terms.
$x = 30$	Divide each side by 6.

▶ The angle measures are 30°, 2(30°) = 60°, and 3(30°) = 90°.

✓ GUIDED PRACTICE for Examples 2 and 3

3. The perimeter of a room is 48 feet and the ratio of its length to its width is 7 : 5. Find the length and width of the room. **14 ft, 10 ft**

4. A triangle's angle measures are in the extended ratio of 1 : 3 : 5. Find the measures of the angles. **20°, 60°, 100°**

PROPORTIONS An equation that states that two ratios are equal is called a **proportion**.

$$\text{extreme} \rightarrow \frac{a}{b} = \frac{c}{d} \leftarrow \text{mean}$$
$$\text{mean} \rightarrow b \qquad d \leftarrow \text{extreme}$$

The numbers b and c are the **means** of the proportion. The numbers a and d are the **extremes** of the proportion.

The property below can be used to solve proportions. To *solve a proportion*, you find the value of any variable in the proportion.

PROPORTIONS
You will learn more properties of proportions on p. 364.

KEY CONCEPT — *For Your Notebook*

A Property of Proportions

1. Cross Products Property In a proportion, the product of the extremes equals the product of the means.

If $\frac{a}{b} = \frac{c}{d}$ where $b \neq 0$ and $d \neq 0$, then $ad = bc$.

$$\frac{2}{3} = \frac{4}{6} \qquad 3 \cdot 4 = 12 \qquad 2 \cdot 6 = 12$$

EXAMPLE 4 Solve proportions

ALGEBRA **Solve the proportion.**

a. $\frac{5}{10} = \frac{x}{16}$ **b.** $\frac{1}{y+1} = \frac{2}{3y}$

Solution

a.

$\frac{5}{10} = \frac{x}{16}$	Write original proportion.
$5 \cdot 16 = 10 \cdot x$	Cross Products Property
$80 = 10x$	Multiply.
$8 = x$	Divide each side by 10.

b.

$\frac{1}{y+1} = \frac{2}{3y}$	Write original proportion.
$1 \cdot 3y = 2(y+1)$	Cross Products Property
$3y = 2y + 2$	Distributive Property
$y = 2$	Subtract $2y$ from each side.

ANOTHER WAY
In part (a), you could multiply each side by the denominator, 16. Then $16 \cdot \frac{5}{10} = 16 \cdot \frac{x}{16}$, so $8 = x$.

✓ GUIDED PRACTICE for Example 4

Solve the proportion.

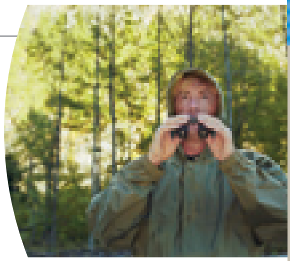
5. $\frac{2}{x} = \frac{5}{8}$ $\frac{16}{5}$

6. $\frac{1}{x-3} = \frac{4}{3x}$ 12

7. $\frac{y-3}{7} = \frac{y}{14}$ 6

Differentiated Instruction

EXAMPLE 5 Solve a real-world problem



SCIENCE As part of an environmental study, you need to estimate the number of trees in a 150 acre area. You count 270 trees in a 2 acre area and you notice that the trees seem to be evenly distributed. Estimate the total number of trees.

Solution

Write and solve a proportion involving two ratios that compare the number of trees with the area of the land.

$\frac{270}{2} = \frac{n}{150}$ ← number of trees / ← area in acres — **Write proportion.**

$270 \cdot 150 = 2 \cdot n$ — **Cross Products Property**

$20{,}250 = n$ — **Simplify.**

▶ There are about 20,250 trees in the 150 acre area.

KEY CONCEPT — *For Your Notebook*

Geometric Mean

The **geometric mean** of two positive numbers a and b is the positive number x that satisfies $\frac{a}{x} = \frac{x}{b}$. So, $x^2 = ab$ and $x = \sqrt{ab}$.

EXAMPLE 6 Find a geometric mean

Find the geometric mean of 24 and 48.

Solution

$x = \sqrt{ab}$ — **Definition of geometric mean**

$= \sqrt{24 \cdot 48}$ — **Substitute 24 for *a* and 48 for *b*.**

$= \sqrt{24 \cdot 24 \cdot 2}$ — **Factor.**

$= 24\sqrt{2}$ — **Simplify.**

▶ The geometric mean of 24 and 48 is $24\sqrt{2} \approx 33.9$.

✓ GUIDED PRACTICE for Examples 5 and 6

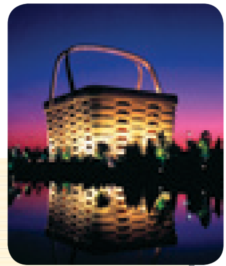
8. **WHAT IF?** In Example 5, suppose you count 390 trees in a 3 acre area of the 150 acre area. Make a new estimate of the total number of trees. **19,500 trees**

Find the geometric mean of the two numbers.

9. 12 and 27 **18**
10. 18 and 54 **$18\sqrt{3}$**
11. 16 and 18 **$12\sqrt{2}$**

Differentiated Instruction

6.2 Use Proportions to Solve Geometry Problems



Before	You wrote and solved proportions.
Now	You will use proportions to solve geometry problems.
Why?	So you can calculate building dimensions, as in Ex. 22.

Key Vocabulary
- **scale drawing**
- **scale**

In Lesson 6.1, you learned to use the Cross Products Property to write equations that are equivalent to a given proportion. Three more ways to do this are given below.

REVIEW RECIPROCALS
For help with reciprocals, see p. 869.

KEY CONCEPT — For Your Notebook

Additional Properties of Proportions

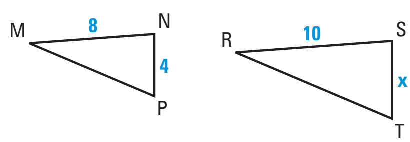
2. **Reciprocal Property** If two ratios are equal, then their reciprocals are also equal. If $\frac{a}{b} = \frac{c}{d}$, then $\frac{b}{a} = \frac{d}{c}$.

3. If you interchange the means of a proportion, then you form another true proportion. If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a}{c} = \frac{b}{d}$.

4. In a proportion, if you add the value of each ratio's denominator to its numerator, then you form another true proportion. If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a+b}{b} = \frac{c+d}{d}$.

EXAMPLE 1 Use properties of proportions

In the diagram, $\frac{MN}{RS} = \frac{NP}{ST}$. Write four true proportions.



Solution

Because $\frac{MN}{RS} = \frac{NP}{ST}$, then $\frac{8}{10} = \frac{4}{x}$.

By the Reciprocal Property, the reciprocals are equal, so $\frac{10}{8} = \frac{x}{4}$.

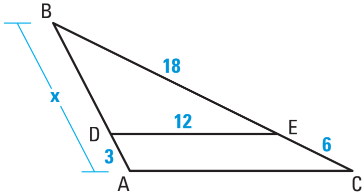
By Property 3, you can interchange the means, so $\frac{8}{4} = \frac{10}{x}$.

By Property 4, you can add the denominators to the numerators, so $\frac{8+10}{10} = \frac{4+x}{x}$, or $\frac{18}{10} = \frac{4+x}{x}$.

