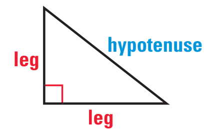
# 7.1 Apply the Pythagorean Theorem

| | |
|---|---|
| **Before** | You learned about the relationships within triangles. |
| **Now** | You will find side lengths in right triangles. |
| **Why?** | So you can find the shortest distance to a campfire, as in Ex. 35. |

**Key Vocabulary**
- **Pythagorean triple**
- **right triangle,** *p. 217*
- **leg of a right triangle,** *p. 241*
- **hypotenuse,** *p. 241*

One of the most famous theorems in mathematics is the Pythagorean Theorem, named for the ancient Greek mathematician Pythagoras (around 500 B.C.). This theorem can be used to find information about the lengths of the sides of a right triangle.

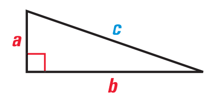



**THEOREM** *For Your Notebook*

**THEOREM 7.1 Pythagorean Theorem**

In a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs.

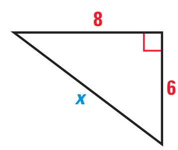
*Proof:* p. 434; Ex. 32, p. 455




$c^2 = a^2 + b^2$

## EXAMPLE 1 Find the length of a hypotenuse

**Find the length of the hypotenuse of the right triangle.**




**Solution**

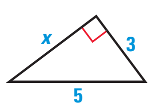
| | |
|---|---|
| $(\text{hypotenuse})^2 = (\text{leg})^2 + (\text{leg})^2$ | Pythagorean Theorem |
| $x^2 = 6^2 + 8^2$ | Substitute. |
| $x^2 = 36 + 64$ | Multiply. |
| $x^2 = 100$ | Add. |
| $x = 10$ | Find the positive square root. |

**ABBREVIATE**
In the equation for the Pythagorean Theorem, "length of hypotenuse" and "length of leg" was shortened to "hypotenuse" and "leg".

### ✓ GUIDED PRACTICE for Example 1

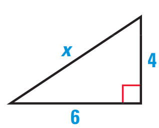
**Identify the unknown side as a *leg* or *hypotenuse*. Then, find the unknown side length of the right triangle. Write your answer in simplest radical form.**

**1.**




leg; 4

**2.**



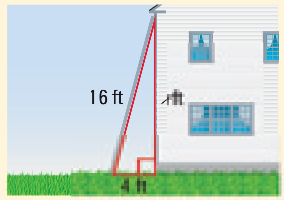

hypotenuse; $2\sqrt{13}$

## EXAMPLE 2 Standardized Test Practice

A 16 foot ladder rests against the side of the house, and the base of the ladder is 4 feet away. Approximately how high above the ground is the top of the ladder?

Ⓐ 240 feet  Ⓑ 20 feet

Ⓒ 16.5 feet  Ⓓ 15.5 feet




### Solution

$$\left(\text{Length of ladder}\right)^2 = \left(\text{Distance from house}\right)^2 + \left(\text{Height of ladder}\right)^2$$

$16^2 = 4^2 + x^2$ **Substitute.**

$256 = 16 + x^2$ **Multiply.**

$240 = x^2$ **Subtract 16 from each side.**

$\sqrt{240} = x$ **Find positive square root.**

$15.491 \approx x$ **Approximate with a calculator.**

The ladder is resting against the house at about 15.5 feet above the ground.

▶ The correct answer is D. Ⓐ Ⓑ Ⓒ Ⓓ

**APPROXIMATE**
In real-world applications, it is usually appropriate to use a calculator to approximate the square root of a number. Round your answer to the nearest tenth.

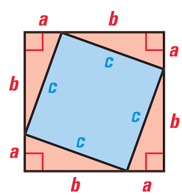
### ✓ GUIDED PRACTICE for Example 2

3. The top of a ladder rests against a wall, 23 feet above the ground. The base of the ladder is 6 feet away from the wall. What is the length of the ladder? **about 23.8 ft**

4. The Pythagorean Theorem is only true for what type of triangle? **right triangle**

**PROVING THE PYTHAGOREAN THEOREM** There are many proofs of the Pythagorean Theorem. An informal proof is shown below. You will write another proof in Exercise 32 on page 455.

In the figure at the right, the four right triangles are congruent, and they form a small square in the middle. The area of the large square is equal to the area of the four triangles plus the area of the smaller square.




$$\left(\text{Area of large square}\right) = \left(\text{Area of four triangles}\right) + \left(\text{Area of smaller square}\right)$$

$(a + b)^2 = 4\left(\frac{1}{2}ab\right) + c^2$ **Use area formulas.**

$a^2 + 2ab + b^2 = 2ab + c^2$ **Multiply.**

$a^2 + b^2 = c^2$ **Subtract $2ab$ from each side.**

**REVIEW AREA**
Recall that the area of a square with side length $s$ is $A = s^2$. The area of a triangle with base $b$ and height $h$ is $A = \frac{1}{2}bh$.

