Save this packet so you can use it on the Final Exam!

Name:

6.1 Ratios, Proportions, and the Geometric Mean

Goal • Solve problems by writing and solving proportions.

Complete the vocab. with definitions or pictures that make sense to you.

Your Notes

Rewrite the Goal as an "I can" statement!

VOCABULARY
Ratio of *a* to *b*
Proportion
Means, extremes
Geometric mean

Example 1 ***Simplify ratios***

Simplify the ratio. ***(See Table of Measures, p. 921)***

a. 76 cm : 8 cm **b.** $\frac{4 \text{ ft}}{24 \text{ in.}}$

Solution

a. Write 76 cm : 8 cm as $\frac{\square}{\square}$. Then divide out the units and simplify.

$\frac{\square}{\square} = \frac{\square}{\square} = \_\_\_ : \_\_\_$

For help with conversion factors, see p. 886.

b. To simplify a ratio with unlike units, multiply by a conversion factor.

$\frac{4 \text{ ft}}{24 \text{ in.}} = \frac{4 \not{\text{ft}}}{24 \not{\text{in.}}} \cdot \frac{\square}{\square} = \frac{\square}{\square} = \frac{\square}{\square}$

Your Notes

Example 2 *Use a ratio to find a dimension*

Painting You are painting barn doors. You know that the perimeter of the doors is 64 feet and that the ratio of the length to the height is 3 : 5. Find the area of the doors.

Solution

Step 1 **Write** expressions for the length and height. Because the ratio of the length to height is 3 : 5, you can represent the length by ___x and the height by ___x.

Step 2 **Solve** an equation to find x.

$2\ell + 2w = P$	**Formula for perimeter**
2(___x) + 2(___x) = ____	**Substitute.**
____x = ____	**Multiply and combine like terms.**
x = ___	**Divide each side by ____.**

Step 3 **Evaluate** the expressions for the length and height. Substitute the value of x into each expression.

Length: ___x = ___(___) = ____

Height: ___x = ___(___) = ____

The doors are ____ feet long and ____ feet high, so the area is ____ • ____ = ________.

Checkpoint **In Exercises 1 and 2, simplify the ratio.**

1. 4 meters to 18 meters	**2.** 33 yd : 9 ft

3. The perimeter of a rectangular table is 21 feet and the ratio of its length to its width is 5 : 2. Find the length and width of the table.

Stop and get the teacher's signature before you move on.

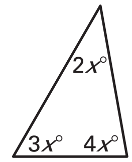
Your Notes

Example 3 *Use extended ratios*

The measures of the angles in $\triangle BCD$ are in the *extended ratio* of 2 : 3 : 4. Find the measures of the angles.

Solution

Begin by sketching the triangle. Then use the extended ratio of 2 : 3 : 4 to label the measures as ____$x°$, ____$x°$, and ____$x°$.



____$x°$ + ____$x°$ + ____$x°$ = 180°	**Triangle Sum Theorem**
____x = 180	**Combine like terms.**
x = ____	**Divide each side by ____.**

The angle measures are 2(____) = ____, 3(____) = ____, and 4(____) = ____.

✔ ***Checkpoint*** **Complete the following exercise.**

4. A triangle's angle measures are in the extended ratio of 1 : 4 : 5. Find the measures of the angles.

Stop and get the teacher's signature before you move on.

A PROPERTY OF PROPORTIONS

1. Cross Products Property In a proportion, the product of the extremes equals the product of the means.

If $\frac{a}{b} = \frac{c}{d}$ where $b \neq 0$ and $d \neq 0$, then ____ = ____.

$\frac{2}{3} = \frac{4}{6}$

3 • ____ = ____

2 • ____ = ____

Your Notes

> In part (a), you could multiply each side by the denominator, 16. Then $16 \cdot \frac{3}{4} = 16 \cdot \frac{x}{16}$ so _____ $= x$.

Example 4 ***Solve proportions***

Solve the proportion.

a. $\frac{3}{4} = \frac{x}{16}$ **Original proportion**

$3 \cdot$ _____ $=$ ____ $\cdot\, x$ **Cross Products Property**

_____ $=$ ____x **Multiply.**

_____ $= x$ **Divide each side by ____.**

b. $\frac{3}{x+1} = \frac{2}{x}$ **Original proportion**

____ $\cdot\, x =$ ____$(x + 1)$ **Cross Products Property**

____$x =$ ____$x +$ ____ **Distributive Property**

$x =$ ____ **Subtract _____ from each side.**

Example 5 ***Solve a real-world problem***

Bowling You want to find the total number of rows of boards that make up 24 lanes at a bowling alley. You know that there are 117 rows in 3 lanes. Find the total number of rows of boards that make up the 24 lanes.

Solution

Write and solve a proportion involving two ratios that compare the number of rows with the number of lanes.

$\frac{\_\_\_\_\_}{\_\_\_\_\_} = \frac{\_\_\_\_\_}{\_\_\_\_\_}$ ← number of rows ← number of lanes **Write proportion.**

______ $\cdot$ _____ $=$ ____ $\cdot$ ____ **Cross Products Property**

______ $= n$ **Simplify.**

There are _____ rows of boards that make up the 24 lanes.

GEOMETRIC MEAN

The geometric mean of two positive numbers a and b is the positive number x that satisfies $\frac{a}{x} = \frac{x}{b}$.

So, $x^2 =$ _____ and $x = \sqrt{\_\_\_\_\_}$.

Your Notes

Example 6 *Find a geometric mean*

Find the geometric mean of 16 and 48.

Solution

$x =$ ______	Definition of geometric mean
$=$ __________	Substitute ____ for a and ____ for b.
$=$ _____________	Factor.
$=$ ______	Simplify.

The geometric mean of 16 and 48 is ______ $\approx$ ______.