EXAMPLE 2 Use proportions with geometric figures

ALGEBRA In the diagram, $\frac{BD}{DA} = \frac{BE}{EC}$.

Find *BA* and *BD*.



Solution

$$\frac{BD}{DA} = \frac{BE}{EC}$$ Given

$$\frac{BD + DA}{DA} = \frac{BE + EC}{EC}$$ Property of Proportions (Property 4)

$$\frac{x}{3} = \frac{18 + 6}{6}$$ Substitution Property of Equality

$$6x = 3(18 + 6)$$ Cross Products Property

$$x = 12$$ Solve for x.

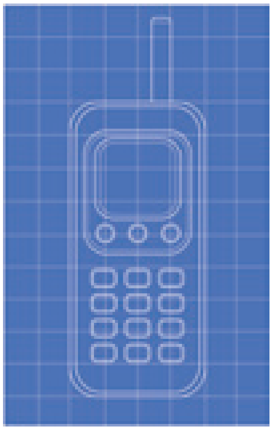
▶ So, $BA = 12$ and $BD = 12 - 3 = 9$.

Animated Geometry at classzone.com

SCALE DRAWING A **scale drawing** is a drawing that is the same shape as the object it represents. The **scale** is a ratio that describes how the dimensions in the drawing are related to the actual dimensions of the object.

EXAMPLE 3 Find the scale of a drawing

BLUEPRINTS The blueprint shows a scale drawing of a cell phone. The length of the antenna on the blueprint is 5 centimeters. The actual length of the antenna is 2 centimeters. What is the scale of the blueprint?



Solution

To find the scale, write the ratio of a length in the drawing to an actual length, then rewrite the ratio so that the denominator is 1.

$$\frac{\text{length on blueprint}}{\text{length of antenna}} = \frac{5\text{ cm}}{2\text{ cm}} = \frac{5 \div 2}{2 \div 2} = \frac{2.5}{1}$$

▶ The scale of the blueprint is 2.5 cm : 1 cm.

✓ GUIDED PRACTICE for Examples 1, 2, and 3

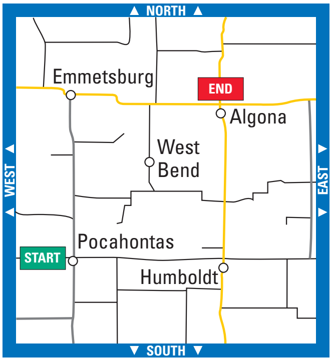
1. In Example 1, find the value of x. **5**

2. In Example 2, $\frac{DE}{AC} = \frac{BE}{BC}$. Find AC. **16**

3. **WHAT IF?** In Example 3, suppose the length of the antenna on the blueprint is 10 centimeters. Find the new scale of the blueprint. $\frac{5\text{ cm}}{1\text{ cm}}$

EXAMPLE 4 Use a scale drawing

MAPS The scale of the map at the right is 1 inch : 26 miles. Find the actual distance from Pocahontas to Algona.



Solution

Use a ruler. The distance from Pocahontas to Algona on the map is about 1.25 inches. Let x be the actual distance in miles.

$$\frac{1.25 \text{ in.}}{x \text{ mi}} = \frac{1 \text{ in.}}{26 \text{ mi}}$$ ← **distance on map** / ← **actual distance**

$x = 1.25(26)$ **Cross Products Property**

$x = 32.5$ **Simplify.**

▶ The actual distance from Pocahontas to Algona is about 32.5 miles.

EXAMPLE 5 Solve a multi-step problem

SCALE MODEL You buy a 3-D scale model of the Reunion Tower in Dallas, TX. The actual building is 560 feet tall. Your model is 10 inches tall, and the diameter of the dome on your scale model is about 2.1 inches.

a. What is the diameter of the actual dome?

b. About how many times as tall as your model is the actual building?

Solution

a. $$\frac{10 \text{ in.}}{560 \text{ ft}} = \frac{2.1 \text{ in.}}{x \text{ ft}}$$ ← **measurement on model** / ← **measurement on actual building**

$10x = 1176$ **Cross Products Property**

$x = 117.6$ **Solve for x.**

▶ The diameter of the actual dome is about 118 feet.

b. To simplify a ratio with unlike units, multiply by a conversion factor.

$$\frac{560 \text{ ft}}{10 \text{ in.}} = \frac{560 \text{ ft}}{10 \text{ in.}} \cdot \frac{12 \text{ in.}}{1 \text{ ft}} = 672$$

▶ The actual building is 672 times as tall as the model.

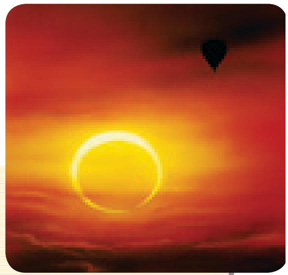
✓ GUIDED PRACTICE for Examples 4 and 5

4. Two cities are 96 miles from each other. The cities are 4 inches apart on a map. Find the scale of the map. **1 in. : 24 mi**

5. **WHAT IF?** Your friend has a model of the Reunion Tower that is 14 inches tall. What is the diameter of the dome on your friend's model? **about 2.95 in.**

Differentiated Instruction

6.3 Use Similar Polygons



Before You used proportions to solve geometry problems.

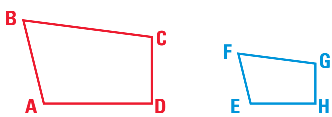
Now You will use proportions to identify similar polygons.

Why? So you can solve science problems, as in Ex. 34.

Key Vocabulary
- **similar polygons**
- **scale factor**

Two polygons are **similar polygons** if corresponding angles are congruent and corresponding side lengths are proportional.

In the diagram below, *ABCD* is similar to *EFGH*. You can write "*ABCD* is similar to *EFGH*" as $ABCD \sim EFGH$. Notice in the similarity statement that the corresponding vertices are listed in the same order.



$ABCD \sim EFGH$

Corresponding angles

$\angle A \cong \angle E$, $\angle B \cong \angle F$, $\angle C \cong \angle G$, and $\angle D \cong \angle H$

Ratios of corresponding sides

$$\frac{AB}{EF} = \frac{BC}{FG} = \frac{CD}{GH} = \frac{DA}{HE}$$

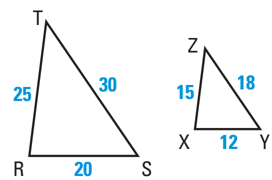
EXAMPLE 1 Use similarity statements

In the diagram, $\triangle RST \sim \triangle XYZ$.

a. List all pairs of congruent angles.

b. Check that the ratios of corresponding side lengths are equal.

c. Write the ratios of the corresponding side lengths in a *statement of proportionality*.



READ VOCABULARY
In a *statement of proportionality*, any pair of ratios forms a true proportion.

Solution

a. $\angle R \cong \angle X$, $\angle S \cong \angle Y$, and $\angle T \cong \angle Z$.

b. $\frac{RS}{XY} = \frac{20}{12} = \frac{5}{3}$ $\frac{ST}{YZ} = \frac{30}{18} = \frac{5}{3}$ $\frac{TR}{ZX} = \frac{25}{15} = \frac{5}{3}$

c. Because the ratios in part (b) are equal, $\frac{RS}{XY} = \frac{ST}{YZ} = \frac{TR}{ZX}$.

