Save this packet so you can use it on the Final Exam!

Name:

4.1 Apply Triangle Sum Properties

Goal • Classsify triangles and find measures of their angles.

Complete the vocab. with definitions or pictures that make sense to you.

Your Notes

Rewrite the Goal as an "I can" statement!

VOCABULARY

Triangle

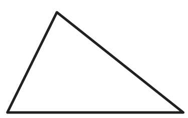
Interior angles

Exterior angles

Corollary to a theorem

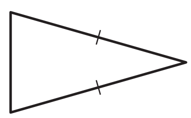
CLASSIFYING TRIANGLES BY SIDES

Scalene Triangle



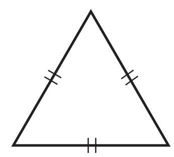
_____ congruent sides

Isosceles Triangle



At least ___ congruent sides

Equilateral Triangle

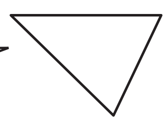


___ congruent sides

CLASSIFYING TRIANGLES BY ANGLES

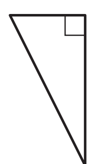
Notice that an equilateral triangle is also isosceles. An equiangular triangle is also acute.

Acute Triangle



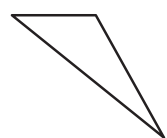
___ acute angles

Right Triangle



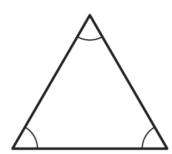
___ right angle

Obtuse Triangle



___ obtuse angle

Equiangular Triangle

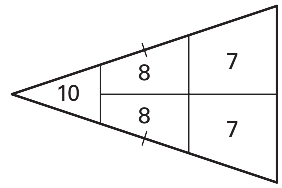


___ congruent angles

Your Notes

Example 1 ***Classify triangles by sides and by angles***

Shuffleboard Classify the triangular shape of the shuffleboard scoring area in the diagram by its sides and by measuring its angles.



Solution

The triangle has a pair of congruent sides, so it is __________. By measuring, the angles are about ______________. It is an ______________ triangle.

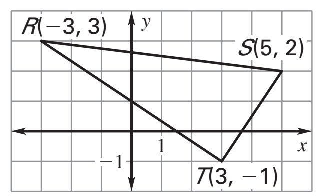
✓ ***Checkpoint*** **Complete the folowing exercise.**

When you draw an isosceles right triangle, the right angle has to be between the 2 congruent sides.

1. Draw an isosceles right triangle and an obtuse scalene triangle.

Example 2 ***Classify a triangle in a coordinate plane***

Classify △*RST* by its sides. Then determine if the triangle is a right triangle.



Solution

Step 1 **Use** the distance formula $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ to find the side lengths.

$RT = \sqrt{\underline{\quad\quad}^2 + \underline{\quad\quad}^2} = \underline{\quad}$

$RS = \sqrt{\underline{\quad\quad}^2 + \underline{\quad\quad}^2} = \underline{\quad}$

$ST = \sqrt{\underline{\quad\quad}^2 + \underline{\quad\quad}^2} = \underline{\quad}$

Step 2 **Check** for right angles. The slope of $\overline{RT}$ is

__________ = _____. The slope of $\overline{ST}$ is

__________ = _____. The product of the slopes is

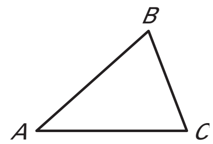
_____, so $\overline{RT} \perp \overline{ST}$ and $\angle RTS$ is a _______ angle.

Therefore, △*RST* is a ______________ triangle.

Your Notes

THEOREM 4.1: TRIANGLE SUM THEOREM

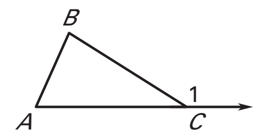
The sum of the measures of the interior angles of a triangle is ______.



$m\angle A + m\angle B + m\angle C =$ ______

THEOREM 4.2: EXTERIOR ANGLE THEOREM

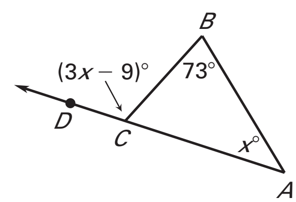
The measure of an exterior angle of a triangle is equal to the sum of the measures of the two ____________________ angles.



$m\angle 1 = m\angle$ ___ $+ m\angle$ ___

Example 3 *Find angle measure*

Use the diagram at the right to find the measure of $\angle DCB$.



Solution

Step 1 **Write** and solve an equation to find the value of x.

$(3x - 9)^\circ =$ __________ **Exterior Angle Theorem**

$x =$ _____ **Solve for x.**

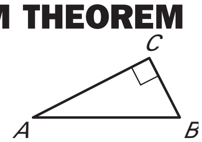
Step 2 **Substitute** ____ for x in $3x - 9$ to find $m\angle DCB$.

$3x - 9 = 3 \cdot$ ____ $- 9 =$ _____

The measure of $\angle DCB$ is ______.

COROLLARY TO THE TRIANGLE SUM THEOREM

The acute angles of a right triangle are _________________.

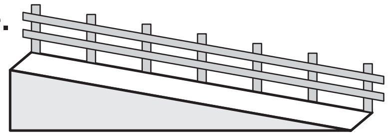


$m\angle A + m\angle B =$ _____

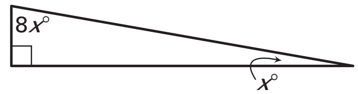
Your Notes

Example 4 ***Find angle measures from a verbal description***

Ramps The front face of the wheelchair ramp shown forms a right triangle. The measure of one acute angle in the triangle is eight times the measure of the other. Find the measure of each acute angle.



Solution



First, sketch a diagram of the situation. Let the measure of the smaller acute angle be $x°$. Then the measure of the larger acute angle is _____.

Use the Corollary to the Triangle Sum Theorem to set up and solve an equation.

$x° +$ _____ $=$ _____ **Corollary to the Triangle Sum Theorem**

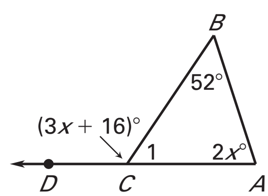
$x =$ _____ **Solve for x.**

So, the measures of the acute angles are _____ and _____.

✔ ***Checkpoint*** **Complete the following exercises.**

2. Triangle JKL has vertices $J(-2, -1)$, $K(1, 3)$, and $L(5, 0)$. Classify it by its sides. Then determine if it is a right triangle.

3. Find the measure of $\angle 1$ in the diagram shown.



4. In Example 4, what is the measure of the obtuse angle formed between the ramp and a segment extending from the horizontal leg?

Stop and get the teacher's signature before you move on.

4.2 Apply Congruence and Triangles

Goal • Identify congruent figures.