AnimatedGeometry at classzone.com



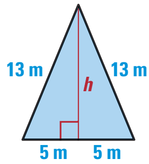
Differentiated Instruction

## EXAMPLE 3 Find the area of an isosceles triangle

**Find the area of the isosceles triangle with side lengths 10 meters, 13 meters, and 13 meters.**

**Solution**

*STEP 1* **Draw** a sketch. By definition, the length of an altitude is the height of a triangle. In an isosceles triangle, the altitude to the base is also a perpendicular bisector. So, the altitude divides the triangle into two right triangles with the dimensions shown.




*STEP 2* **Use** the Pythagorean Theorem to find the height of the triangle.

| | |
|---|---|
| $c^2 = a^2 + b^2$ | Pythagorean Theorem |
| $13^2 = 5^2 + h^2$ | Substitute. |
| $169 = 25 + h^2$ | Multiply. |
| $144 = h^2$ | Subtract 25 from each side. |
| $12 = h$ | Find the positive square root. |

*STEP 3* **Find** the area.

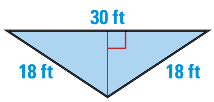
$$\text{Area} = \frac{1}{2}(\text{base})(\text{height}) = \frac{1}{2}(10)(12) = 60 \text{ m}^2$$

▶ The area of the triangle is 60 square meters.

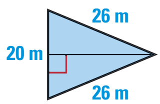
**READ TABLES**
You may find it helpful to use the Table of Squares and Square Roots on p. 924.

## ✓ GUIDED PRACTICE for Example 3

**Find the area of the triangle.**

5. 


about 149.2 ft$^2$

6. 


240 m$^2$

**PYTHAGOREAN TRIPLES** A **Pythagorean triple** is a set of three positive integers $a$, $b$, and $c$ that satisfy the equation $c^2 = a^2 + b^2$.

## KEY CONCEPT *For Your Notebook*

### Common Pythagorean Triples and Some of Their Multiples

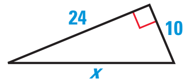
| **3, 4, 5** | **5, 12, 13** | **8, 15, 17** | **7, 24, 25** |
|---|---|---|---|
| 6, 8, 10 | 10, 24, 26 | 16, 30, 34 | 14, 48, 50 |
| 9, 12, 15 | 15, 36, 39 | 24, 45, 51 | 21, 72, 75 |
| 30, 40, 50 | 50, 120, 130 | 80, 150, 170 | 70, 240, 250 |
| $3x, 4x, 5x$ | $5x, 12x, 13x$ | $8x, 15x, 17x$ | $7x, 24x, 25x$ |

The most common Pythagorean triples are in bold. The other triples are the result of multiplying each integer in a bold face triple by the same factor.

**STANDARDIZED TESTS**
You may find it helpful to memorize the basic Pythagorean triples, shown in **bold**, for standardized tests.

## EXAMPLE 4 Find the length of a hypotenuse using two methods

**Find the length of the hypotenuse of the right triangle.**




**Solution**

**Method 1:** Use a Pythagorean triple.

A common Pythagorean triple is 5, 12, 13. Notice that if you multiply the lengths of the legs of the Pythagorean triple by 2, you get the lengths of the legs of this triangle: $5 \cdot 2 = 10$ and $12 \cdot 2 = 24$. So, the length of the hypotenuse is $13 \cdot 2 = 26$.

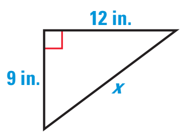
**Method 2:** Use the Pythagorean Theorem.

| | |
|---|---|
| $x^2 = 10^2 + 24^2$ | **Pythagorean Theorem** |
| $x^2 = 100 + 576$ | **Multiply.** |
| $x^2 = 676$ | **Add.** |
| $x = 26$ | **Find the positive square root.** |

## ✓ GUIDED PRACTICE for Example 4

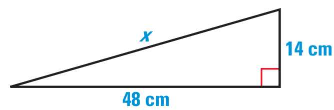
**Find the unknown side length of the right triangle using the Pythagorean Theorem. Then use a Pythagorean triple.**

**7.**




15 in.

**8.**




50 cm

# 7.1 EXERCISES

**HOMEWORK KEY**

◯ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 9, 11, and 33

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 17, 27, 33, and 36

◆ = **MULTIPLE REPRESENTATIONS** Ex. 35

## SKILL PRACTICE

A

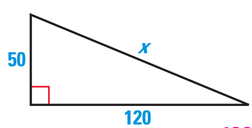
1. **VOCABULARY** Copy and complete: A set of three positive integers $a$, $b$, and $c$ that satisfy the equation $c^2 = a^2 + b^2$ is called a _?_. **Pythagorean triple**

2. ★ **WRITING** *Describe* the information you need to have in order to use the Pythagorean Theorem to find the length of a side of a triangle. **A right triangle, the measure of a leg of the triangle, and the measure of either the hypotenuse or the other leg.**