✓ GUIDED PRACTICE for Example 1

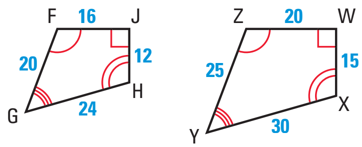
1. Given $\triangle JKL \sim \triangle PQR$, list all pairs of congruent angles. Write the ratios of the corresponding side lengths in a statement of proportionality.

$\angle J \cong \angle P$, $\angle K \cong \angle Q$, $\angle L \cong \angle R$; $\frac{JK}{PQ} = \frac{KL}{QR} = \frac{LJ}{RP}$

SCALE FACTOR If two polygons are similar, then the ratio of the lengths of two corresponding sides is called the **scale factor**. In Example 1, the common ratio of $\frac{5}{3}$ is the scale factor of $\triangle RST$ to $\triangle XYZ$.

EXAMPLE 2 Find the scale factor

Determine whether the polygons are similar. If they are, write a similarity statement and find the scale factor of *ZYXW* to *FGHJ*.



Solution

STEP 1 **Identify** pairs of congruent angles. From the diagram, you can see that $\angle Z \cong \angle F$, $\angle Y \cong \angle G$, and $\angle X \cong \angle H$. Angles W and J are right angles, so $\angle W \cong \angle J$. So, the corresponding angles are congruent.

STEP 2 **Show** that corresponding side lengths are proportional.

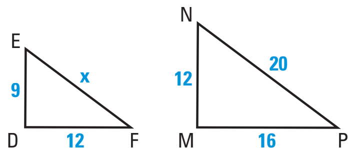
$$\frac{ZY}{FG} = \frac{25}{20} = \frac{5}{4} \qquad \frac{YX}{GH} = \frac{30}{24} = \frac{5}{4} \qquad \frac{XW}{HJ} = \frac{15}{12} = \frac{5}{4} \qquad \frac{WZ}{JF} = \frac{20}{16} = \frac{5}{4}$$

The ratios are equal, so the corresponding side lengths are proportional.

▶ So $ZYXW \sim FGHJ$. The scale factor of $ZYXW$ to $FGHJ$ is $\frac{5}{4}$.

EXAMPLE 3 Use similar polygons

ALGEBRA In the diagram, $\triangle DEF \sim \triangle MNP$. Find the value of x.



Solution

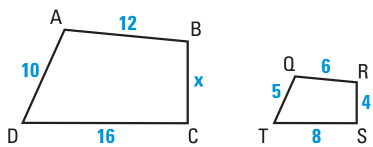
The triangles are similar, so the corresponding side lengths are proportional.

$\frac{MN}{DE} = \frac{NP}{EF}$	Write proportion.
$\frac{12}{9} = \frac{20}{x}$	Substitute.
$12x = 180$	Cross Products Property
$x = 15$	Solve for x.

ANOTHER WAY
There are several ways to write the proportion. For example, you could write $\frac{DF}{MP} = \frac{EF}{NP}$.

✓ GUIDED PRACTICE for Examples 2 and 3

In the diagram, *ABCD* ~ *QRST*.



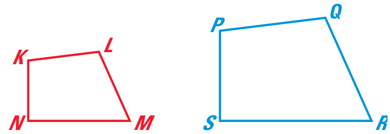
2. What is the scale factor of $QRST$ to $ABCD$? $\frac{1}{2}$
3. Find the value of x. 8

PERIMETERS The ratios of lengths in similar polygons is the same as the scale factor. Theorem 6.1 shows this is true for the perimeters of the polygons.

THEOREM *For Your Notebook*

THEOREM 6.1 Perimeters of Similar Polygons

If two polygons are similar, then the ratio of their perimeters is equal to the ratios of their corresponding side lengths.



If $KLMN \sim PQRS$, then $\frac{KL + LM + MN + NK}{PQ + QR + RS + SP} = \frac{KL}{PQ} = \frac{LM}{QR} = \frac{MN}{RS} = \frac{NK}{SP}$.

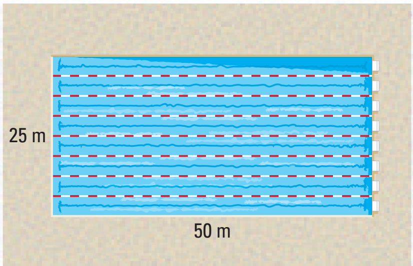
Proof: Ex. 38, p. 379

EXAMPLE 4 Find perimeters of similar figures

SWIMMING A town is building a new swimming pool. An Olympic pool is rectangular with length 50 meters and width 25 meters. The new pool will be similar in shape, but only 40 meters long.

a. Find the scale factor of the new pool to an Olympic pool.

b. Find the perimeter of an Olympic pool and the new pool.



Solution

ANOTHER WAY
Another way to solve Example 4 is to write the scale factor as the decimal 0.8. Then, multiply the perimeter of the Olympic pool by the scale factor to get the perimeter of the new pool:
0.8(150) = 120.

a. Because the new pool will be similar to an Olympic pool, the scale factor is the ratio of the lengths, $\frac{40}{50} = \frac{4}{5}$.

b. The perimeter of an Olympic pool is $2(50) + 2(25) = 150$ meters. You can use Theorem 6.1 to find the perimeter x of the new pool.

$\frac{x}{150} = \frac{4}{5}$ **Use Theorem 6.1 to write a proportion.**

$x = 120$ **Multiply each side by 150 and simplify.**

▶ The perimeter of the new pool is 120 meters.

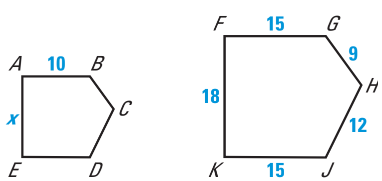
GUIDED PRACTICE for Example 4

In the diagram, *ABCDE* ~ *FGHJK*.

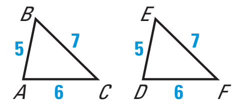
4. Find the scale factor of *FGHJK* to *ABCDE*. $\frac{3}{2}$

5. Find the value of x. **12**

6. Find the perimeter of *ABCDE*. **46**



SIMILARITY AND CONGRUENCE Notice that any two congruent figures are also similar. Their scale factor is 1 : 1. In $\triangle ABC$ and $\triangle DEF$, the scale factor is $\frac{5}{5} = 1$. You can write $\triangle ABC \sim \triangle DEF$ and $\triangle ABC \cong \triangle DEF$.



READ VOCABULARY

For example, *corresponding lengths* in similar triangles include side lengths, altitudes, medians, midsegments, and so on.

CORRESPONDING LENGTHS You know that perimeters of similar polygons are in the same ratio as corresponding side lengths. You can extend this concept to other segments in polygons.

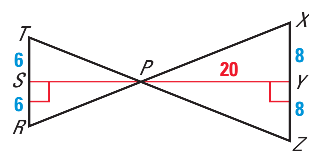
KEY CONCEPT *For Your Notebook*

Corresponding Lengths in Similar Polygons

If two polygons are similar, then the ratio of any two corresponding lengths in the polygons is equal to the scale factor of the similar polygons.