Complete the vocab. with definitions or pictures that make sense to you.

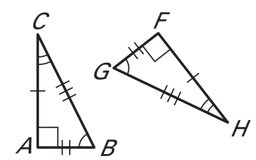
Your Notes

Rewrite the Goal as an "I can" statement!

VOCABULARY
Congruent figures
Corresponding parts

Example 1 ***Identify congruent parts***

Write a congruence statement for the triangles. Identify all pairs of congruent corresponding parts.



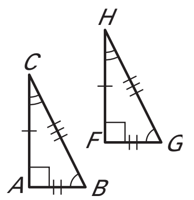
Solution

The diagram indicates that $\triangle ABC \cong \triangle$______.

Corresponding angles $\angle A \cong$ _____, $\angle B \cong$ _____, $\angle C \cong$ _____

Corresponding sides $\overline{AB} \cong$ _____, $\overline{BC} \cong$ _____, $\overline{CA} \cong$ _____

To help you identify corresponding parts, turn $\triangle FGH$.

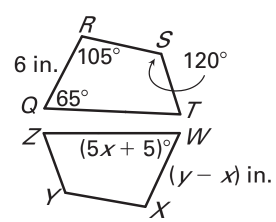


Example 2 ***Use properties of congruent figures***

In the diagram, $QRST \cong WXYZ$.

a. Find the value of x.

b. Find the value of y.



Solution

a. You know $\angle Q \cong \angle W$.

$m\angle Q =$ _______

$65° =$ __________

_____ $=$ _____

_____ $= x$

b. You know $\overline{QR} \cong \overline{WX}$.

$QR =$ _____

$6 =$ _______

$6 =$ ________

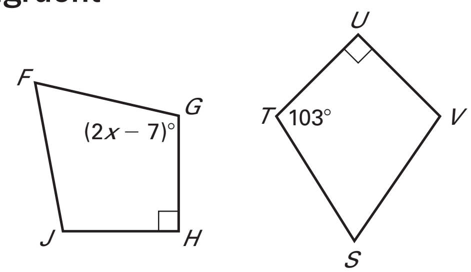
_____ $= y$

Your Notes

Stop and get the teacher's signature before you move on.

Checkpoint In Exercises 1 and 2, use the diagram shown in which $FGHJ \cong STUV$.

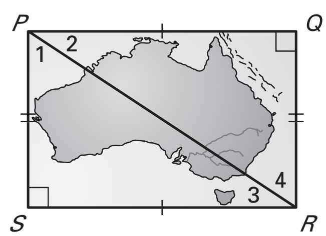
1. Identify all pairs of congruent corresponding parts.



2. Find the value of x and find $m\angle G$.

Example 3 ***Show that figures are congruent***

Maps If you cut the map in half along $\overline{PR}$, will the sections of the map be the same size and shape? *Explain.*



Solution

From the diagram, $\angle S \cong$ _____ because all right angles are congruent. Also, by the Lines Perpendicular to a Transversal Theorem, $\overline{PQ} \parallel$ _____. Then $\angle 1 \cong$ _____ and $\angle 2 \cong$ _____ by the ______________________ __________. So, all pairs of corresponding angles are ____________.

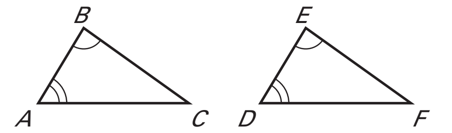
The diagram shows $\overline{PQ} \cong$ _____ and $\overline{QR} \cong$ _____. By the ____________________, $\overline{PR} \cong \overline{RP}$. All corresponding parts are ____________, so $\triangle PQR \cong$ ________.

______, the two sections will be the same ______ and ________.

Your Notes

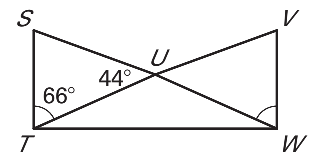
THEOREM 4.3: THIRD ANGLES THEOREM

If two angles of one triangle are congruent to two angles of another triangle, then the third angles are also ___________.



Example 4 *Use the Third Angles Theorem*

Find $m\angle V$.



Solution

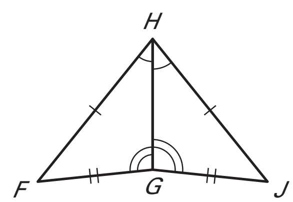
$\angle SUT \cong \angle VUW$ by the _________________________. The diagram shows that $\angle STU \cong$ ________, so by the Third Angles Theorem, $\angle S \cong$ _____. By the Triangle Sum Theorem, $m\angle S =$ _________________ $=$ _____. So, $m\angle S = m\angle V =$ _____ by the definition of congruent angles.

Example 5 *Prove that triangles are congruent*

Write a proof.

Given $\overline{FH} \cong \overline{JH}$, $\overline{FG} \cong \overline{JG}$, $\angle FHG \cong \angle JHG$, $\angle FGH \cong \angle JGH$

Prove $\triangle FGH \cong \triangle JGH$



Plan for Proof

a. Use the Reflexive Property to show ___________.

b. Use the Third Angles Theorem to show __________.

Plan in Action

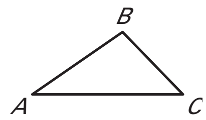
	Statements	Reasons
	1. $\overline{FH} \cong \overline{JH}$, $\overline{FG} \cong \overline{JG}$	**1.** ________
a.	**2.** ___________	**2.** Reflexive Property of Congruence
	3. $\angle FHG \cong \angle JHG$, $\angle FGH \cong \angle JGH$	**3.** ________
b.	**4.** __________	**4.** Third Angles Theorem
	5. $\triangle FGH \cong \triangle JGH$	**5.** ___________________

Your Notes

THEOREM 4.4: PROPERTIES OF CONGRUENT TRIANGLES

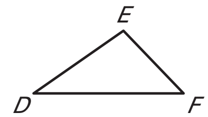
Reflexive Property of Congruent Triangles

For any triangle ABC, $\triangle ABC \cong$ ________.



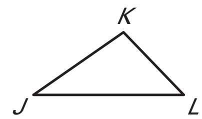
Symmetric Property of Congruent Triangles

If $\triangle ABC \cong \triangle DEF$, then ________________.



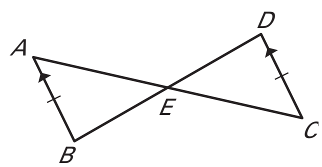
Transitive Property of Congruent Triangles

If $\triangle ABC \cong \triangle DEF$ and $\triangle DEF \cong \triangle JKL$, then ________________.