**EXAMPLE 1** on p. 433 for Exs. 3–7

**xy ALGEBRA Find the length of the hypotenuse of the right triangle.**

**3.**

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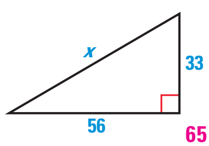
**4.**

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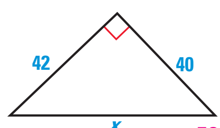
**5.**

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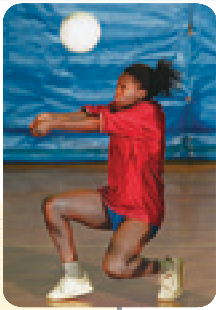
# 7.2 Use the Converse of the Pythagorean Theorem



**Before** You used the Pythagorean Theorem to find missing side lengths.
**Now** You will use its converse to determine if a triangle is a right triangle.
**Why?** So you can determine if a volleyball net is set up correctly, as in Ex. 38.

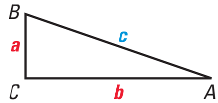
**Key Vocabulary**
- **acute triangle,** *p. 217*
- **obtuse triangle,** *p. 217*

The converse of the Pythagorean Theorem is also true. You can use it to verify that a triangle with given side lengths is a right triangle.

**THEOREM** *For Your Notebook*

**THEOREM 7.2 Converse of the Pythagorean Theorem**

If the square of the length of the longest side of a triangle is equal to the sum of the squares of the lengths of the other two sides, then the triangle is a right triangle.



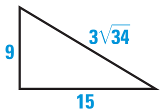

If $c^2 = a^2 + b^2$, then $\triangle ABC$ is a right triangle.

*Proof:* Ex. 42, p. 446

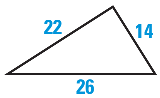
## EXAMPLE 1 Verify right triangles

**Tell whether the given triangle is a right triangle.**

**a.**




**b.**




Let $c$ represent the length of the longest side of the triangle. Check to see whether the side lengths satisfy the equation $c^2 = a^2 + b^2$.

**a.** $(3\sqrt{34})^2 \stackrel{?}{=} 9^2 + 15^2$

$9 \cdot 34 \stackrel{?}{=} 81 + 225$

$306 = 306$ ✓

The triangle is a right triangle.

**b.** $26^2 \stackrel{?}{=} 22^2 + 14^2$

$676 \stackrel{?}{=} 484 + 196$

$676 \neq 680$

The triangle is not a right triangle.

**REVIEW ALGEBRA**
Use a square root table or a calculator to find the decimal representation. So, $3\sqrt{34} \approx 17.493$ is the length of the longest side in part (a).

✓ **GUIDED PRACTICE** for Example 1

**Tell whether a triangle with the given side lengths is a right triangle.**

**1.** 4, $4\sqrt{3}$, 8 right triangle

**2.** 10, 11, and 14 not a right triangle

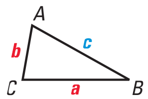
**3.** 5, 6, and $\sqrt{61}$ right triangle

**CLASSIFYING TRIANGLES** The Converse of the Pythagorean Theorem is used to verify that a given triangle is a right triangle. The theorems below are used to verify that a given triangle is acute or obtuse.

## THEOREMS *For Your Notebook*

### THEOREM 7.3

If the square of the length of the longest side of a triangle is less than the sum of the squares of the lengths of the other two sides, then the triangle *ABC* is an acute triangle.

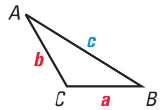



If $c^2 < a^2 + b^2$, then the triangle *ABC* is acute.

*Proof:* Ex. 40, p. 446

### THEOREM 7.4

If the square of the length of the longest side of a triangle is greater than the sum of the squares of the lengths of the other two sides, then the triangle *ABC* is an obtuse triangle.




If $c^2 > a^2 + b^2$, then triangle *ABC* is obtuse.

*Proof:* Ex. 41, p. 446

## EXAMPLE 2 Classify triangles

**Can segments with lengths of 4.3 feet, 5.2 feet, and 6.1 feet form a triangle? If so, would the triangle be *acute, right,* or *obtuse*?**

**Solution**

**APPLY THEOREMS**
The Triangle Inequality Theorem on page 330 states that the sum of the lengths of any two sides of a triangle is greater than the length of the third side.

***STEP 1*** **Use** the Triangle Inequality Theorem to check that the segments can make a triangle.

$4.3 + 5.2 = 9.5$ $\quad 4.3 + 6.1 = 10.4$ $\quad 5.2 + 6.1 = 11.3$

$9.5 > 6.1$ $\quad 10.4 > 5.2$ $\quad 11.3 > 4.3$

▶ The side lengths 4.3 feet, 5.2 feet, and 6.1 feet can form a triangle.

***STEP 2*** **Classify** the triangle by comparing the square of the length of the longest side with the sum of squares of the lengths of the shorter sides.