EXAMPLE 5 Use a scale factor

In the diagram, $\triangle TPR \sim \triangle XPZ$. Find the length of the altitude $\overline{PS}$.



Solution

First, find the scale factor of $\triangle TPR$ to $\triangle XPZ$.

$$\frac{TR}{XZ} = \frac{6+6}{8+8} = \frac{12}{16} = \frac{3}{4}$$

Because the ratio of the lengths of the altitudes in similar triangles is equal to the scale factor, you can write the following proportion.

$\frac{PS}{PY} = \frac{3}{4}$ **Write proportion.**

$\frac{PS}{20} = \frac{3}{4}$ **Substitute 20 for *PY*.**

$PS = 15$ **Multiply each side by 20 and simplify.**

▶ The length of the altitude $\overline{PS}$ is 15.

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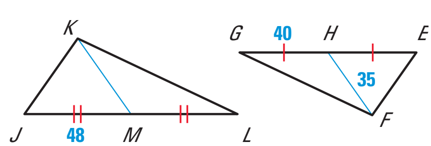
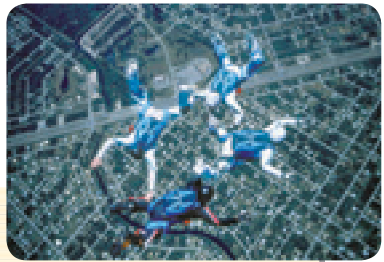
✓ **GUIDED PRACTICE** for Example 5

7. In the diagram, $\triangle JKL \sim \triangle EFG$. Find the length of the median $\overline{KM}$. 42



6.4 Prove Triangles Similar by AA



Before You used the AAS Congruence Theorem.

Now You will use the AA Similarity Postulate.

Why? So you can use similar triangles to understand aerial photography, as in Ex. 34.

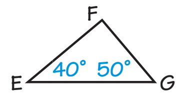
Key Vocabulary
- **similar polygons,** *p. 372*

ACTIVITY ANGLES AND SIMILAR TRIANGLES

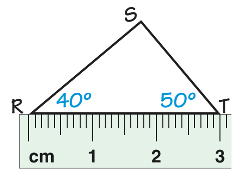
QUESTION What can you conclude about two triangles if you know two pairs of corresponding angles are congruent?

Materials:
- protractor
- metric ruler

STEP 1 **Draw** $\triangle EFG$ so that $m\angle E = 40°$ and $m\angle G = 50°$.



STEP 2 **Draw** $\triangle RST$ so that $m\angle R = 40°$ and $m\angle T = 50°$, and $\triangle RST$ is not congruent to $\triangle EFG$.



STEP 3 **Calculate** $m\angle F$ and $m\angle S$ using the Triangle Sum Theorem. Use a protractor to check that your results are true.

STEP 4 **Measure** and record the side lengths of both triangles. Use a metric ruler.

DRAW CONCLUSIONS

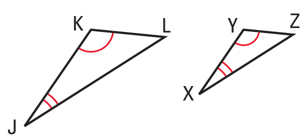
1. Are the triangles similar? Explain your reasoning.
2. Repeat the steps above using different angle measures. Make a conjecture about two triangles with two pairs of congruent corresponding angles.

TRIANGLE SIMILARITY The Activity suggests that two triangles are similar if two pairs of corresponding angles are congruent. In other words, you do not need to know the measures of the sides or the third pair of angles.

POSTULATE *For Your Notebook*

POSTULATE 22 Angle-Angle (AA) Similarity Postulate

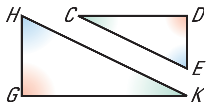
If two angles of one triangle are congruent to two angles of another triangle, then the two triangles are similar.



$\triangle JKL \sim \triangle XYZ$

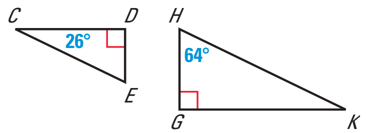
EXAMPLE 1 Use the AA Similarity Postulate

DRAW DIAGRAMS



Use colored pencils to show congruent angles. This will help you write similarity statements.

Determine whether the triangles are similar. If they are, write a similarity statement. Explain your reasoning.



Solution

Because they are both right angles, $\angle D$ and $\angle G$ are congruent.

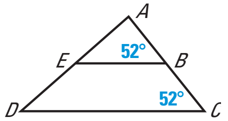
By the Triangle Sum Theorem, $26° + 90° + m\angle E = 180°$, so $m\angle E = 64°$. Therefore, $\angle E$ and $\angle H$ are congruent.

▶ So, $\triangle CDE \sim \triangle KGH$ by the AA Similarity Postulate.

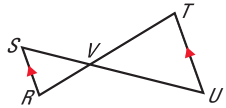
EXAMPLE 2 Show that triangles are similar

Show that the two triangles are similar.

a. $\triangle ABE$ and $\triangle ACD$



b. $\triangle SVR$ and $\triangle UVT$



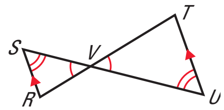
Solution

a. You may find it helpful to redraw the triangles separately.

Because $m\angle ABE$ and $m\angle C$ both equal 52°, $\angle ABE \cong \angle C$. By the Reflexive Property, $\angle A \cong \angle A$.

▶ So, $\triangle ABE \sim \triangle ACD$ by the AA Similarity Postulate.

b. You know $\angle SVR \cong \angle UVT$ by the Vertical Angles Congruence Theorem. The diagram shows $\overline{RS} \parallel \overline{UT}$ so $\angle S \cong \angle U$ by the Alternate Interior Angles Theorem.



▶ So, $\triangle SVR \sim \triangle UVT$ by the AA Similarity Postulate.

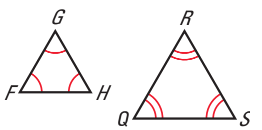
1. In each triangle all three angles measure 60°, so by the AA Similarity Postulate the triangles are similar; $\triangle FGH \sim \triangle QRS$.

2. Since $m\angle CDF = 58°$ by the Triangle Sum Theorem and $m\angle DFE = 90°$ by the Linear Pair Postulate the two triangles are similar by the AA Similarity Postulate; $\triangle CDF \sim \triangle DEF$.

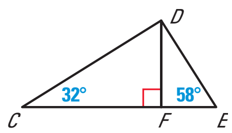
✓ GUIDED PRACTICE for Examples 1 and 2

Show that the triangles are similar. Write a similarity statement.

1. $\triangle FGH$ and $\triangle RQS$



2. $\triangle CDF$ and $\triangle DEF$



3. **REASONING** Suppose in Example 2, part (b), $\overline{SR} \nparallel \overline{TU}$. Could the triangles still be similar? *Explain.*

Yes; if $\angle S \cong \angle T$, the triangles are similar by the AA Similarity Postulate.

INDIRECT MEASUREMENT In Lesson 4.6, you learned a way to use congruent triangles to find measurements indirectly. Another useful way to find measurements indirectly is by using similar triangles.

EXAMPLE 3 Standardized Test Practice

A flagpole casts a shadow that is 50 feet long. At the same time, a woman standing nearby who is five feet four inches tall casts a shadow that is 40 inches long. How tall is the flagpole to the nearest foot?