✔ ***Checkpoint*** **Complete the following exercises.**

5. Solve $\frac{8}{y} = \frac{2}{5}$.

6. Solve $\frac{x - 3}{3} = \frac{2x}{9}$.

7. A small gymnasium contains 10 sets of bleachers. You count 192 spectators in 3 sets of bleachers and the spectators seem to be evenly distributed. Estimate the total number of spectators.

8. Find the geometric mean of 14 and 16.

Stop and get the teacher's signature before you move on.

Homework

6.2 Use Proportions to Solve Geometry Problems

Goal • Use proportions to solve geometry problems.

Your Notes

Rewrite the Goal as an "I can" statement!

VOCABULARY

Scale drawing

Scale

ADDITIONAL PROPERTIES OF PROPORTIONS

2. **Reciprocal Property** If two ratios are equal, then their reciprocals are also equal.

If $\frac{a}{b} = \frac{c}{d}$, then $\frac{b}{a} =$ ____.

3. If you interchange the means of a proportion, then you form another true proportion.

If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a}{c} =$ ____.

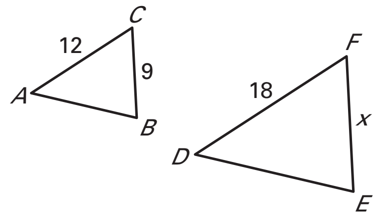
4. In a proportion, if you add the value of each ratio's denominator to its numerator, then you form another true proportion.

If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a + b}{b} =$ ________.

Your Notes

Example 1 *Use properties of proportions*

In the diagram, $\frac{AC}{DF} = \frac{BC}{EF}$. Write four true proportions.



Because $\frac{AC}{DF} = \frac{BC}{EF}$, then $\frac{12}{18} =$ ___.

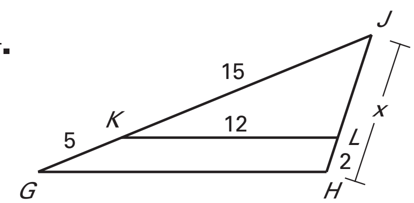
Reciprocal Property: The reciprocals are equal, so $\frac{18}{12} =$ ___.

Property 3: You can interchange the means, so $\frac{12}{9} =$ ___.

Property 4: You can add the denominators to the numerators, so ___ = ___.

Example 2 *Use proportions with geometric figures*

In the diagram, $\frac{JL}{LH} = \frac{JK}{KG}$. Find JH and JL.



$\frac{JL}{LH} = \frac{JK}{KG}$	**Given**
___ = ___	**Property of Proportions (Property 4)**
___ = ___	**Substitution Property of Equality**
___ = ___	**Cross Products Property**
$x =$ ___	**Solve for x.**

So $JH =$ ___ and $JL =$ ___ = ___.

✔ ***Checkpoint*** **Complete the following exercises.**

1. In Example 1, find the value of x.	**2.** In Example 2, $\frac{KL}{GH} = \frac{JK}{JG}$. Find GH.

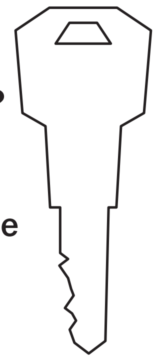
Stop and get the teacher's signature before you move on.

Your Notes

Stop and get the teacher's signature before you move on.

Example 3 *Find the scale of a drawing*

Keys The length of the key in the scale drawing is 7 centimeters. The length of the actual key is 4 centimeters. What is the scale of the drawing?



Solution

To find the scale, write the ratio of a length in the drawing to ______________, then rewrite the ratio so that the ______________ is 1.

$\frac{\text{length in drawing}}{\boxed{}} = \underline{} = \underline{} = \underline{}$

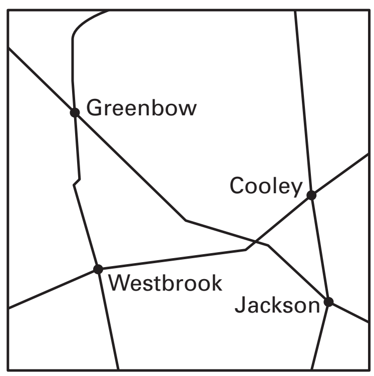
The scale of the drawing is ______________.

Checkpoint Complete the following exercise.

3. In Example 3, suppose the length of the key in the scale drawing is 6 centimeters. Find the new scale of the drawing.

Example 4 *Use a scale drawing*

Maps The scale of the map at the right is 1 inch : 8 miles. Find the actual distance from Westbrook to Cooley.



Solution

Use a ruler. The distance from Westbrook to Cooley on the map is about ____________. Let x be the actual distance in miles.

$\underline{} = \frac{1 \text{ in.}}{8 \text{ mi}}$ ← distance on map / ← actual distance

$x = \underline{}$ **Cross Products Property**

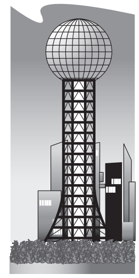
$x = \underline{}$ **Simplify.**

The actual distance from Westbrook to Cooley is about __________.

Your Notes

Example 5 *Solve a multi-step problem*

Scale Model You buy a 3-D scale model of the Sunsphere in Knoxville, TN. The actual building is 266 feet tall. Your model is 20 inches tall, and the diameter of the dome on your scale model is about 5.6 inches.



a. What is the diameter of the actual dome?

b. How many times as tall as your model is the actual building?

Solution

a. $\frac{20 \text{ in.}}{266 \text{ ft}} = \frac{5.6 \text{ in.}}{x \text{ ft}}$ ← measurement on model / ← measurement on actual building

$\_\_\_\_x = \_\_\_\_\_\_\_\_$ **Cross Products Property**

$x \approx \_\_\_\_\_\_$ **Divide each side by ____.**

The diameter of the actual dome is about ______ feet.

b. To simplify a ratio with unlike units, multiply by a conversion factor.