$c^2 \ \underline{?} \ a^2 + b^2$ **Compare $c^2$ with $a^2 + b^2$.**

$6.1^2 \ \underline{?} \ 4.3^2 + 5.2^2$ **Substitute.**

$37.21 \ \underline{?} \ 18.49 + 27.04$ **Simplify.**

$37.21 < 45.53$ **$c^2$ is less than $a^2 + b^2$.**

▶ The side lengths 4.3 feet, 5.2 feet, and 6.1 feet form an acute triangle.

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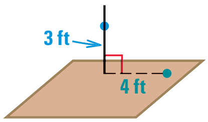
## EXAMPLE 3 Use the Converse of the Pythagorean Theorem

**CATAMARAN** You are part of a crew that is installing the mast on a catamaran. When the mast is fastened properly, it is perpendicular to the trampoline deck. How can you check that the mast is perpendicular using a tape measure?

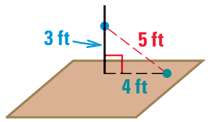
### Solution

To show a line is perpendicular to a plane you must show that the line is perpendicular to two lines in the plane.

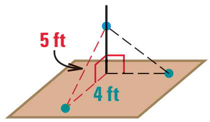
Think of the mast as a line and the deck as a plane. Use a 3-4-5 right triangle and the Converse of the Pythagorean Theorem to show that the mast is perpendicular to different lines on the deck.




First place a mark 3 feet up the mast and a mark on the deck 4 feet from the mast.




Use the tape measure to check that the distance between the two marks is 5 feet. The mast makes a right angle with the line on the deck.




Finally, repeat the procedure to show that the mast is perpendicular to another line on the deck.

## ✓ GUIDED PRACTICE for Example 2 and 3

**4.** Show that segments with lengths 3, 4, and 6 can form a triangle and classify the triangle as *acute, right,* or *obtuse*. **$3 + 4 > 6, 4 + 6 > 3, 6 + 3 > 4$, obtuse**

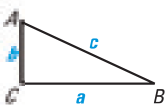
**5.** **WHAT IF?** In Example 3, could you use triangles with side lengths 2, 3, and 4 to verify that you have perpendicular lines? *Explain*. **No; in order to verify that you have perpendicular lines, the triangle would have to be a right triangle, and a 2-3-4 triangle is not a right triangle.**

**CLASSIFYING TRIANGLES** You can use the theorems from this lesson to classify a triangle as acute, right, or obtuse based on its side lengths.

## CONCEPT SUMMARY *For Your Notebook*

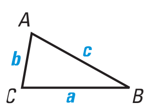
### Methods for Classifying a Triangle by Angles Using its Side Lengths

**Theorem 7.2**



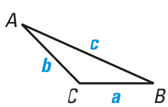

If $c^2 = a^2 + b^2$, then $m\angle C = 90°$ and $\triangle ABC$ is a right triangle.

**Theorem 7.3**




If $c^2 < a^2 + b^2$, then $m\angle C < 90°$ and $\triangle ABC$ is an acute triangle.

**Theorem 7.4**

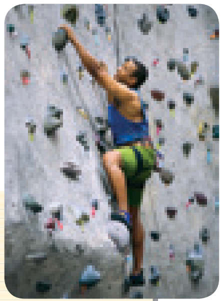



If $c^2 > a^2 + b^2$, then $m\angle C > 90°$ and $\triangle ABC$ is an obtuse triangle.



Differentiated Instruction

# 7.3 Use Similar Right Triangles



| | |
|---|---|
| **Before** | You identified the altitudes of a triangle. |
| **Now** | You will use properties of the altitude of a right triangle. |
| **Why?** | So you can determine the height of a wall, as in Example 4. |

**Key Vocabulary**

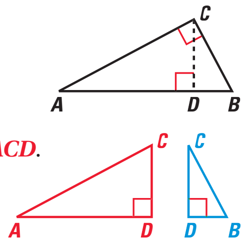
- **altitude of a triangle,** *p. 320*
- **geometric mean,** *p. 359*
- **similar polygons,** *p. 372*

When the altitude is drawn to the hypotenuse of a right triangle, the two smaller triangles are similar to the original triangle and to each other.

**THEOREM** *For Your Notebook*

**THEOREM 7.5**

If the altitude is drawn to the hypotenuse of a right triangle, then the two triangles formed are similar to the original triangle and to each other.




$\triangle CBD \sim \triangle ABC$, $\triangle ACD \sim \triangle ABC$, and $\triangle CBD \sim \triangle ACD$.

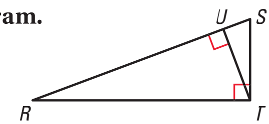
*Proof:* below; Ex. 35, p. 456

**Plan for Proof of Theorem 7.5** First prove that $\triangle CBD \sim \triangle ABC$. Each triangle has a right angle and each triangle includes $\angle B$. The triangles are similar by the AA Similarity Postulate. Use similar reasoning to show that $\triangle ACD \sim \triangle ABC$.