Ⓐ 12 feet Ⓑ 40 feet

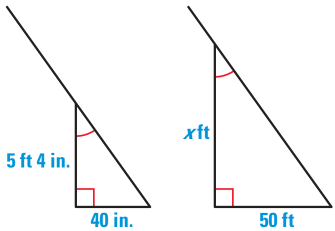
Ⓒ 80 feet Ⓓ 140 feet

ELIMINATE CHOICES

Notice that the woman's height is greater than her shadow's length. So the flagpole must be taller than its shadow's length. Eliminate choices A and B.

Solution

The flagpole and the woman form sides of two right triangles with the ground, as shown below. The sun's rays hit the flagpole and the woman at the same angle. You have two pairs of congruent angles, so the triangles are similar by the AA Similarity Postulate.



You can use a proportion to find the height x. Write 5 feet 4 inches as 64 inches so that you can form two ratios of feet to inches.

$\frac{x \text{ ft}}{64 \text{ in.}} = \frac{50 \text{ ft}}{40 \text{ in.}}$ **Write proportion of side lengths.**

$40x = 64(50)$ **Cross Products Property**

$x = 80$ **Solve for *x*.**

▶ The flagpole is 80 feet tall. The correct answer is C. Ⓐ Ⓑ Ⓒ Ⓓ

✓ GUIDED PRACTICE for Example 3

4. **WHAT IF?** A child who is 58 inches tall is standing next to the woman in Example 3. How long is the child's shadow? **36.25 in.**

5. You are standing in your backyard, and you measure the lengths of the shadows cast by both you and a tree. Write a proportion showing how you could find the height of the tree. ***Sample answer:*** $\frac{\text{tree height}}{\text{your height}} = \frac{\text{length of tree shadow}}{\text{length of your shadow}}$

Differentiated Instruction

6.5 Prove Triangles Similar by SSS and SAS

Before You used the AA Similarity Postulate to prove triangles similar.

Now You will use the SSS and SAS Similarity Theorems.

Why? So you can show that triangles are similar, as in Ex. 28.

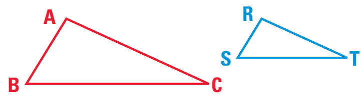
Key Vocabulary
- **ratio,** *p. 356*
- **proportion,** *p. 358*
- **similar polygons,** *p. 372*

In addition to using congruent corresponding angles to show that two triangles are similar, you can use proportional corresponding side lengths.

THEOREM *For Your Notebook*

THEOREM 6.2 Side-Side-Side (SSS) Similarity Theorem

If the corresponding side lengths of two triangles are proportional, then the triangles are similar.

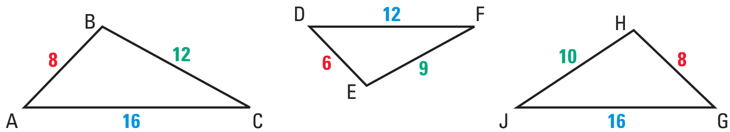


If $\frac{AB}{RS} = \frac{BC}{ST} = \frac{CA}{TR}$, then $\triangle ABC \sim \triangle RST$.

Proof: p. 389

EXAMPLE 1 Use the SSS Similarity Theorem

Is either $\triangle DEF$ or $\triangle GHJ$ similar to $\triangle ABC$?



Solution

APPLY THEOREMS
When using the SSS Similarity Theorem, compare the shortest sides, the longest sides, and then the remaining sides.

Compare $\triangle ABC$ and $\triangle DEF$ by finding ratios of corresponding side lengths.

Shortest sides	Longest sides	Remaining sides
$\frac{AB}{DE} = \frac{8}{6} = \frac{4}{3}$	$\frac{CA}{FD} = \frac{16}{12} = \frac{4}{3}$	$\frac{BC}{EF} = \frac{12}{9} = \frac{4}{3}$

▶ All of the ratios are equal, so $\triangle ABC \sim \triangle DEF$.

Compare $\triangle ABC$ and $\triangle GHJ$ by finding ratios of corresponding side lengths.

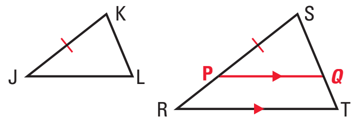
Shortest sides	Longest sides	Remaining sides
$\frac{AB}{GH} = \frac{8}{8} = 1$	$\frac{CA}{JG} = \frac{16}{16} = 1$	$\frac{BC}{HJ} = \frac{12}{10} = \frac{6}{5}$

▶ The ratios are not all equal, so $\triangle ABC$ and $\triangle GHJ$ are not similar.

PROOF SSS Similarity Theorem

GIVEN ▶ $\frac{RS}{JK} = \frac{ST}{KL} = \frac{TR}{LJ}$

PROVE ▶ $\triangle RST \sim \triangle JKL$



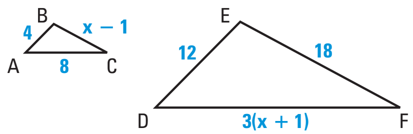
USE AN AUXILIARY LINE
The Parallel Postulate allows you to draw an auxiliary line $\overleftrightarrow{PQ}$ in $\triangle RST$. There is only one line through point P parallel to $\overleftrightarrow{RT}$, so you are able to draw it.

Locate P on $\overline{RS}$ so that $PS = JK$. Draw $\overline{PQ}$ so that $\overline{PQ} \parallel \overline{RT}$. Then $\triangle RST \sim \triangle PSQ$ by the AA Similarity Postulate, and $\frac{RS}{PS} = \frac{ST}{SQ} = \frac{TR}{QP}$.

You can use the given proportion and the fact that $PS = JK$ to deduce that $SQ = KL$ and $QP = LJ$. By the SSS Congruence Postulate, it follows that $\triangle PSQ \cong \triangle JKL$. Finally, use the definition of congruent triangles and the AA Similarity Postulate to conclude that $\triangle RST \sim \triangle JKL$.

EXAMPLE 2 Use the SSS Similarity Theorem

ALGEBRA Find the value of x that makes $\triangle ABC \sim \triangle DEF$.



Solution

STEP 1 **Find** the value of x that makes corresponding side lengths proportional.

CHOOSE A METHOD
You can use either $\frac{AB}{DE} = \frac{BC}{EF}$ or $\frac{AB}{DE} = \frac{AC}{DF}$ in Step 1.

$\frac{4}{12} = \frac{x-1}{18}$	**Write proportion.**
$4 \cdot 18 = 12(x - 1)$	**Cross Products Property**
$72 = 12x - 12$	**Simplify.**
$7 = x$	**Solve for *x*.**

STEP 2 **Check** that the side lengths are proportional when $x = 7$.

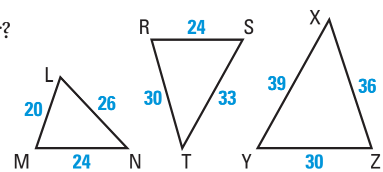
$BC = x - 1 = 6$ $\qquad$ $DF = 3(x + 1) = 24$

$\frac{AB}{DE} \stackrel{?}{=} \frac{BC}{EF} \rightarrow \frac{4}{12} = \frac{6}{18}$ ✓ $\qquad$ $\frac{AB}{DE} \stackrel{?}{=} \frac{AC}{DF} \rightarrow \frac{4}{12} = \frac{8}{24}$ ✓

▶ When $x = 7$, the triangles are similar by the SSS Similarity Theorem.