$\frac{266 \text{ ft}}{20 \text{ in.}} = \_\_\_\_\_\_\_\_\_\_\_\_ = \_\_\_\_\_\_\_$

The actual building is _______ times as tall as the model.

✓ ***Checkpoint*** **Complete the following exercises.**

Stop and get the teacher's signature before you move on.

4. Two landmarks are 130 miles from each other. The landmarks are 6.5 inches apart on a map. Find the scale of the map.

5. Your friend has a model of the Sunsphere that is 5 inches tall. What is the approximate diameter of the dome on your friend's model?

Homework

6.3 Use Similar Polygons

Goal • Use proportions to identify similar polygons.

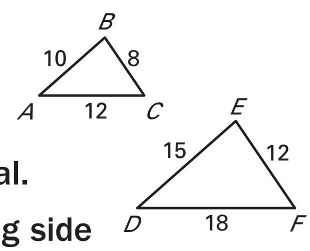
Your Notes

Rewrite the Goal as an "I can" statement!

VOCABULARY
Similar polygons
Scale factor of two similar polygons

Example 1 ***Use similarity statements***

In the diagram, $\triangle ABC \sim \triangle DEF$.



a. List all pairs of congruent angles.

b. Check that the ratios of corresponding side lengths are equal.

c. Write the ratios of the corresponding side lengths in a *statement of proportionality*.

In a *statement of proportionality*, any pair of ratios forms a true proportion.

Solution

a. $\angle A \cong \angle$ ___, $\angle B \cong \angle$ ___, $\angle C \cong \angle$ ___

b. $\frac{AB}{DE} = \frac{\_\_\_}{\_\_\_} = \_\_\_$ $\qquad \frac{BC}{EF} = \frac{\_\_\_}{\_\_\_} = \_\_\_$

$\frac{CA}{FD} = \frac{\_\_\_}{\_\_\_} = \_\_\_$

c. The ratios in part (b) are equal, so

$\frac{\_\_\_}{\_\_\_} = \frac{\_\_\_}{\_\_\_} = \frac{\_\_\_}{\_\_\_}$.

✔ ***Checkpoint*** **Complete the following exercise.**

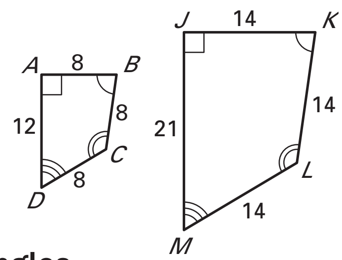
1. Given $\triangle PQR \sim \triangle XYZ$, list all pairs of congruent angles. Write the ratios of the corresponding side lengths in a statement of proportionality.

Stop and get the teacher's signature before you move on.

Your Notes

Example 2 *Find the scale factor*

Determine whether the polygons are similar. If they are, write a similarity statement and find the scale factor of *ABCD* to *JKLM*.



Solution

Step 1 **Identify** pairs of congruent angles.
From the diagram, you can see that $\angle B \cong \angle$___, $\angle C \cong \angle$___, and $\angle D \cong \angle$___. Angles ___ and ___ are right angles, so $\angle$___ $\cong \angle$___.
So, the corresponding angles are __________.

Step 2 **Show** that corresponding side lengths are proportional.

$\frac{AB}{JK} = \frac{\_\_\_}{\_\_\_} = \_\_\_$ $\qquad$ $\frac{BC}{KL} = \frac{\_\_\_}{\_\_\_} = \_\_\_$

$\frac{CD}{LM} = \frac{\_\_\_}{\_\_\_} = \_\_\_$ $\qquad$ $\frac{AD}{JM} = \frac{\_\_\_}{\_\_\_} = \_\_\_$

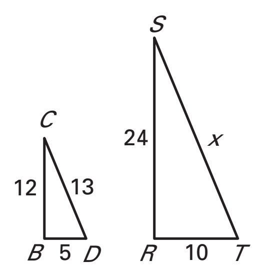
The ratios are equal, so the corresponding side lengths are __________.

So *ABCD* ~ _______. The scale factor of *ABCD* to *JKLM* is ___.

Example 3 *Use similar polygons*

In the diagram, $\triangle BCD \sim \triangle RST$. Find the value of *x*.

Solution



The triangles are similar, so the corresponding side lengths are __________.

$\frac{BC}{\square} = \frac{\square}{ST}$ **Write proportion.**

$\frac{12}{\square} = \frac{\square}{x}$ **Substitute.**

$12x =$ _____ **Cross Products Property**

$x =$ _____ **Solve for *x*.**

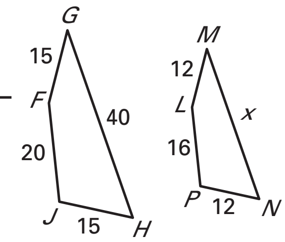
There are several ways to write the proportion. For example, you could write $\frac{BD}{RT} = \frac{CD}{ST}$.

Your Notes

Stop and get the teacher's signature before you move on.

Checkpoint In the diagram, *FGHJ* ~ *LMNP*.

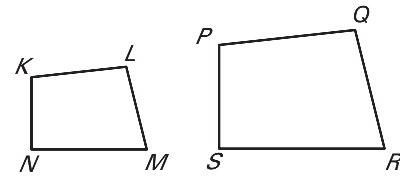
2. What is the scale factor of *LMNP* to *FGHJ*?



3. Find the value of *x*.

THEOREM 6.1: PERIMETERS OF SIMILAR POLYGONS

If two polygons are similar, then the ratio of their perimeters is equal to the ratios of their corresponding side lengths.