To show $\angle CBD \sim \triangle ACD$, begin by showing $\angle ACD \cong \angle B$ because they are both complementary to $\angle DCB$. Each triangle also has a right angle, so you can use the AA Similarity Postulate.

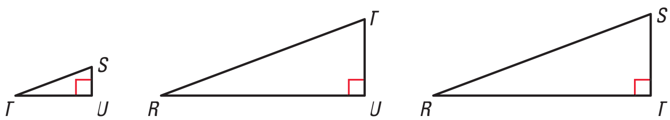
## EXAMPLE 1 Identify similar triangles

**Identify the similar triangles in the diagram.**




**Solution**

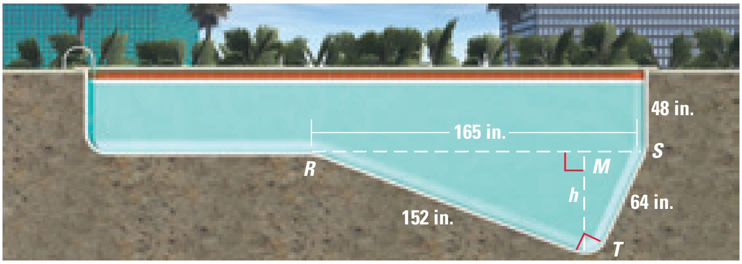
Sketch the three similar right triangles so that the corresponding angles and sides have the same orientation.




▸ $\triangle TSU \sim \triangle RTU \sim \triangle RST$

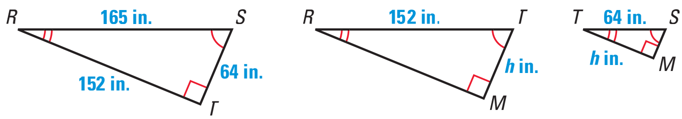
## EXAMPLE 2 Find the length of the altitude to the hypotenuse

**SWIMMING POOL** The diagram below shows a cross-section of a swimming pool. What is the maximum depth of the pool?




### Solution

***STEP 1*** **Identify** the similar triangles and sketch them.




$\triangle RST \sim \triangle RTM \sim \triangle TSM$

***STEP 2*** **Find** the value of $h$. Use the fact that $\triangle RST \sim \triangle RTM$ to write a proportion.

| | |
|---|---|
| $\frac{TM}{ST} = \frac{TR}{SR}$ | **Corresponding side lengths of similar triangles are in proportion.** |
| $\frac{h}{64} = \frac{152}{165}$ | **Substitute.** |
| $165h = 64(152)$ | **Cross Products Property** |
| $h \approx 59$ | **Solve for *h*.** |

***STEP 3*** **Read** the diagram above. You can see that the maximum depth of the pool is $h + 48$, which is about $59 + 48 = 107$ inches.

▶ The maximum depth of the pool is about 107 inches.

**Animated Geometry** at classzone.com

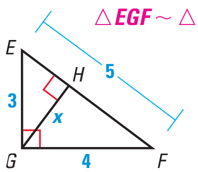
**AVOID ERRORS**

Notice that if you tried to write a proportion using $\triangle RTM$ and $\triangle TSM$, there would be two unknowns, so you would not be able to solve for $h$.

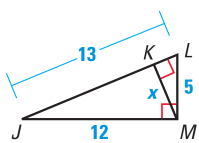
## ✓ GUIDED PRACTICE for Examples 1 and 2

**Identify the similar triangles. Then find the value of *x*.**

**1.** $\triangle EGF \sim \triangle GHF \sim \triangle EHG; \frac{12}{5}$

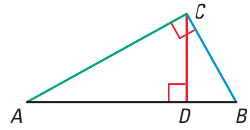



**2.**

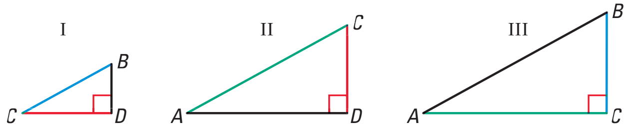



$\triangle LMJ \sim \triangle MKJ \sim \triangle LKM; \frac{60}{13}$

**GEOMETRIC MEANS** In Lesson 6.1, you learned that the *geometric mean* of two numbers $a$ and $b$ is the positive number $x$ such that $\frac{a}{x} = \frac{x}{b}$. Consider right $\triangle ABC$. From Theorem 7.5, you know that altitude $\overline{CD}$ forms two smaller triangles so that $\triangle CBD \sim \triangle ACD \sim \triangle ABC$.




**READ SYMBOLS**
Remember that an altitude is defined as a segment. So, $\overline{CD}$ refers to an altitude in $\triangle ABC$ and $CD$ refers to its length.




Notice that $\overline{CD}$ is the longer leg of $\triangle CBD$ and the shorter leg of $\triangle ACD$. When you write a proportion comparing the leg lengths of $\triangle CBD$ and $\triangle ACD$, you can see that $CD$ is the geometric mean of $BD$ and $AD$. As you see below, $CB$ and $AC$ are also geometric means of segment lengths in the diagram.