✓ GUIDED PRACTICE for Examples 1 and 2

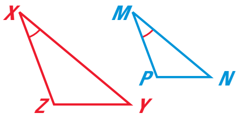
1. Which of the three triangles are similar? Write a similarity statement.
$\triangle MLN \sim \triangle ZYX$
2. The shortest side of a triangle similar to $\triangle RST$ is 12 units long. Find the other side lengths of the triangle. **15, 16.5**



THEOREM — *For Your Notebook*

THEOREM 6.3 Side-Angle-Side (SAS) Similarity Theorem

If an angle of one triangle is congruent to an angle of a second triangle and the lengths of the sides including these angles are proportional, then the triangles are similar.

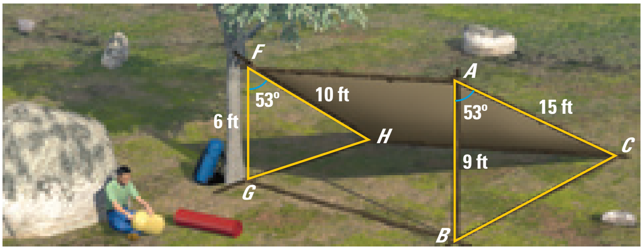


If $\angle X \cong \angle M$ and $\frac{ZX}{PM} = \frac{XY}{MN}$, then $\triangle XYZ \sim \triangle MNP$.

Proof: Ex. 37, p. 395

EXAMPLE 3 Use the SAS Similarity Theorem

LEAN-TO SHELTER You are building a lean-to shelter starting from a tree branch, as shown. Can you construct the right end so it is similar to the left end using the angle measure and lengths shown?



Solution

Both $m\angle A$ and $m\angle F$ equal 53°, so $\angle A \cong \angle F$. Next, compare the ratios of the lengths of the sides that include $\angle A$ and $\angle F$.

Shorter sides $\frac{AB}{FG} = \frac{9}{6} = \frac{3}{2}$ **Longer sides** $\frac{AC}{FH} = \frac{15}{10} = \frac{3}{2}$

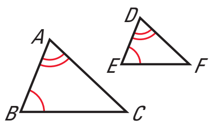
The lengths of the sides that include $\angle A$ and $\angle F$ are proportional.

▶ So, by the SAS Similarity Theorem, $\triangle ABC \sim \triangle FGH$. Yes, you can make the right end similar to the left end of the shelter.

CONCEPT SUMMARY — *For Your Notebook*

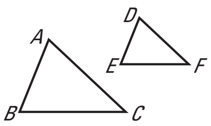
Triangle Similarity Postulate and Theorems

AA Similarity Postulate



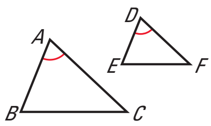
If $\angle A \cong \angle D$ and $\angle B \cong \angle E$, then $\triangle ABC \sim \triangle DEF$.

SSS Similarity Theorem



If $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$, then $\triangle ABC \sim \triangle DEF$.

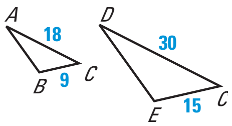
SAS Similarity Theorem



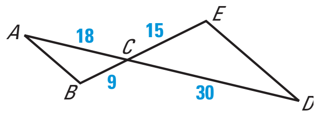
If $\angle A \cong \angle D$ and $\frac{AB}{DE} = \frac{AC}{DF}$, then $\triangle ABC \sim \triangle DEF$.

EXAMPLE 4 Choose a method

VISUAL REASONING
To identify corresponding parts, redraw the triangles so that the corresponding parts have the same orientation.



Tell what method you would use to show that the triangles are similar.



Solution

Find the ratios of the lengths of the corresponding sides.

Shorter sides $\frac{BC}{EC} = \frac{9}{15} = \frac{3}{5}$ **Longer sides** $\frac{CA}{CD} = \frac{18}{30} = \frac{3}{5}$

The corresponding side lengths are proportional. The included angles $\angle ACB$ and $\angle DCE$ are congruent because they are vertical angles. So, $\triangle ACB \sim \triangle DCE$ by the SAS Similarity Theorem.

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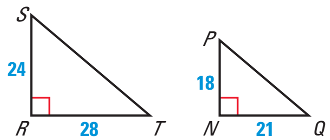
✓ GUIDED PRACTICE for Examples 3 and 4

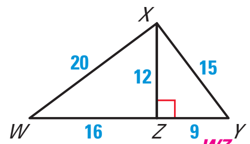
Explain **how to show that the indicated triangles are similar.**

3. $\triangle SRT \sim \triangle PNQ$



$\angle R \cong \angle N$ and $\frac{SR}{PN} = \frac{RT}{NQ} = \frac{4}{3}$ therefore the triangles are similar by the SAS Similarity Theorem.

4. $\triangle XZW \sim \triangle YZX$



$\angle WZX \cong \angle XZY$ and $\frac{WZ}{XZ} = \frac{XZ}{YZ} = \frac{WX}{XY} = \frac{4}{3}$ therefore the triangles are similar by either SSS or SAS Similarity Theorems.

6.5 EXERCISES

HOMEWORK KEY
○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 3, 7, and 31
★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 14, 32, 34, and 36

SKILL PRACTICE

A

1. **VOCABULARY** You plan to prove that $\triangle ACB$ is similar to $\triangle PXQ$ by the SSS Similarity Theorem. Copy and complete the proportion that is needed to use this theorem: $\frac{AC}{?} = \frac{?}{XQ} = \frac{AB}{?}$.
   PX CB PQ

2. ★ **WRITING** If you know two triangles are similar by the SAS Similarity Theorem, what additional piece(s) of information would you need to know to show that the triangles are congruent?
   You would need to know that one pair of corresponding sides is congruent.

EXAMPLES 1 and 2 on pp. 388–389 for Exs. 3–6

SSS SIMILARITY THEOREM **Verify that $\triangle ABC \sim \triangle DEF$. Find the scale factor of $\triangle ABC$ to $\triangle DEF$.**

3. $\triangle ABC$: $BC = 18$, $AB = 15$, $AC = 12$
   $\triangle DEF$: $EF = 12$, $DE = 10$, $DF = 8$
   $\frac{18}{12} = \frac{15}{10} = \frac{12}{8}; \frac{3}{2}$

4. $\triangle ABC$: $AB = 10$, $BC = 16$, $CA = 20$
   $\triangle DEF$: $DE = 25$, $EF = 40$, $FD = 50$
   $\frac{10}{25} = \frac{16}{40} = \frac{20}{50}; \frac{2}{5}$

6.6 Use Proportionality Theorems

Before You used proportions with similar triangles.

Now You will use proportions with a triangle or parallel lines.

Why? So you can use perspective drawings, as in Ex. 28.

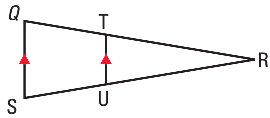
Key Vocabulary
- **corresponding angles,** *p. 147*
- **ratio,** *p. 356*
- **proportion,** *p. 358*

The Midsegment Theorem, which you learned on page 295, is a special case of the Triangle Proportionality Theorem and its converse.