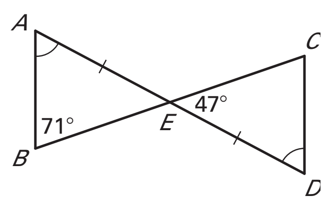


Checkpoint **Complete the following exercises.**

3. In the diagram at the right, E is the midpoint of $\overline{AC}$ and $\overline{BD}$. Show that $\triangle ABE \cong \triangle CDE$.



4. In the diagram, what is the measure of $\angle D$?



5. By the definition of congruence, what additional information is needed to know that $\triangle ABE \cong \triangle DCE$ in Exercise 4?

Stop and get the teacher's signature before you move on.

Homework

4.3 Prove Triangles Congruent by SSS

Goal • Use side lengths to prove triangles are congruent.

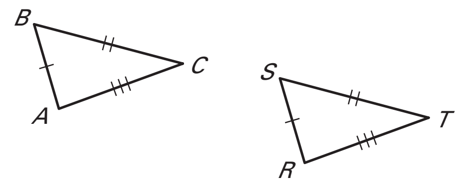
Complete the vocab. with definitions or pictures that make sense to you.

Your Notes

Rewrite the Goal as an "I can" statement!

POSTULATE 19: SIDE-SIDE-SIDE (SSS) CONGRUENCE POSTULATE

If three sides of one triangle are congruent to three sides of a second triangle, then the two triangles are congruent.



If Side $\overline{AB} \cong$ _____,

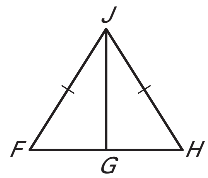
Side $\overline{BC} \cong$ _____, and

Side $\overline{CA} \cong$ _____,

then $\triangle ABC \cong$ ________.

Example 1 ***Use the SSS Congruence Postulate***

Write a proof.



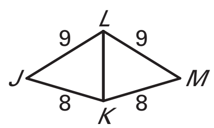
Given $\overline{FJ} \cong \overline{HJ}$,
G is the midpoint of $\overline{FH}$.

Prove $\triangle FGJ \cong \triangle HGJ$

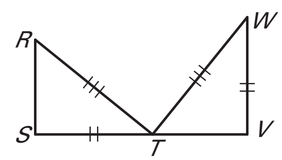
Proof It is given that $\overline{FJ} \cong$ _____. Point G is the midpoint of $\overline{FH}$, so ____________. By the Reflexive Property, ____________. So, by the ________________________________, $\triangle FGJ \cong \triangle HGJ$.

Checkpoint **Decide whether the congruence statement is true. *Explain* your reasoning.**

1. $\triangle JKL \cong \triangle MKL$



2. $\triangle RST \cong \triangle TVW$

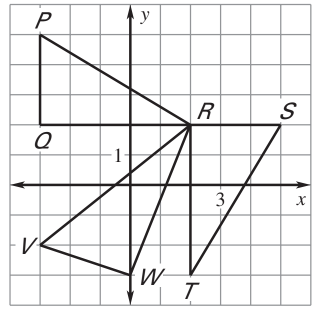


Stop and get the teacher's signature before you move on.

Your Notes

Example 2 ***Congruence in the coordinate plane***

Determine whether $\triangle PQR$ is congruent to the other triangles shown at the right.



Solution

By counting, $PQ = 3$ and $QR = 5$. Use the distance formula to find PR.

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

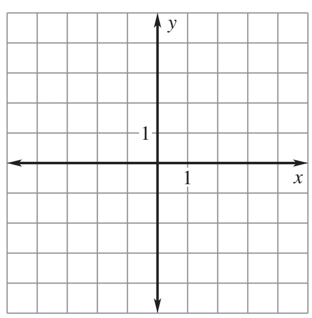
$PR = \sqrt{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_} = \sqrt{\_\_\_\_}$

By the SSS Congruence Postulate, any triangle with side lengths ___, ___, and _____ will be congruent to $\triangle PQR$. The distance from R to S is ___. The distance from R to T is ___. The distance from S to T is $\sqrt{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_} = \sqrt{\_\_\_\_}$. So, $\triangle PQR \cong$ _______.

The distance from W to V is $\sqrt{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_} = \sqrt{\_\_\_\_}$. No side of $\triangle PQR$ has a length of $\sqrt{\_\_\_\_}$, so $\triangle PQR$ ___ $\triangle VWR$.

✔ ***Checkpoint*** **Complete the following exercise.**

3. $\triangle DFG$ has vertices $D(-2, 4)$, $F(4, 4)$, and $G(-2, 2)$. $\triangle LMN$ has vertices $L(-3, -3)$, $M(-3, 3)$, and $N(-1, -3)$. Graph the triangles in the same coordinate plane and show that they are congruent.

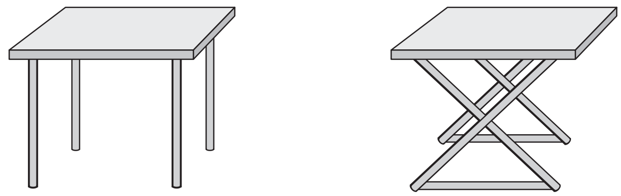


Stop and get the teacher's signature before you move on.

Your Notes

Example 3 *Solve a real-world problem*

Stability *Explain* why the table with the diagonal legs is stable, while the one without the diagonal legs can collapse.

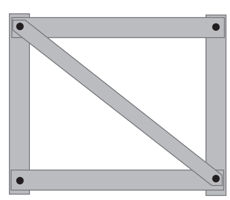


Solution

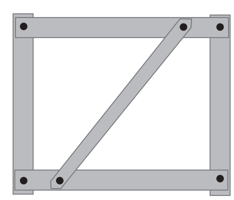
The table with the diagonal legs forms triangles with _______ side lengths. By the SSS Congruence Postulate, these triangles ________________________, so the table is ________. The table without the diagonal legs is ____________ because there are many possible quadrilaterals with the given side lengths.

Checkpoint **Determine whether the figure is stable. *Explain* your reasoning.**

4.



5.



Stop and get the teacher's signature before you move on.

Homework

4.4 Prove Triangles Congruent by SAS and HL

Goal • Use sides and angles to prove congruence.

Complete the vocab. with definitions or pictures that make sense to you.

Your Notes

Rewrite the Goal as an "I can" statement!

VOCABULARY
Leg of a right triangle
Hypotenuse

POSTULATE 20: SIDE-ANGLE-SIDE (SAS) CONGRUENCE POSTULATE

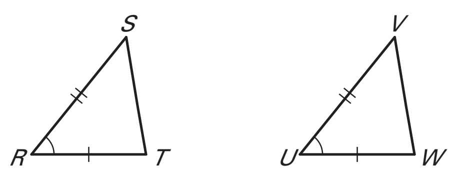
If two sides and the included angle of one triangle are congruent to two sides and the included angle of a second triangle, then the two triangles are congruent.