**Proportions Involving Geometric Means in Right $\triangle ABC$**

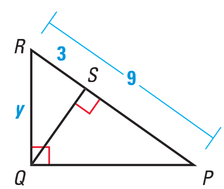
length of shorter leg of I / length of shorter leg of II → $\frac{BD}{CD} = \frac{CD}{AD}$ ← length of longer leg of I / length of longer leg of II

length of hypotenuse of III / length of hypotenuse of I → $\frac{AB}{CB} = \frac{CB}{DB}$ ← length of shorter leg of III / length of shorter leg of I

length of hypotenuse of III / length of hypotenuse of II → $\frac{AB}{AC} = \frac{AC}{AD}$ ← length of longer leg of III / length of longer leg of II

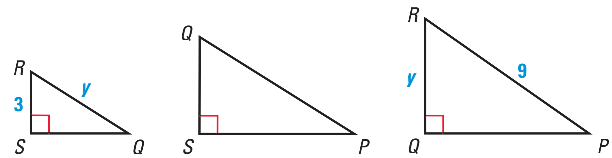
## EXAMPLE 3 Use a geometric mean

**Find the value of $y$. Write your answer in simplest radical form.**




**Solution**

**STEP 1** **Draw** the three similar triangles.




**REVIEW SIMILARITY**
Notice that $\triangle RQS$ and $\triangle RPQ$ both contain the side with length $y$, so these are the similar pair of triangles to use to solve for $y$.

**STEP 2** **Write** a proportion.

$$\frac{\text{length of hyp. of } \triangle RPQ}{\text{length of hyp. of } \triangle RQS} = \frac{\text{length of shorter leg of } \triangle RPQ}{\text{length of shorter leg of } \triangle RQS}$$

$\frac{9}{y} = \frac{y}{3}$ **Substitute.**

$27 = y^2$ **Cross Products Property**

$\sqrt{27} = y$ **Take the positive square root of each side.**

$3\sqrt{3} = y$ **Simplify.**

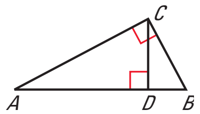


Differentiated Instruction

**WRITE PROOFS**
In Exercise 32 on page 455, you will use the geometric mean theorems to prove the Pythagorean Theorem.

## THEOREMS *For Your Notebook*

### THEOREM 7.6 Geometric Mean (Altitude) Theorem



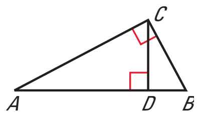

$$\frac{BD}{CD} = \frac{CD}{AD}$$

In a right triangle, the altitude from the right angle to the hypotenuse divides the hypotenuse into two segments.

The length of the altitude is the geometric mean of the lengths of the two segments.

*Proof:* Ex. 36, p. 456

### THEOREM 7.7 Geometric Mean (Leg) Theorem




$$\frac{AB}{CB} = \frac{CB}{DB} \text{ and } \frac{AB}{AC} = \frac{AC}{AD}$$

In a right triangle, the altitude from the right angle to the hypotenuse divides the hypotenuse into two segments.

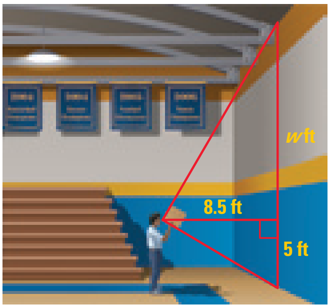
The length of each leg of the right triangle is the geometric mean of the lengths of the hypotenuse and the segment of the hypotenuse that is adjacent to the leg.

*Proof:* Ex. 37, p. 456

## EXAMPLE 4 Find a height using indirect measurement

**ROCK CLIMBING WALL** To find the cost of installing a rock wall in your school gymnasium, you need to find the height of the gym wall.

You use a cardboard square to line up the top and bottom of the gym wall. Your friend measures the vertical distance from the ground to your eye and the distance from you to the gym wall. Approximate the height of the gym wall.




**Solution**

By Theorem 7.6, you know that 8.5 is the geometric mean of $w$ and 5.

$\frac{w}{8.5} = \frac{8.5}{5}$ **Write a proportion.**

$w \approx 14.5$ **Solve for *w*.**

▶ So, the height of the wall is $5 + w \approx 5 + 14.5 = 19.5$ feet.

✓ **GUIDED PRACTICE** for Examples 3 and 4

**3.** In Example 3, which theorem did you use to solve for $y$? *Explain.*

**3. Theorem 7.7; set the ratios hypotenuse of the large triangle to the shorter leg and the hypotenuse of the small triangle to the shorter leg equal to each other.**

**4.** Mary is 5.5 feet tall. How far from the wall in Example 4 would she have to stand in order to measure its height? **about 8.93 ft**

# 7.4 Special Right Triangles

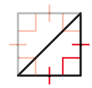
**Before** You found side lengths using the Pythagorean Theorem.