THEOREMS *For Your Notebook*

THEOREM 6.4 Triangle Proportionality Theorem

If a line parallel to one side of a triangle intersects the other two sides, then it divides the two sides proportionally.

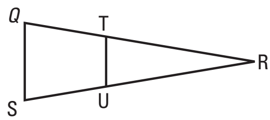


If $\overline{TU} \parallel \overline{QS}$, then $\frac{RT}{TQ} = \frac{RU}{US}$.

Proof: Ex. 22, p. 402

THEOREM 6.5 Converse of the Triangle Proportionality Theorem

If a line divides two sides of a triangle proportionally, then it is parallel to the third side.

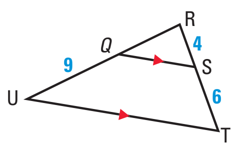


If $\frac{RT}{TQ} = \frac{RU}{US}$, then $\overline{TU} \parallel \overline{QS}$.

Proof: Ex. 26, p. 402

EXAMPLE 1 Find the length of a segment

In the diagram, $\overline{QS} \parallel \overline{UT}$, $RS = 4$, $ST = 6$, and $QU = 9$. What is the length of $\overline{RQ}$?



Solution

$\frac{RQ}{QU} = \frac{RS}{ST}$ **Triangle Proportionality Theorem**

$\frac{RQ}{9} = \frac{4}{6}$ **Substitute.**

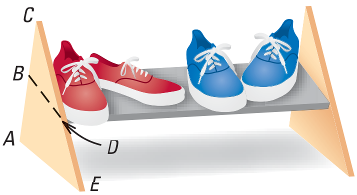
$RQ = 6$ **Multiply each side by 9 and simplify.**

REASONING Theorems 6.4 and 6.5 also tell you that if the lines are *not* parallel, then the proportion is *not* true, and vice-versa.

So if $\overline{TU} \nparallel \overline{QS}$, then $\frac{RT}{TQ} \neq \frac{RU}{US}$. Also, if $\frac{RT}{TQ} \neq \frac{RU}{US}$, then $\overline{TU} \nparallel \overline{QS}$.

EXAMPLE 2 Solve a real-world problem

SHOERACK On the shoerack shown, $AB = 33$ cm, $BC = 27$ cm, $CD = 44$ cm, and $DE = 25$ cm. *Explain* why the gray shelf is not parallel to the floor.



Solution

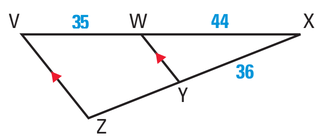
Find and simplify the ratios of lengths determined by the shoerack.

$$\frac{CD}{DE} = \frac{44}{25} \qquad \frac{CB}{BA} = \frac{27}{33} = \frac{9}{11}$$

▶ Because $\frac{44}{25} \neq \frac{9}{11}$, $\overline{BD}$ is not parallel to $\overline{AE}$. So, the shelf is not parallel to the floor.

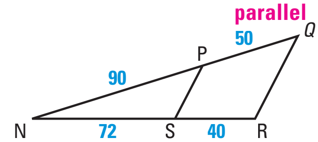
✓ GUIDED PRACTICE for Examples 1 and 2

1. Find the length of $\overline{YZ}$. $\frac{315}{11}$



2. Determine whether $\overline{PS} \parallel \overline{QR}$.

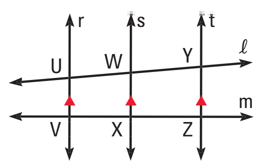
parallel



THEOREMS For Your Notebook

THEOREM 6.6

If three parallel lines intersect two transversals, then they divide the transversals proportionally.

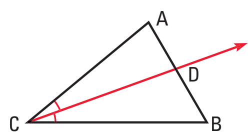


$$\frac{UW}{WY} = \frac{VX}{XZ}$$

Proof: Ex. 23, p. 402

THEOREM 6.7

If a ray bisects an angle of a triangle, then it divides the opposite side into segments whose lengths are proportional to the lengths of the other two sides.

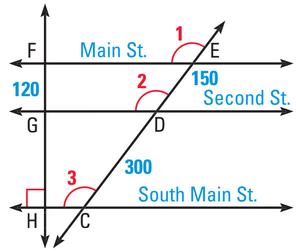


$$\frac{AD}{DB} = \frac{CA}{CB}$$

Proof: Ex. 27, p. 403

EXAMPLE 3 Use Theorem 6.6

CITY TRAVEL In the diagram, $\angle 1$, $\angle 2$, and $\angle 3$ are all congruent and $GF = 120$ yards, $DE = 150$ yards, and $CD = 300$ yards. Find the distance HF between Main Street and South Main Street.



ANOTHER WAY
For alternative methods for solving the problem in Example 3, turn to page 404 for the **Problem Solving Workshop**.

Solution

Corresponding angles are congruent, so $\overleftrightarrow{FE}$, $\overleftrightarrow{GD}$, and $\overleftrightarrow{HC}$ are parallel. Use Theorem 6.6.

$\frac{HG}{GF} = \frac{CD}{DE}$ — **Parallel lines divide transversals proportionally.**

$\frac{HG + GF}{GF} = \frac{CD + DE}{DE}$ — **Property of proportions (Property 4)**

$\frac{HF}{120} = \frac{300 + 150}{150}$ — **Substitute.**

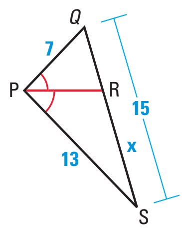
$\frac{HF}{120} = \frac{450}{150}$ — **Simplify.**

$HF = 360$ — **Multiply each side by 120 and simplify.**

▶ The distance between Main Street and South Main Street is 360 yards.

EXAMPLE 4 Use Theorem 6.7

In the diagram, $\angle QPR \cong \angle RPS$. Use the given side lengths to find the length of $\overline{RS}$.



Solution

Because $\overrightarrow{PR}$ is an angle bisector of $\angle QPS$, you can apply Theorem 6.7. Let $RS = x$. Then $RQ = 15 - x$.

$\frac{RQ}{RS} = \frac{PQ}{PS}$ — **Angle bisector divides opposite side proportionally.**

$\frac{15 - x}{x} = \frac{7}{13}$ — **Substitute.**

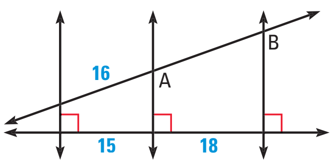
$7x = 195 - 13x$ — **Cross Products Property**

$x = 9.75$ — **Solve for *x*.**

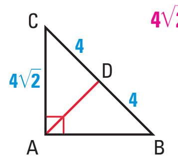
✓ GUIDED PRACTICE for Examples 3 and 4

Find the length of $\overline{AB}$.

3.



4.



4√2

6.7 Perform Similarity Transformations

Before You performed congruence transformations.

Now You will perform dilations.

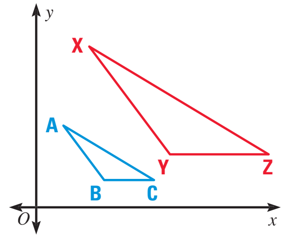
Why? So you can solve problems in art, as in Ex. 26.