If **Side** $\overline{RS} \cong$ _____,

Angle $\angle R \cong$ _____, and

Side $\overline{RT} \cong$ _____,

then $\triangle RST \cong$ _________.

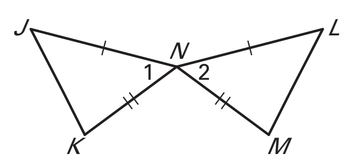


Example 1 ***Use the SAS Congruence Postulate***

Write a proof.

Given $\overline{JN} \cong \overline{LN}$, $\overline{KN} \cong \overline{MN}$

Prove $\triangle JKN \cong \triangle LMN$

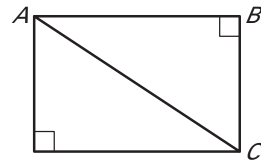


Statements	**Reasons**
1. $\overline{JN} \cong$ _____, $\overline{KN} \cong$ _____	**1.** Given
2. $\angle 1 \cong \angle 2$	**2.** ______________________
3. $\triangle JKN \cong \triangle LMN$	**3.** ______________________

Your Notes

Example 2 *Use SAS and properties of shapes*

In the diagram, *ABCD* is a rectangle. What can you conclude about △*ABC* and △*CDA*?



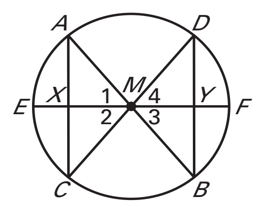
Solution

By the ______________________________,

∠*B* ≅ ∠*D*. Opposite sides of a rectangle are congruent, so _________ and _________.

△*ABC* and △*CDA* are congruent by the _______________ ________.

Checkpoint **In the diagram, $\overline{AB}$, $\overline{CD}$, and $\overline{EF}$ pass through the center *M* of the circle. Also, ∠1 ≅ ∠2 ≅ ∠3 ≅ ∠4.**



1. Prove that △*DMY* ≅ △*BMY*.

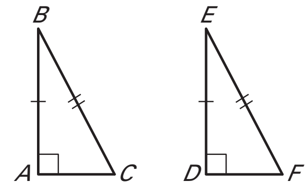
2. What can you conclude about $\overline{AC}$ and $\overline{BD}$?

Stop and get the teacher's signature before you move on.

Your Notes

THEOREM 4.5: HYPOTENUSE-LEG CONGRUENCE THEOREM

If the hypotenuse and a leg of a right triangle are congruent to the hypotenuse and a leg of a second triangle, then the two triangles are __________.

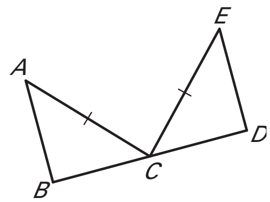


Example 3 *Use the Hypotenuse-Leg Theorem*

Write a proof.

Given $\overline{AC} \cong \overline{EC}$, $\overline{AB} \perp \overline{BD}$, $\overline{ED} \perp \overline{BD}$, $\overline{AC}$ is a bisector of $\overline{BD}$.

Prove $\triangle ABC \cong \triangle EDC$

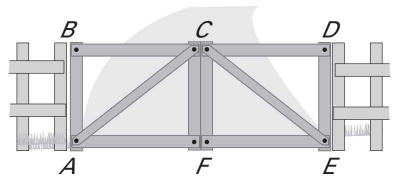


	Statements	Reasons
H	1. $\overline{AC} \cong \overline{EC}$	1. ________
	2. $\overline{AB} \perp \overline{BD}$, $\overline{ED} \perp \overline{BD}$	2. ________
	3. $\angle B$ and $\angle D$ are __________.	3. Definition of $\perp$ lines
	4. $\triangle ABC$ and $\triangle EDC$ are __________.	4. Definition of a ______ ________
	5. $\overline{AC}$ is a bisector of $\overline{BD}$.	5. ________
L	6. $\overline{BC} \cong \overline{DC}$	6. Definition of segment bisector
	7. $\triangle ABC \cong \triangle EDC$	7. ______________ __________

Your Notes

Example 4 ***Choose a postulate or theorem***

Gate The entrance to a ranch has a rectangular gate as shown in the diagram. You know that $\triangle AFC \cong \triangle EFC$. What postulate or theorem can you use to conclude that $\triangle ABC \cong \triangle EDC$?



Solution

You are given that *ABDE* is a rectangle, so $\angle B$ and $\angle D$ are ______________. Because opposite sides of a rectangle are ______________, $\overline{AB} \cong$ _____. You are also given that $\triangle AFC \cong \triangle EFC$, so $\overline{AC} \cong$ _____. The hypotenuse and a leg of each triangle is congruent.

You can use the ___________________________ to conclude that $\triangle ABC \cong \triangle EDC$.

✔ ***Checkpoint*** **Complete the following exercises.**

3. *Explain* why a diagonal of a rectangle forms a pair of congruent triangles.

4. In Example 4, suppose it is given that *ABCF* and *EDCF* are squares. What postulate or theorem can you use to conclude that $\triangle ABC \cong \triangle EDC$? *Explain*.

Stop and get the teacher's signature before you move on.

Homework

4.5 Prove Triangles Congruent by ASA and AAS

Complete the vocab. with definitions or pictures that make sense to you.

Goal • Use two more methods to prove congruences.

Your Notes

Rewrite the Goal as an "I can" statement!

VOCABULARY

Flow proof

POSTULATE 21: ANGLE-SIDE-ANGLE (ASA) CONGRUENCE POSTULATE

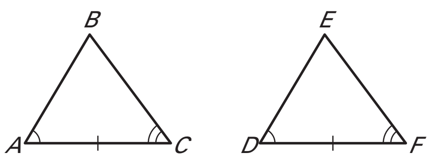
If two angles and the included side of one triangle are congruent to two angles and the included side of a second triangle, then the two triangles are congruent.

If Angle $\angle A \cong$ _____,

Side $\overline{AC} \cong$ _____, and

Angle $\angle C \cong$ _____,

then $\triangle ABC \cong$ ________.



THEOREM 4.6: ANGLE-ANGLE-SIDE (AAS) CONGRUENCE THEOREM

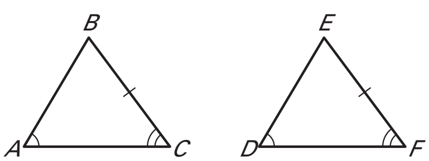
If two angles and a non-included side of one triangle are congruent to two angles and the corresponding non-included side of a second triangle, then the two triangles are congruent.

If Angle $\angle A \cong$ _____,

Angle $\angle C \cong$ _____, and

Side $\overline{BC} \cong$ _____,

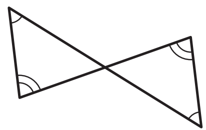
then $\triangle ABC \cong$ ________.