If *KLMN* ~ *PQRS*, then

$$\frac{KL + LM + MN + NK}{PQ + QR + RS + SP} = \_\_\_ = \_\_\_ = \_\_\_ = \_\_\_.$$

Example 4 *Find perimeters of similar figures*

Basketball A larger cement court is being poured for a basketball hoop in place of a smaller one. The court will be 20 feet wide and 25 feet long. The old court was similar in shape, but only 16 feet wide.

a. Find the scale factor of the new court to the old court.

b. Find the perimeters of the new court and the old court.

Solution

a. Because the new court will be similar to the old court, the scale factor is the ratio of the widths, ____ = ____.

b. The new court's perimeter is ________________ = _____ feet. Use Theorem 6.1 to find the perimeter *x* of the old court.

$\frac{90}{x} =$ ____ **Use Theorem 6.1 to write a proportion.**

$x =$ _____ **Simplify.**

The perimeter of the old court was _____ feet.

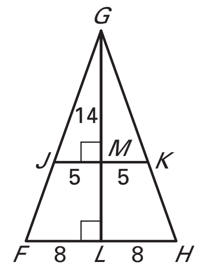
Your Notes

CORRESPONDING LENGTHS IN SIMILAR POLYGONS

If two polygons are similar, then the ratio of any two corresponding lengths in the polygons is equal to the ______________ of the similar polygons.

Example 5 *Use a scale factor*

In the diagram, △*FGH* ~ △*JGK*. Find the length of the altitude $\overline{GL}$.



Solution

First, find the scale factor of △*FGH* to △*JGK*.

$\frac{FH}{\square}$ = ______ = ____ = ___

Because the ratio of the lengths of the altitudes in similar triangles is equal to the scale factor, you can write the following proportion.

$\frac{GL}{GM}$ = ___ **Write proportion.**

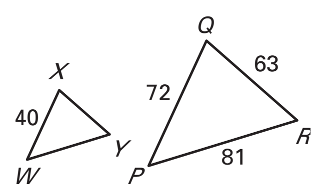
$\frac{GL}{\square}$ = ___ **Substitute _____ for *GM*.**

GL = ______ **Multiply each side by _____ and simplify.**

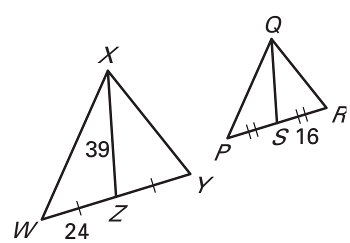
The length of altitude $\overline{GL}$ is ______.

Checkpoint In the diagrams, △*PQR* ~ △*WXY*.

4. Find the perimeter of △*WXY*.



5. Find the length of median $\overline{QS}$.



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6.4 Prove Triangles Similar by AA

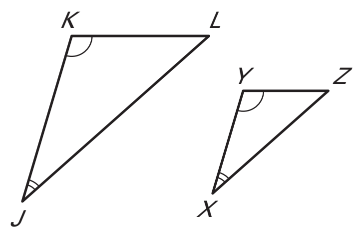
Goal • Use the AA Similarity Postulate.

Your Notes

Rewrite the Goal as
an "I can" statement!

POSTULATE 22: ANGLE-ANGLE (AA) SIMILARITY POSTULATE

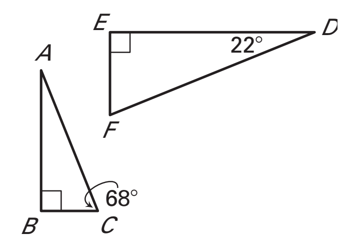
If two angles of one triangle are congruent to two angles of another triangle, then the two triangles are similar.



$\triangle JKL \sim \triangle XYZ$

Example 1 ***Use the AA Similarity Postulate***

Determine whether the triangles are similar. If they are, write a similarity statement. *Explain* your reasoning.



Solution

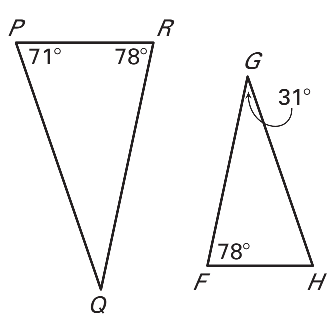
Because they are both right angles, $\angle$___ and $\angle$___ are congruent.

By the Triangle Sum Theorem,
_____ + _____ + $m\angle A = 180°$, so $m\angle A =$ _____.
Therefore, $\angle A$ and $\angle$___ are congruent.

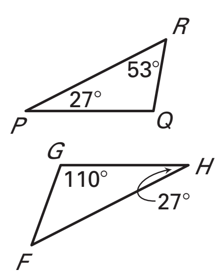
So, $\triangle ABC \sim \triangle DEF$ by the ____________________________.

Checkpoint **Determine whether the triangles are similar. If they are, write a similarity statement.**

1.



2.



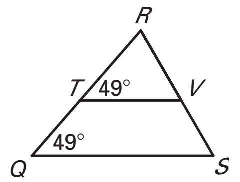
Stop and get the
teacher's signature
before you move on.

Your Notes

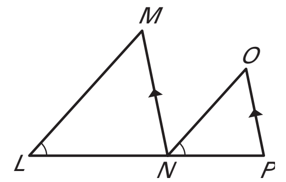
Example 2 *Show that triangles are similar*

Show that the two triangles are similar.

a. $\triangle RTV$ and $\triangle RQS$



b. $\triangle LMN$ and $\triangle NOP$



Solution

a. You may find it helpful to redraw the triangles separately.

Because $m\angle$______ and $m\angle$____ both equal 49°, $\angle$______ $\cong \angle$____. By the Reflexive Property, $\angle R \cong \angle$____.