Key Vocabulary
- **dilation**
- **center of dilation**
- **scale factor of a dilation**
- **reduction**
- **enlargement**
- **transformation,** *p. 272*

A **dilation** is a transformation that stretches or shrinks a figure to create a similar figure. A dilation is a type of *similarity transformation*.

In a dilation, a figure is enlarged or reduced with respect to a fixed point called the **center of dilation**.

The **scale factor of a dilation** is the ratio of a side length of the image to the corresponding side length of the original figure. In the figure shown, $\triangle XYZ$ is the image of $\triangle ABC$. The center of dilation is (0, 0) and the scale factor is $\frac{XY}{AB}$.



KEY CONCEPT *For Your Notebook*

Coordinate Notation for a Dilation

You can describe a dilation with respect to the origin with the notation $(x, y) \rightarrow (kx, ky)$, where k is the scale factor.

If $0 < k < 1$, the dilation is a **reduction**. If $k > 1$, the dilation is an **enlargement**.

EXAMPLE 1 **Draw a dilation with a scale factor greater than 1**

READ DIAGRAMS
All of the dilations in this lesson are in the coordinate plane and each center of dilation is the origin.

Draw a dilation of quadrilateral *ABCD* with vertices *A*(2, 1), *B*(4, 1), *C*(4, −1), and *D*(1, −1). Use a scale factor of 2.

Solution

First draw *ABCD*. Find the dilation of each vertex by multiplying its coordinates by 2. Then draw the dilation.

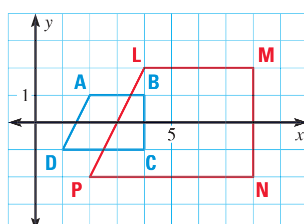
$$(x, y) \rightarrow (2x, 2y)$$

$$A(2, 1) \rightarrow L(4, 2)$$

$$B(4, 1) \rightarrow M(8, 2)$$

$$C(4, -1) \rightarrow N(8, -2)$$

$$D(1, -1) \rightarrow P(2, -2)$$



EXAMPLE 2 Verify that a figure is similar to its dilation

A triangle has the vertices $A(4, -4)$, $B(8, 2)$, and $C(8, -4)$. The image of $\triangle ABC$ after a dilation with a scale factor of $\frac{1}{2}$ is $\triangle DEF$.

a. Sketch $\triangle ABC$ and $\triangle DEF$.

b. Verify that $\triangle ABC$ and $\triangle DEF$ are similar.

Solution

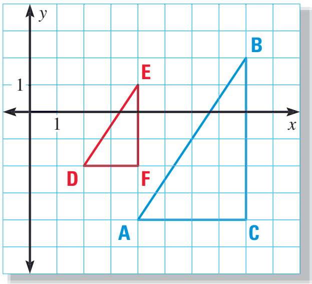
a. The scale factor is less than one, so the dilation is a reduction.

$$(x, y) \rightarrow \left(\frac{1}{2}x, \frac{1}{2}y\right)$$

$$A(4, -4) \rightarrow D(2, -2)$$

$$B(8, 2) \rightarrow E(4, 1)$$

$$C(8, -4) \rightarrow F(4, -2)$$



b. Because $\angle C$ and $\angle F$ are both right angles, $\angle C \cong \angle F$. Show that the lengths of the sides that include $\angle C$ and $\angle F$ are proportional. Find the horizontal and vertical lengths from the coordinate plane.

$$\frac{AC}{DF} \stackrel{?}{=} \frac{BC}{EF} \rightarrow \frac{4}{2} = \frac{6}{3} \checkmark$$

So, the lengths of the sides that include $\angle C$ and $\angle F$ are proportional.

▶ Therefore, $\triangle ABC \sim \triangle DEF$ by the SAS Similarity Theorem.

✓ GUIDED PRACTICE for Examples 1 and 2

Find the coordinates of L, M, and N so that $\triangle LMN$ is a dilation of $\triangle PQR$ with a scale factor of k. Sketch $\triangle PQR$ and $\triangle LMN$. 1, 2. See margin for art.

1. $P(-2, -1)$, $Q(-1, 0)$, $R(0, -1)$; $k = 4$
   L(−8, −4), M(−4, 0), N(0, −4)

2. $P(5, -5)$, $Q(10, -5)$, $R(10, 5)$; $k = 0.4$
   L(2, −2), M(4, −2), N(4, 2)

EXAMPLE 3 Find a scale factor

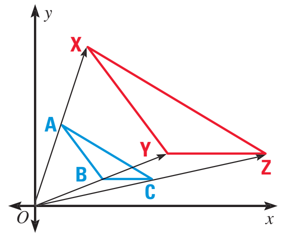
PHOTO STICKERS You are making your own photo stickers. Your photo is 4 inches by 4 inches. The image on the stickers is 1.1 inches by 1.1 inches. What is the scale factor of the reduction?

Solution

The scale factor is the ratio of a side length of the sticker image to a side length of the original photo, or $\frac{1.1 \text{ in.}}{4 \text{ in.}}$. In simplest form, the scale factor is $\frac{11}{40}$.

Differentiated Instruction

READING DIAGRAMS Generally, for a center of dilation at the origin, a point of the figure and its image lie on the same ray from the origin. However, if a point of the figure *is* the origin, its image is also the origin.



EXAMPLE 4 Standardized Test Practice

You want to create a quadrilateral *EFGH* that is similar to quadrilateral *PQRS*. What are the coordinates of *H*?

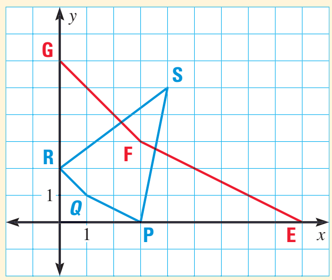
ELIMINATE CHOICES You can eliminate choice A, because you can tell by looking at the graph that *H* is in Quadrant I. The point (12, −15) is in Quadrant II.

Ⓐ (12, −15)

Ⓑ (7, 8)

Ⓒ (12, 15)

Ⓓ (15, 18)



Solution

Determine if *EFGH* is a dilation of *PQRS* by checking whether the same scale factor can be used to obtain *E*, *F*, and *G* from *P*, *Q*, and *R*.

$(x, y) \to (kx, ky)$

$P(3, 0) \to E(9, 0)$ **$k = 3$**

$Q(1, 1) \to F(3, 3)$ **$k = 3$**

$R(0, 2) \to G(0, 6)$ **$k = 3$**

Because *k* is the same in each case, the image is a dilation with a scale factor of 3. So, you can use the scale factor to find the image *H* of point *S*.

$S(4, 5) \to H(3 \cdot 4, 3 \cdot 5) = H(12, 15)$

▶ The correct answer is C. Ⓐ Ⓑ Ⓒ Ⓓ

CHECK Draw rays from the origin through each point and its image.

GUIDED PRACTICE for Examples 3 and 4

3. WHAT IF? In Example 3, what is the scale factor of the reduction if your photo is 5.5 inches by 5.5 inches? $\frac{1}{5}$

4. Suppose a figure containing the origin is dilated. *Explain* why the corresponding point in the image of the figure is also the origin.
A dilation with respect to the origin and scale factor k can be described as $(x, y) \to (kx, ky)$. If $(x, y) = (0, 0)$, then $(kx, ky) = (k \cdot 0, k \cdot 0) = (0, 0)$.