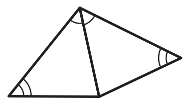


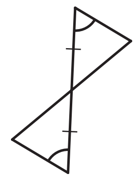
Your Notes

Example 1 *Identify congruent triangles*

Can the triangles be proven congruent with the information given in the diagram? If so, state the postulate or theorem you would use.

a. 

b. 

c. 

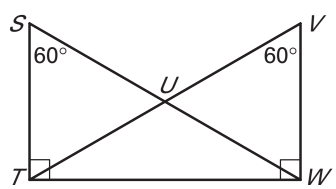
Solution

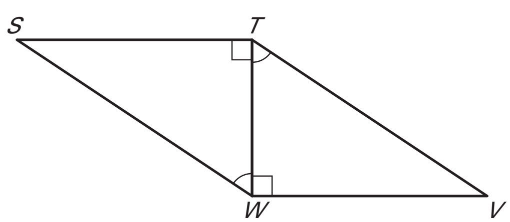
a. There is not enough information to prove the triangles are congruent, because no _______ are known to be congruent.

b. Two pairs of angles and a _______________ pair of sides are congruent. The triangles are congruent by the ______________________________.

c. The vertical angles are congruent, so two pairs of angles and their ________________ are congruent. The triangles are congruent by the __________________ __________.

Checkpoint **Can △*STW* and △*VWT* be proven congruent with the information given in the diagram? If so, state the postulate or theorem you would use.**

1. 

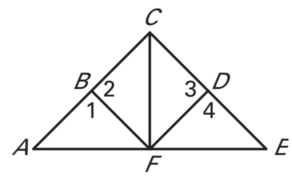
2. 

Stop and get the teacher's signature before you move on.

Your Notes

Example 2 *Write a flow proof*

In the diagram, $\angle 1 \cong \angle 4$ and $\overline{CF}$ bisects $\angle ACE$. Write a flow proof to show $\triangle CBF \cong \triangle CDF$.



Solution

Given $\angle 1 \cong \angle 4$, $\overline{CF}$ bisects $\angle ACE$.

Prove $\triangle CBF \cong \triangle CDF$

$\angle 1 \cong \angle 4$

$\angle 1$ and $\angle 2$ are ______________.
$\angle 3$ and $\angle 4$ are ______________.

$\overline{CF}$ bisects. $\angle ACE$.

________ Def. of ______________ angles ________

$\angle 2 \cong$ ______

$\overline{CF} \cong \overline{CF}$

$\angle ACF \cong$ ________

Congruent Supps. Thm.

____

Def. of $\angle$ bisector

$\triangle CBF \cong \triangle CDF$

____

Checkpoint Complete the following exercise.

3. In Example 2, suppose it is given that $\overline{CF}$ bisects $\angle ACE$ and $\angle BFD$. Write a flow proof to show $\triangle CBF \cong \triangle CDF$.

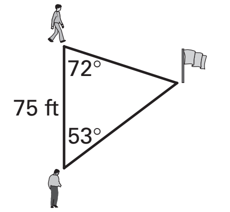
Stop and get the teacher's signature before you move on.

Your Notes

Example 3 ***Choose a postulate or theorem***

Games You and a friend are trying to find a flag hidden in the woods. Your friend is standing 75 feet away from you. When facing each other, the angle from you to the flag is 72° and the angle from your friend to the flag is 53°. Is there enough information to locate the flag?

Solution

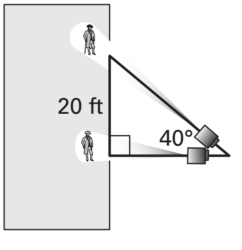


The locations of you, your friend, and the flag form a triangle. The measures of ____________ and an ______________ of the triangle are known.

By the ___________________________, all triangles with these measures are congruent. So, the triangle formed is unique and the flag location is given by the _____ _______.

Checkpoint **Complete the following exercise.**

4. Theater You are working two spotlights for a play. Two actors are standing apart from each other on the end of the stage. The spotlights are located and pointed as shown in the diagram. Can one of the actors move without requiring the spotlight to move and without changing the distance between the other actor?



Stop and get the teacher's signature before you move on.

Homework

4.6 Use Congruent Triangles

Complete the vocab. with definitions or pictures that make sense to you.

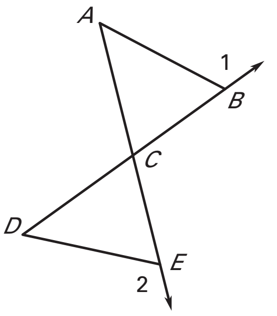
Goal • Use congruent triangles to prove corresponding parts congruent.

Your Notes

Rewrite the Goal as an "I can" statement!

Example 1 ***Use congruent triangles***

Explain **how you can use the given information to prove that the triangles are congruent.**



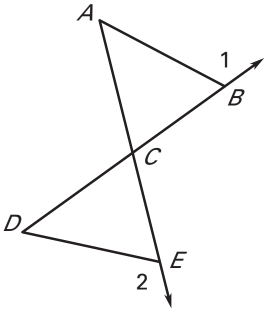
Given $\angle 1 \cong \angle 2$, $\overline{AB} \cong \overline{DE}$

Prove $\overline{DC} \cong \overline{AC}$

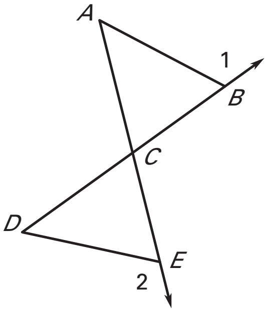
Solution

If you can show that ________________, you will know that $\overline{DC} \cong \overline{AC}$. First, copy the diagram and mark the given information. Then add the information that you can deduce. In this case, $\angle ABC$ and $\angle DEC$ are ________________ to congruent angles, so $\angle$______ $\cong$ $\angle$______. Also, $\angle ACB \cong$ ________.

Mark given information.



Add deduced information.



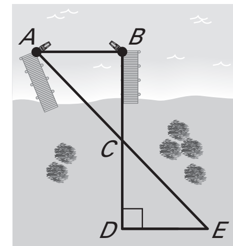
Two angle pairs and a ______________ side are congruent, so by the ___________________________, $\triangle ABC \cong \triangle DEC$. Because ______________________ of congruent triangles are congruent, $\overline{DC} \cong \overline{AC}$.

Your Notes

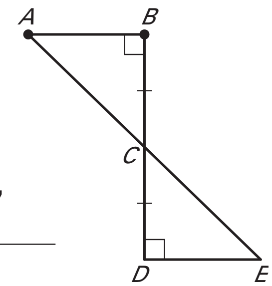
When you cannot easily measure a length directly, you can make conclusions about the length *indirectly*, usually by calculations based on known lengths.