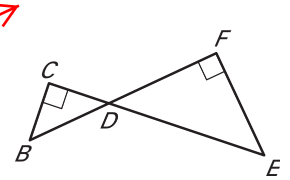
So, $\triangle RTV \sim \triangle RQS$ by the ________________________.

b. The diagram shows $\angle L \cong \angle$______. It also shows that $\overline{MN} \parallel$ _____ so $\angle$_______ $\cong \angle$___ by the Corresponding Angles Postulate.

So, $\triangle LMN \sim \triangle NOP$ by the ________________________.

✔ ***Checkpoint*** **Complete the following exercise.**

3. Show that $\triangle BCD \sim \triangle EFD$.



This means "Write a proof"

Stop and get the teacher's signature before you move on.

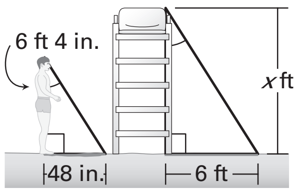
Your Notes

Example 3 *Using similar triangles*

Height A lifeguard is standing beside the lifeguard chair on a beach. The lifeguard is 6 feet 4 inches tall and casts a shadow that is 48 inches long. The chair casts a shadow that is 6 feet long. How tall is the chair?

Solution

The lifeguard and the chair form sides of two right triangles with the ground, as shown below. The sun's rays hit the lifeguard and the chair at the same angle. You have two pairs of congruent ________, so the triangles are similar by the ________________________.



You can use a proportion to find the height x. Write 6 feet 4 inches as ____ inches so you can form two ratios of feet to inches.

$$\frac{x \text{ ft}}{\square} = \frac{\square}{\square}$$ **Write proportion of side lengths.**

$\_\_\_\_ x = \_\_\_\_\_$ **Cross Products Property**

$x = \_\_\_\_\_$ **Solve for x.**

The chair is _____ feet tall.

✔ ***Checkpoint*** **Complete the following exercise.**

4. In Example 3, how long is the shadow of a person that is 4 feet 9 inches tall?

Homework

Stop and get the teacher's signature before you move on.

6.5 Prove Triangles Similar by SSS and SAS

Goal • Use the SSS and SAS Similarity Theorems.

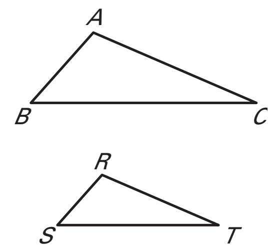
Your Notes

Rewrite the Goal as an "I can" statement!

THEOREM 6.2: SIDE-SIDE-SIDE (SSS) SIMILARITY THEOREM

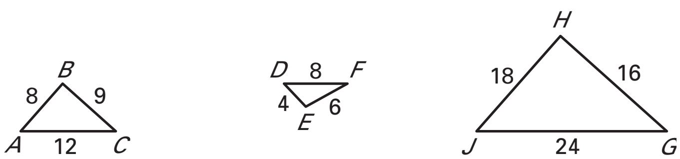
If the corresponding side lengths of two triangles are ______________, then the triangles are similar.

If $\frac{AB}{RS} = \frac{BC}{ST} = \frac{CA}{TR}$, then $\triangle ABC \sim \triangle RST$.



Example 1 ***Use the SSS Similarity Theorem***

Is either $\triangle DEF$ or $\triangle GHJ$ similar to $\triangle ABC$?



Solution

Compare $\triangle ABC$ and $\triangle DEF$ by finding ratios of corresponding side lengths.

When using the SSS Similarity Theorem, compare the shortest sides, the longest sides, and then the remaining sides.

Shortest sides	**Longest sides**	**Remaining sides**
$\frac{AB}{DE} = \underline{\quad\quad} = \underline{\quad\quad}$	$\frac{CA}{FD} = \underline{\quad\quad} = \underline{\quad\quad}$	$\frac{BC}{EF} = \underline{\quad\quad} = \underline{\quad\quad}$

All the ratios are ____________, so $\triangle ABC$ and $\triangle DEF$ are ______________.

Compare $\triangle ABC$ and $\triangle GHJ$ by finding ratios of corresponding side lengths.

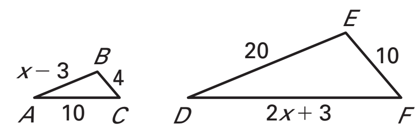
Shortest sides	**Longest sides**	**Remaining sides**
$\frac{AB}{GH} = \underline{\quad\quad} = \underline{\quad\quad}$	$\frac{CA}{JG} = \underline{\quad\quad} = \underline{\quad\quad}$	$\frac{BC}{HJ} = \underline{\quad\quad} = \underline{\quad\quad}$

All the ratios are ________, so $\triangle$________ $\sim$ $\triangle$______.

Your Notes

Example 2 *Use the SSS Similarity Theorem*

Find the value of x that makes $\triangle ABC \sim \triangle DEF$.



Solution

Step 1 **Find** the value of x that makes corresponding side lengths proportional.

$\frac{4}{\square} = \frac{x - 3}{\square}$ **Write proportion.**

$4 \cdot$ ____ $=$ ____$(x - 3)$ **Cross Products Property**

____ $=$ ____$x -$ ____ **Simplify.**

____ $= x$ **Solve for x.**

Step 2 **Check** that the side lengths are proportional when $x =$ ____.

$AB = x - 3 =$ ____ $DF = 2x + 3 =$ ____

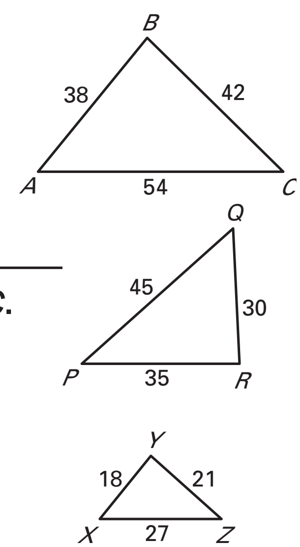
$\frac{BC}{EF} \stackrel{?}{=} \frac{AB}{DE}$ ____ $=$ ____ ✓ $\frac{BC}{EF} \stackrel{?}{=} \frac{AC}{DF}$ ____ $=$ ____ ✓

When $x =$ ____, the triangles are similar by the ____________________.