**Now** You will use the relationships among the sides in special right triangles.

**Why?** So you can find the height of a drawbridge, as in Ex. 28.

**Key Vocabulary**
- **isosceles triangle,** *p. 217*

A 45°-45°-90° triangle is an *isosceles right triangle* that can be formed by cutting a square in half as shown.



**USE RATIOS**
The extended ratio of the side lengths of a 45°-45°-90° triangle is $1:1:\sqrt{2}$.

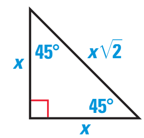
**THEOREM** *For Your Notebook*

**THEOREM 7.8 45°-45°-90° Triangle Theorem**

In a 45°-45°-90° triangle, the hypotenuse is $\sqrt{2}$ times as long as each leg.

hypotenuse = leg • $\sqrt{2}$

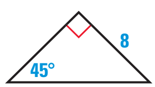
*Proof:* Ex. 30, p. 463




## EXAMPLE 1 Find hypotenuse length in a 45°-45°-90° triangle

**Find the length of the hypotenuse.**

**a.**




**b.**


**Solution**

**a.** By the Triangle Sum Theorem, the measure of the third angle must be 45°. Then the triangle is a 45°-45°-90° triangle, so by Theorem 7.8, the hypotenuse is $\sqrt{2}$ times as long as each leg.

| | |
|---|---|
| hypotenuse = leg • $\sqrt{2}$ | **45°-45°-90° Triangle Theorem** |
| $= 8\sqrt{2}$ | **Substitute.** |

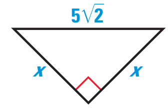
**REVIEW ALGEBRA**
Remember the following properties of radicals:
$\sqrt{a} \cdot \sqrt{b} = \sqrt{a \cdot b}$
$\sqrt{a \cdot a} = a$
For a review of radical expressions, see p. 874.

**b.** By the Base Angles Theorem and the Corollary to the Triangle Sum Theorem, the triangle is a 45°-45°-90° triangle.

| | |
|---|---|
| hypotenuse = leg • $\sqrt{2}$ | **45°-45°-90° Triangle Theorem** |
| $= 3\sqrt{2} \cdot \sqrt{2}$ | **Substitute.** |
| $= 3 \cdot 2$ | **Product of square roots** |
| $= 6$ | **Simplify.** |

## EXAMPLE 2 Find leg lengths in a 45°-45°-90° triangle

**Find the lengths of the legs in the triangle.**



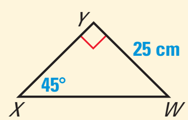

### Solution

By the Base Angles Theorem and the Corollary to the Triangle Sum Theorem, the triangle is a 45°-45°-90° triangle.

| | |
|---|---|
| $\text{hypotenuse} = \text{leg} \cdot \sqrt{2}$ | 45°-45°-90° Triangle Theorem |
| $5\sqrt{2} = x \cdot \sqrt{2}$ | Substitute. |
| $\frac{5\sqrt{2}}{\sqrt{2}} = \frac{x\sqrt{2}}{\sqrt{2}}$ | Divide each side by $\sqrt{2}$. |
| $5 = x$ | Simplify. |

## ★ EXAMPLE 3 Standardized Test Practice

**Triangle *WXY* is a right triangle. Find the length of $\overline{WX}$.**




Ⓐ 50 cm  Ⓑ $25\sqrt{2}$ cm

Ⓒ 25 cm  Ⓓ $\frac{25\sqrt{2}}{2}$ cm

**ELIMINATE CHOICES**
You can eliminate choices C and D because the hypotenuse has to be longer than the leg.

### Solution

By the Corollary to the Triangle Sum Theorem, the triangle is a 45°-45°-90° triangle.

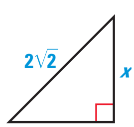
| | |
|---|---|
| $\text{hypotenuse} = \text{leg} \cdot \sqrt{2}$ | 45°-45°-90° Triangle Theorem |
| $WX = 25\sqrt{2}$ | Substitute. |

▶ The correct answer is B. Ⓐ Ⓑ Ⓒ Ⓓ

## ✓ GUIDED PRACTICE for Examples 1, 2, and 3

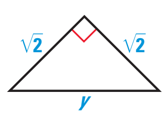
**Find the value of the variable.**

**1.**



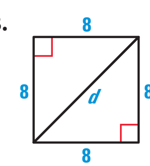

2

**2.**




2

**3.**




$8\sqrt{2}$

**4.** Find the leg length of a 45°-45°-90° triangle with a hypotenuse length of 6. $3\sqrt{2}$



Differentiated Instruction

A 30°-60°-90° triangle can be formed by dividing an equilateral triangle in half.