Example 2 *Use congruent triangles for measurement*

Boats Use the following method to find the distance between two docked boats, from point A to point B.



- Place a marker at D so that $\overline{AB} \perp \overline{BD}$.
- Find C, the midpoint of $\overline{BD}$.
- Locate the point E so that $\overline{BD} \perp \overline{DE}$ and A, C, and E are collinear.
- *Explain* how this plan allows you to find the distance.

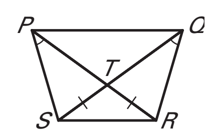


Solution

Because $\overline{AB} \perp \overline{BD}$ and $\overline{BD} \perp \overline{DE}$, _____ and _____ are congruent right angles. Because C is the midpoint of $\overline{BD}$, _____ $\cong$ _____. The vertical angles ________ and ________ are congruent. So, $\triangle CBA \cong$ ________ by the ______________________________. Then, because corresponding parts of congruent triangles are congruent, $BA =$ _____. So, you can find the distance AB between the boats by measuring _____.

Checkpoint Complete the following exercises.

1. *Explain* how you can prove that $\overline{PR} \cong \overline{QS}$.



2. In Example 2, does it matter how far away from point B you place a marker at point D? *Explain*.

Stop and get the teacher's signature before you move on.

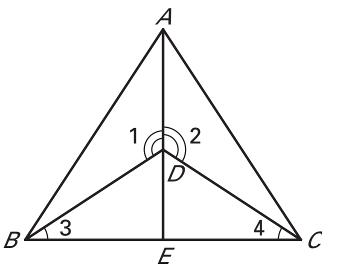
Your Notes

Example 3 *Plan a proof involving pairs of triangles*

Use the given information to write a plan for proof.

Given $\angle 1 \cong \angle 2$, $\angle 3 \cong \angle 4$

Prove $\triangle ABD \cong \triangle ACD$



Solution

In $\triangle ABD$ and $\triangle ACD$, you know that $\angle 1 \cong$ _____ and $\overline{AD} \cong \overline{AD}$. If you can show that $\overline{BD} \cong \overline{CD}$, you can use the ______________________.

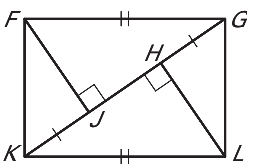
To prove that $\overline{BD} \cong \overline{CD}$, you can first prove that $\triangle BED \cong$ ________. You are given $\angle 1 \cong \angle 2$ and $\angle 3 \cong \angle 4$. $\overline{ED} \cong \overline{ED}$ by the Reflexive Property and $\angle BDE \cong$ ________ by the Congruent Supplements Theorem. You can use the ______________________ to prove that $\triangle BED \cong$ ________.

Plan for Proof Use the ______________________ to prove that $\triangle BED \cong$ ________. Then state that $\overline{BD} \cong \overline{CD}$. Use the ______________________ to prove that $\triangle ABD \cong \triangle ACD$.

✔ ***Checkpoint*** **Use the given information to write a plan for proof.**

3. Given $\overline{GH} \cong \overline{KJ}$, $\overline{FG} \cong \overline{LK}$, $\angle FJG$ and $\angle LHK$ are rt. $\angle$s.

Prove $\triangle FJK \cong \triangle LHG$

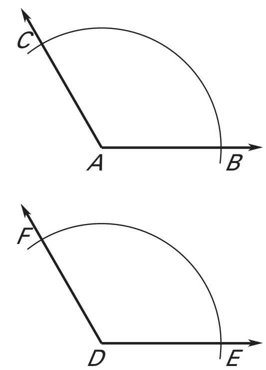


Stop and get the teacher's signature before you move on.

Your Notes

Example 4 *Prove a construction*

Write a proof to verify that the construction for copying an obtuse angle is valid.



Solution

Add $\overline{BC}$ and $\overline{EF}$ to the diagram. In the construction, $\overline{AB}$, ______, ______, and ______ are determined by the same compass setting, as are $\overline{BC}$ and ______. So, you can assume the following as given statements.

Given $\overline{AB} \cong$ ______, $\overline{AC} \cong$ ______, $\overline{BC} \cong$ ______

Prove $\angle D \cong$ ______

Plan for Proof Show that $\triangle CAB \cong$ ________, so you can conclude that the corresponding parts $\angle D$ and ______ are congruent.

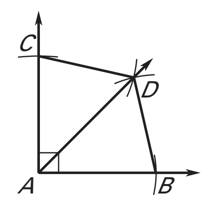
Plan for Action

Statements	Reasons
1. $\overline{AB} \cong$ ______, $\overline{AC} \cong$ ______, $\overline{BC} \cong$ ______	**1.** ________
2. $\triangle CAB \cong$ ________	**2.** SSS Congruence Postulate
3. $\angle D \cong$ ______	**3.** Corresp. parts of $\cong$ triangles are $\cong$.

Stop and get the teacher's signature before you move on.

Checkpoint **Complete the following exercise.**

4. Write a paragraph proof to verify that the construction for bisecting a right angle is valid.



Homework

4.7 Use Isosceles and Equilateral Triangles

Complete the vocab. with definitions or pictures that make sense to you.

Goal • Use theorems about isosceles and equilateral triangles.

Your Notes

Rewrite the Goal as an "I can" statement!

VOCABULARY

Legs

Vertex angle

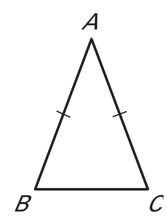
Base

Base angles

THEOREM 4.7: BASE ANGLES THEOREM

If two sides of a triangle are congruent, then the angles opposite them are congruent.

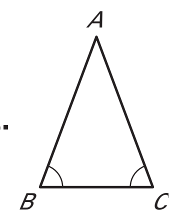
If $\overline{AB} \cong \overline{AC}$, then $\angle B \cong$ _____.



THEOREM 4.8: CONVERSE OF BASE ANGLES THEOREM

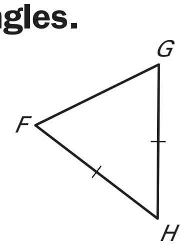
If two angles of a triangle are congruent, then the sides opposite them are congruent.

If $\angle B \cong \angle C$, then $\overline{AB} \cong$ _____.



Example 1 ***Apply the Base Angles Theorem***

In $\triangle FGH$, $\overline{FH} \cong \overline{GH}$. Name two congruent angles.



Solution

$\overline{FH} \cong \overline{GH}$, so by the Base Angles Theorem, $\angle$___ $\cong \angle$___.