✓ ***Checkpoint*** **Complete the following exercises.**

1. Which of the three triangles are similar?

2. Suppose AB is not given in $\triangle ABC$. What length for AB would make $\triangle ABC$ similar to $\triangle QRP$?

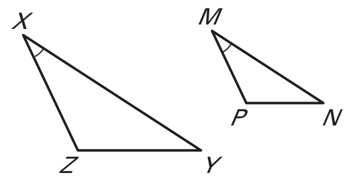


Stop and get the teacher's signature before you move on.

Your Notes

THEOREM 6.3: SIDE-ANGLE-SIDE (SAS) SIMILARITY THEOREM

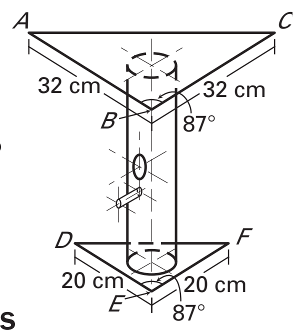
If an angle of one triangle is congruent to an angle of a second triangle and the lengths of the sides including these angles are ______________, then the triangles are similar.



If $\angle X \cong \angle M$, and $\frac{ZX}{PM} = \frac{XY}{MN}$, then $\triangle XYZ \sim \triangle MNP$.

Example 3 ***Use the SAS Similarity Theorem***

Birdfeeder You are drawing a design for a birdfeeder. Can you construct the top so it is similar to the bottom using the angle measure and lengths shown?



Solution

Both $m\angle$___ and $m\angle$___ equal 87°, so $\angle$___ $\cong$ $\angle$___. Next, compare the ratios of the lengths of the sides that include $\angle B$ and $\angle E$.

$$\frac{AB}{\square} = \frac{\square}{EF} = \frac{\_\_\_}{\_\_\_} = \_\_\_$$

The lengths of the sides that include $\angle B$ and $\angle E$ are ______________.

So, by the ________________________, $\triangle ABC \sim \triangle DEF$. Yes, you can make the top similar to the bottom.

✔ ***Checkpoint*** **Complete the following exercise.**

3. In Example 3, suppose you use equilateral triangles on the top and bottom. Are the top and bottom similar? *Explain.*

Stop and get the teacher's signature before you move on.

Your Notes

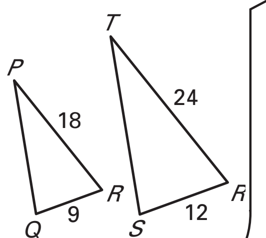
TRIANGLE SIMILARITY POSTULATE AND THEOREMS

AA Similarity Postulate If $\angle A \cong \angle D$ and $\angle B \cong \angle E$, then $\triangle ABC \sim \triangle DEF$.

SSS Similarity Theorem If $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$, then $\triangle ABC \sim \triangle DEF$.

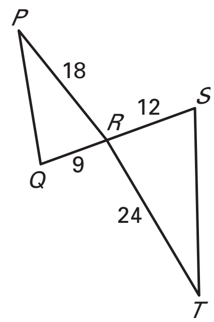
SAS Similarity Theorem If $\angle A \cong \angle D$ and $\frac{AB}{DE} = \frac{AC}{DF}$, then $\triangle ABC \sim \triangle DEF$.

To identify corresponding parts, redraw the triangles so that the corresponding parts have the same orientation.



Example 4 *Choose a method*

Tell what method you would use to show that the triangles are similar.



Solution

Find the ratios of the lengths of the corresponding sides.

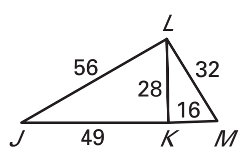
Shorter sides ______________

Longer sides ______________

The corresponding side lengths are ______________.
The included angles $\angle PRQ$ and $\angle TRS$ are ______________ because they are __________ angles. So, $\triangle PQR \sim \triangle TSR$ by the ________________________.

Checkpoint **Complete the following exercise.**

4. Show that $\triangle JKL \sim \triangle LKM$.



Stop and get the teacher's signature before you move on.

6.6 Use Proportionality Theorems

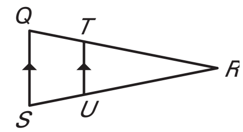
Goal • Use proportions with a triangle or parallel lines.

Your Notes

Rewrite the Goal as an "I can" statement!

THEOREM 6.4: TRIANGLE PROPORTIONALITY THEOREM

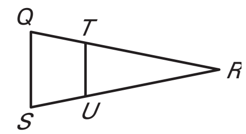
If a line parallel to one side of a triangle intersects the other two sides, then it divides the two sides ______________.



If $\overline{TU} \parallel \overline{QS}$, then $\frac{\_\_\_}{\_\_\_} = \frac{\_\_\_}{\_\_\_}$.

THEOREM 6.5: CONVERSE OF THE TRIANGLE PROPORTIONALITY THEOREM

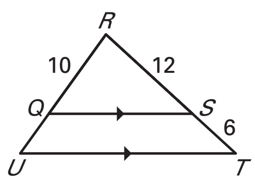
If a line divides two sides of a triangle proportionally, then it is parallel to the __________.



If $\frac{RT}{TQ} = \frac{RU}{US}$, then _____ $\parallel$ _____.