Your Notes

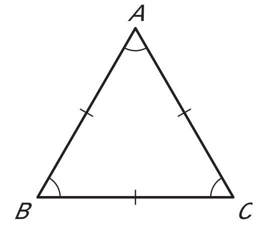
The corollaries state that a triangle is *equilateral* if and only if it is *equiangular*.

COROLLARY TO THE BASE ANGLES THEOREM

If a triangle is equilateral, then it is ______________.

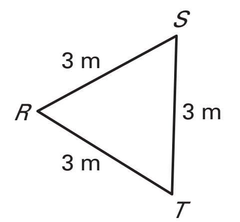
COROLLARY TO THE CONVERSE OF BASE ANGLES THEOREM

If a triangle is equiangular, then it is ____________.



Example 2 *Find measures in a triangle*

Find the measures of $\angle R$, $\angle S$, and $\angle T$.



Solution

The diagram shows that $\triangle RST$ is ____________. Therefore, by the Corollary to the Base Angles Theorem, $\triangle RST$ is _____________. So, $m\angle R = m\angle S = m\angle T$.

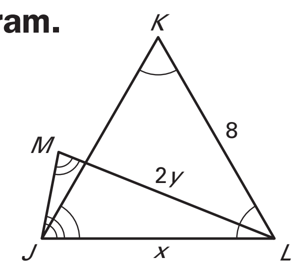
$3(m\angle R) =$ ______ **Triangle Sum Theorem**

$m\angle R =$ _____ **Divide each side by 3.**

The measures of $\angle R$, $\angle S$, and $\angle T$ are all _____.

Example 3 *Use isosceles and equilateral triangles*

Find the values of x and y in the diagram.



Solution

You cannot use $\angle J$ to refer to $\angle LJM$ because three angles have J as their vertex.

Step 1 **Find** the value of x. Because $\triangle JKL$ is _____________, it is also _____________ and $\overline{KL} \cong$ ____. Therefore, $x =$ ___.

Step 2 Find the value of y. Because $\angle JML \cong$ ________, $\overline{LM} \cong$ _____, and $\triangle LMJ$ is isosceles. You know that $LJ =$ ___.

$LM =$ _____ **Definition of congruent segments**

$2y =$ ___ **Substitute $2y$ for LM and ___ for LJ.**

$y =$ ___ **Divide each side by 2.**

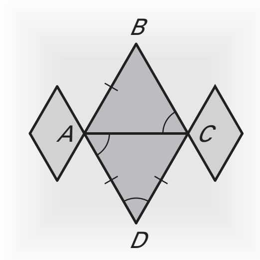
Your Notes

Example 4 ***Solve a multi-step problem***

Quilting The pattern at the right is present in a quilt.

a. *Explain* why $\triangle ADC$ is equilateral.

b. Show that $\triangle CBA \cong \triangle ADC$.



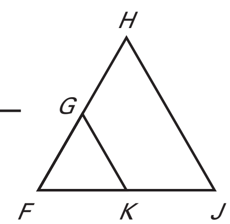
Solution

a. By the Base Angles Theorem, $\angle DAC \cong$ ________. So, $\triangle ADC$ is ______________. By the ________________ ____________________________________, $\triangle ADC$ is equilateral.

b. By the Base Angles Theorem, $\angle ABC \cong$ ________. So, $\triangle CBA \cong \triangle ADC$ by the ____________________________.

✔ ***Checkpoint*** **Complete the following exercises.**

1. Copy and complete the statement: If $\overline{FH} \cong \overline{FJ}$, then $\angle$? $\cong \angle$? .



2. Copy and complete the statement: If $\triangle FGK$ is equiangular and $FG = 15$, then $GK =$? .

3. Use parts (a) and (b) in Example 4 to show that $m\angle BAD = 120°$.

Stop and get the teacher's signature before you move on.

Homework

4.8 Perform Congruence Transformations

Complete the vocab. with definitions or pictures that make sense to you.

Goal • Create an image congruent to a given triangle.

Your Notes

Rewrite the Goal as an "I can" statement!

VOCABULARY

Transformation

Image

Translation

Reflection

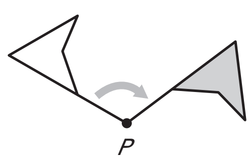
Rotation

Congruence Transformation

Example 1 ***Identify transformations***

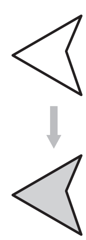
Name the type of transformation demonstrated in each picture.

a.



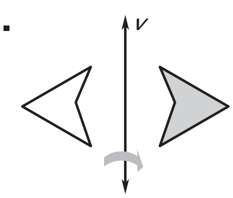
__________ about a point

b.



__________ in a straight path

c.



__________ in a vertical line

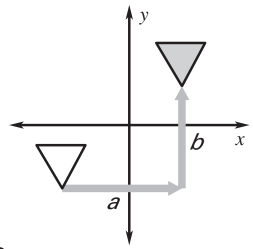
Your Notes

COORDINATE NOTATION FOR A TRANSLATION

You can describe a translation by the notation

$(x, y) \rightarrow (x + a, y + b)$

which shows that each point (x, y) of the unshaded figure is translated horizontally a units and vertically b units.



Example 2 ***Translate a figure in the coordinate plane***

Figure $ABCD$ has the vertices $A(1, 2)$, $B(3, 3)$, $C(4, -1)$, and $D(1, -2)$. Sketch $ABCD$ and its image after the translation $(x, y) \rightarrow (x - 4, y + 2)$.

Solution

First draw $ABCD$. Find the translation of each vertex by ________________ from its x-coordinate and ____________ to its y-coordinate. Then draw $ABCD$ and its image.

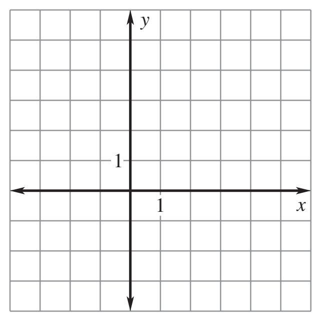
$(x, y) \rightarrow (x - 4, y + 2)$

$A(1, 2) \rightarrow$ __________

$B(3, 3) \rightarrow$ __________

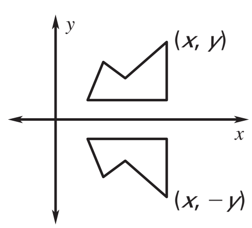
$C(4, -1) \rightarrow$ ________

$D(1, -2) \rightarrow$ __________



COORDINATE NOTATION FOR A REFLECTION

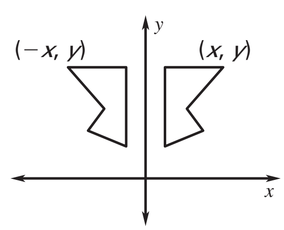
Reflection in the x-axis



Multiply y-coordinate by -1.

$(x, y) \rightarrow (x, -y)$

Reflection in the y-axis



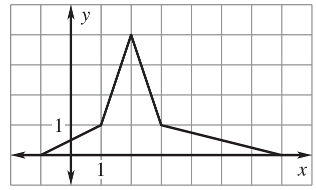
Multiply x-coordinate by -1.

$(x, y) \rightarrow (-x, y)$

Your Notes

Example 3 *Reflect a figure in the x-axis*

Shapes You are cutting figures out of paper. Use a reflection in the *x*-axis to draw the other half of the figure.



Solution

Multiply the ______________ of each vertex by -1 to find the corresponding vertex in the image. Then draw the image.

$(x, y) \rightarrow$ ________

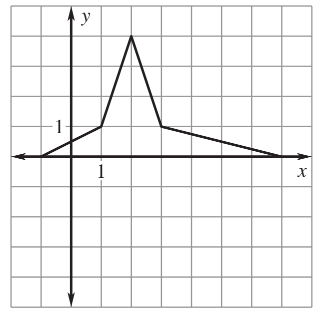
$(-1, 0) \rightarrow$ ________

$(1, 1) \rightarrow$ ________

$(2, 4) \rightarrow$ ________

$(3, 1) \rightarrow$ ________

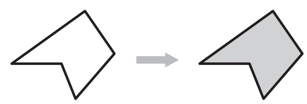
$(7, 0) \rightarrow$ ______



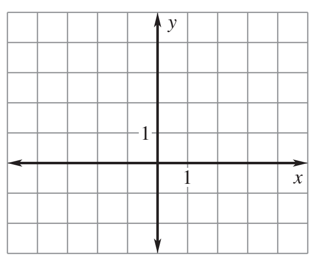
You can check your results by looking to see if each original point and its image are the same distance from the ________.

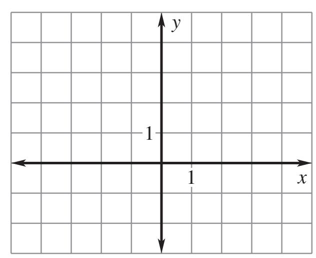
✔ ***Checkpoint*** **Complete the following exercises.**

1. Name the type of transformation shown.



2. Figure *FGHJ* has the vertices $F(0, 2)$, $G(2, 3)$, $H(3, 3)$, and $J(0, -2)$. Sketch *FGHJ* and its image after (a) the translation $(x, y) \rightarrow (x - 3, y + 1)$ and (b) a reflection in the *y*-axis.





Stop and get the teacher's signature before you move on.

Your Notes

Example 4 ***Identify a rotation***

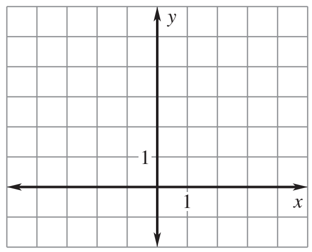
Graph $\overline{JK}$ and $\overline{LM}$. Tell whether $\overline{LM}$ is a rotation of $\overline{JK}$ about the origin. If so, give the angle and direction of rotation.

a. $J(3, 1)$, $K(1, 4)$, $L(-1, 3)$, $M(-4, 1)$

b. $J(-2, 1)$, $K(-1, 5)$, $L(1, 1)$, $M(2, 5)$

Solution

a.



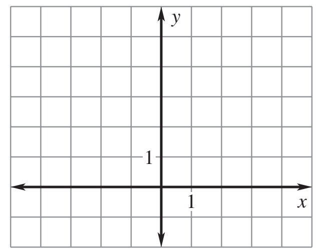
$m\angle JOL$ ____ $m\angle KOM$

____ ______

____

____

b.

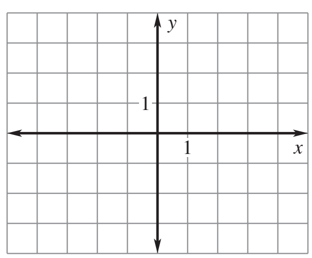


$m\angle JOL$ ____ $m\angle KOM$

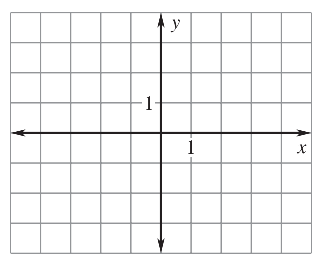
____

Checkpoint **Graph $\overline{RS}$ and $\overline{TV}$. Tell whether $\overline{TV}$ is a rotation of $\overline{RS}$ about the origin. If so, give the angle of rotation.**

3. $R(-3, -2)$, $S(-3, 2)$, $T(-1, 2)$, $V(3, 2)$



4. $R(-1, 1)$, $S(-4, 2)$, $T(1, -1)$, $V(4, -2)$



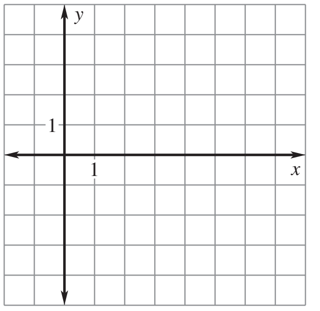
Stop and get the teacher's signature before you move on.

Your Notes

Example 5 *Verify congruence*

The vertices of △*PQR* are *P*(2, 2), *Q*(3, 4), and *R*(5, 2). The notation (*x*, *y*) → (*x* + 1, *y* − 6) describes the translation of △*PQR* to △*XYZ*. Show that △*PQR* ≅ △*XYZ* to verify that the translation is a congruence transformation.

Solution



S You can see that *PR* = _____ = ___, so $\overline{PR}$ ≅ _____.

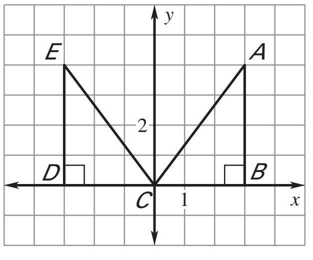
A Using the slopes, $\overline{PQ}$ ∥ _____ and $\overline{QR}$ ∥ _____. If you extend $\overline{PQ}$ and $\overline{XZ}$ to form ∠*V*, the Corresponding Angles Postulate gives you ________ ≅ ∠*V* and ∠*V* ≅ ________. Then, ________ ≅ ________ by the Transitive Property of Congruence.

S Using the distance formula, *PQ* = _____ = _____ so $\overline{PQ}$ ≅ _____. So, △*PQR* ≅ △*XYZ* by the ______ ________________________.

Because △*PQR* ≅ △*XYZ*, the translation is a congruence transformation.

Checkpoint **Complete the following exercise.**

5. Show that △*ABC* ≅ △*EDC* to verify that the transformation is a congruence transformation.



Stop and get the teacher's signature before you move on.

Homework