Example 1 ***Find the length of a segment***

In the diagram, $\overline{QS} \parallel \overline{UT}$, $RQ = 10$, $RS = 12$, and $ST = 6$. What is the length of $\overline{QU}$?



Solution

$\frac{RQ}{QU} = \frac{RS}{ST}$ ______________________________ ____________

$\frac{\square}{QU} = \frac{\square}{\square}$ **Substitute.**

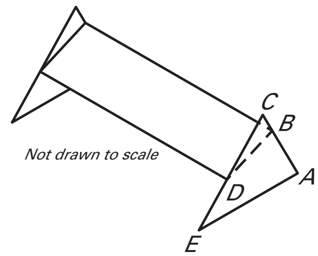
_____ = _____ • QU **Cross Products Property**

____ = QU **Divide each side by _____.**

Your Notes

Example 2 ***Solve a real-world problem***

Aerodynamics A spoiler for a remote controlled car is shown where $AB = 31$ mm, $BC = 19$ mm, $CD = 27$ mm, and $DE = 23$ mm. *Explain* why $\overline{BD}$ is not parallel to $\overline{AE}$.



Solution

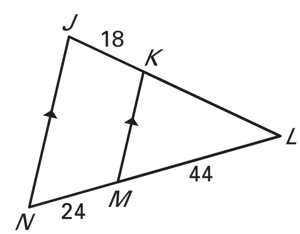
Find and simplify the ratios of lengths determined by $\overline{BD}$.

$\frac{CD}{DE} =$ ____ $\frac{CB}{BA} =$ ____

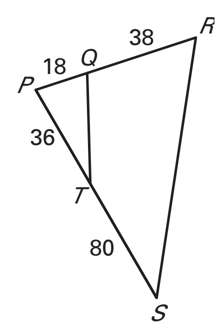
Because ____ ≠ ____, $\overline{BD}$ is not parallel to $\overline{AE}$.

Checkpoint **Complete the following exercises.**

1. Find the length of $\overline{KL}$.



2. Determine whether $\overline{QT} \parallel \overline{RS}$.

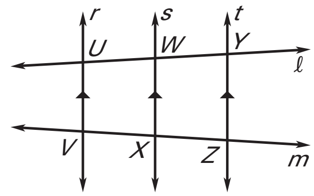


Stop and get the teacher's signature before you move on.

Your Notes

THEOREM 6.6

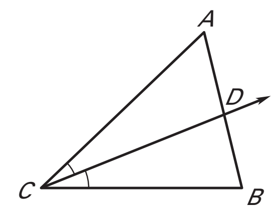
If three parallel lines intersect two transversals, then they divide the transversals ______________.



$\frac{UW}{WY} =$ ____

THEOREM 6.7

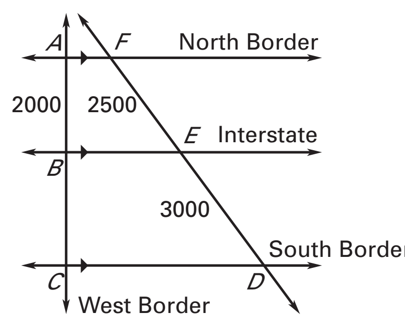
If a ray bisects an angle of a triangle, then it divides the opposite side into segments whose lengths are ____________ to the lengths of the other two sides.



$\frac{AD}{DB} =$ ____

Example 3 ***Use Theorem 6.6***

Farming A farmer's land is divided by a newly constructed interstate. The distances shown are in meters. Find the distance *CA* between the north border and the south border of the farmer's land.



Use Theorem 6.6.

$\frac{CB}{BA} = \frac{DE}{EF}$ **Parallel lines divide transversals proportionally.**

$\frac{\square}{BA} = \frac{\square}{EF}$ **Property of proportions (Property 4)**

$\frac{CA}{\square} =$ ______ **Substitute.**

$\frac{CA}{\square} =$ ____ **Simplify.**

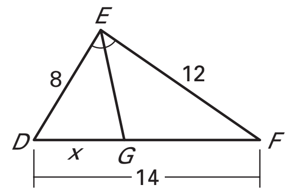
$CA =$ ____ **Multiply each side by ____ and simplify.**

The distance between the north border and the south border is ____ meters.

Your Notes

Example 4 *Use Theorem 6.7*

In the diagram, $\angle DEG \cong \angle GEF$. Use the given side lengths to find the length of $\overline{DG}$.



Solution

Because $\overrightarrow{EG}$ is an angle bisector of $\angle DEF$, you can apply Theorem 6.7. Let $GD = x$. Then $GF =$ ________.

$\frac{GF}{GD} = \frac{EF}{ED}$ — **Angle bisector divides opposite side proportionally.**

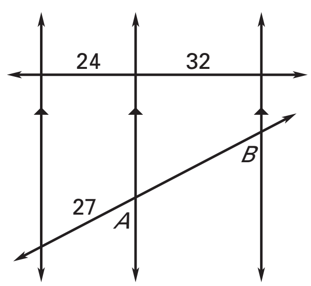
$\frac{\boxed{}}{x} = \frac{\_\_\_}{\_\_\_}$ — **Substitute.**

$\_\_\_ x = \_\_\_ - \_\_\_ x$ — **Cross Products Property**

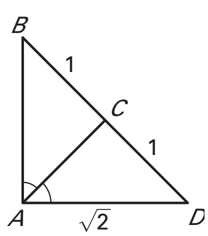
$x = \_\_\_$ — **Solve for *x*.**

✔ ***Checkpoint*** **Find the length of $\overline{AB}$.**

3.



4.



Homework

Stop and get the teacher's signature before you move on.

6.7 Perform Similarity Transformations

Goal • Perform dilations.

Your Notes

Rewrite the Goal as
an "I can" statement!

VOCABULARY

Dilation

Center of dilation

Scale factor of a dilation

Reduction

Enlargement

COORDINATE NOTATION FOR A DILATION

You can describe a dilation with respect to the origin with the notation $(x, y) \rightarrow (kx, ky)$, where k is the scale factor.

If $0 < k < 1$, the dilation is a ____________. If $k > 1$, the dilation is an _______________.

Your Notes

All of the dilations in this lesson are in the coordinate plane and each center of dilation is the origin.

Example 1 ***Draw a dilation with a scale factor greater than 1***

Draw a dilation of quadrilateral *ABCD* with vertices *A*(2, 0), *B*(6, −4), *C*(8, 2), and *D*(6, 4). Use a scale factor of $\frac{1}{2}$.

First draw *ABCD*. Find the dilation of each vertex by multiplying its coordinates by ____. Then draw the dilation.

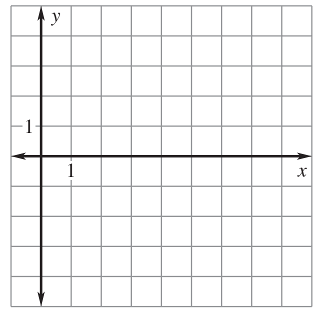
(*x*, *y*) → (____*x*, ____*y*)

A(2, 0) → *L*________

B(6, −4) → *M*________

C(8, 2) → *N*________

D(6, 4) → *P*________



Example 2 ***Verify that a figure is similar to its dilation***

A triangle has the vertices *A*(2, −1), *B*(4, −1), and *C*(4, 2). The image of △*ABC* after a dilation with a scale factor of 2 is △*DEF*.

a. Sketch △*ABC* and △*DEF*.

b. Verify that △*ABC* and △*DEF* are similar.

Solution

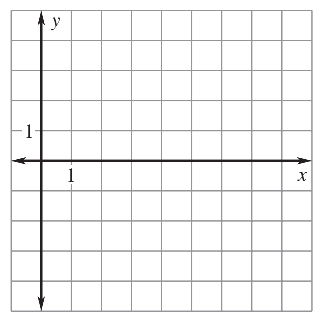
a. The scale factor is greater than 1, so the dilation is an ______________.

(*x*, *y*) → (____*x*, ____*y*)

A(2, −1) → ________

B(4, −1) → ________

C(4, 2) → ________



b. Because ∠____ and ∠____ are both right angles, ∠____ ≅ ∠____. Show that the lengths of the sides that include ∠____ and ∠____ are proportional.

$\frac{AB}{\square} \stackrel{?}{=} \frac{BC}{\square}$ $\frac{2}{\square} = \frac{3}{\square}$ ✓

The lengths are proportional. So, △*ABC* ~ △*DEF* by the ______________________.

Your Notes

Example 3 *Find a scale factor*

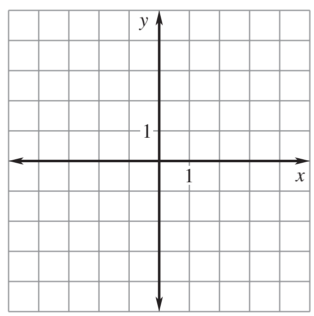
Magnets You are making your own photo magnets. Your photo is 8 inches by 10 inches. The image on the magnet is 2.8 inches by 3.5 inches. What is the scale factor of the reduction?

Solution

The scale factor is the ratio of a side length of the ______________ to a side length of the ______________, or $\frac{\square}{8 \text{ in.}}$. In simplest form, the scale factor is ____.

✔ ***Checkpoint*** **Complete the following exercises.**

1. A triangle has the vertices $B(-1, -1)$, $C(0, 1)$, and $D(1, 0)$. Find the coordinates of L, M, and N so that $\triangle LMN$ is a dilation of $\triangle BCD$ with a scale factor of 4. Sketch $\triangle BCD$ and $\triangle LMN$.



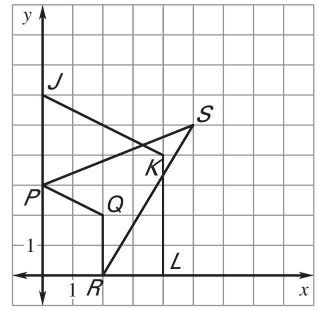
2. In Example 3, what is the scale factor of the reduction if your photo is 4 inches by 5 inches?

Stop and get the teacher's signature before you move on.

Your Notes

Example 4 ***Find missing coordinates***

You want to create a quadrilateral *JKLM* that is similar to quadrilateral *PQRS*. What are the coordinates of *M*?



Solution

Determine if *JKLM* is a dilation of *PQRS* by checking whether the same scale factor can be used to obtain *J*, *K*, and *L* from *P*, *Q*, and *R*.

$(x, y) \rightarrow (kx, ky)$

$P($_____$) \rightarrow J($_____$)$ **k =** ___

$Q($_____$) \rightarrow K($_____$)$ **k =** ___

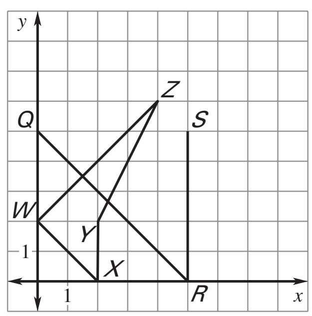
$R($_____$) \rightarrow L($_____$)$ **k =** ___

Because *k* is the same in each case, the image is a _________ with a scale factor of ___. So, you can use the scale factor to find the image *M* of point *S*.

$S($_____$) \rightarrow M($___ • ___, ___ • ___$) = M($________$)$

Checkpoint **Complete the following exercise.**

3. You want to create a quadrilateral *QRST* that is similar to quadrilateral *WXYZ*. What are the coordinates of *T*?



Homework

Stop and get the teacher's signature before you move on.