**THEOREM** *For Your Notebook*

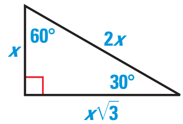
**THEOREM 7.9 30°-60°-90° Triangle Theorem**

In a 30°-60°-90° triangle, the hypotenuse is twice as long as the shorter leg, and the longer leg is $\sqrt{3}$ times as long as the shorter leg.

hypotenuse = 2 • shorter leg

longer leg = shorter leg • $\sqrt{3}$

*Proof:* Ex. 32, p. 463




**USE RATIOS**
The extended ratio of the side lengths of a 30°-60°-90° triangle is $1 : \sqrt{3} : 2$.

## EXAMPLE 4 Find the height of an equilateral triangle

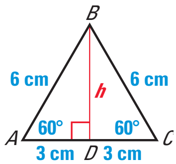
**LOGO** The logo on the recycling bin at the right resembles an equilateral triangle with side lengths of 6 centimeters. What is the approximate height of the logo?

**Solution**

Draw the equilateral triangle described. Its altitude forms the longer leg of two 30°-60°-90° triangles. The length $h$ of the altitude is approximately the height of the logo.

longer leg = shorter leg • $\sqrt{3}$

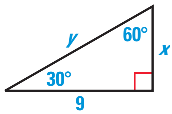
$h = 3 \cdot \sqrt{3} \approx 5.2$ cm




**REVIEW MEDIAN**
Remember that in an equilateral triangle, the altitude to a side is also the median to that side. So, altitude $\overline{BD}$ bisects $\overline{AC}$.

## EXAMPLE 5 Find lengths in a 30°-60°-90° triangle

**xy Find the values of $x$ and $y$. Write your answer in simplest radical form.**




**STEP 1** **Find** the value of $x$.

| | |
|---|---|
| longer leg = shorter leg • $\sqrt{3}$ | **30°-60°-90° Triangle Theorem** |
| $9 = x\sqrt{3}$ | **Substitute.** |
| $\frac{9}{\sqrt{3}} = x$ | **Divide each side by $\sqrt{3}$.** |
| $\frac{9}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = x$ | **Multiply numerator and denominator by $\sqrt{3}$.** |
| $\frac{9\sqrt{3}}{3} = x$ | **Multiply fractions.** |
| $3\sqrt{3} = x$ | **Simplify.** |

**STEP 2** **Find** the value of $y$.

| | |
|---|---|
| hypotenuse = 2 • shorter leg | **30°-60°-90° Triangle Theorem** |
| $y = 2 \cdot 3\sqrt{3} = 6\sqrt{3}$ | **Substitute and simplify.** |



Differentiated Instruction

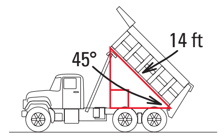
## EXAMPLE 6 Find a height

**DUMP TRUCK** The body of a dump truck is raised to empty a load of sand. How high is the 14 foot body from the frame when it is tipped upward at the given angle?

**a.** 45° angle **b.** 60° angle

### Solution

**a.** When the body is raised 45° above the frame, the height $h$ is the length of a leg of a 45°-45°-90° triangle. The length of the hypotenuse is 14 feet.

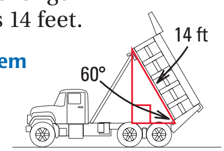



| | |
|---|---|
| $14 = h \cdot \sqrt{2}$ | **45°-45°-90° Triangle Theorem** |
| $\frac{14}{\sqrt{2}} = h$ | **Divide each side by $\sqrt{2}$.** |
| $9.9 \approx h$ | **Use a calculator to approximate.** |

**REWRITE MEASURES**
To write 9.9 ft in feet and inches, multiply the decimal part by 12.
$12 \cdot 0.9 = 10.8$
So, 9.9 ft is about 9 feet 11 inches.

▶ When the angle of elevation is 45°, the body is about 9 feet 11 inches above the frame.

**b.** When the body is raised 60°, the height $h$ is the length of the longer leg of a 30°-60°-90° triangle. The length of the hypotenuse is 14 feet.



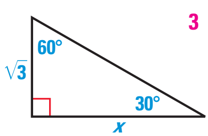

| | |
|---|---|
| hypotenuse $= 2 \cdot$ shorter leg | **30°-60°-90° Triangle Theorem** |
| $14 = 2 \cdot s$ | **Substitute.** |
| $7 = s$ | **Divide each side by 2.** |
| longer leg $=$ shorter leg $\cdot \sqrt{3}$ | **30°-60°-90° Triangle Theorem** |
| $h = 7\sqrt{3}$ | **Substitute.** |
| $h \approx 12.1$ | **Use a calculator to approximate.** |

▶ When the angle of elevation is 60°, the body is about 12 feet 1 inch above the frame.

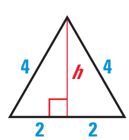
Animated Geometry at classzone.com

## ✓ GUIDED PRACTICE for Examples 4, 5, and 6

**Find the value of the variable.**

**5.** 

**3**

**6.** 

**$2\sqrt{3}$**

**7.** **WHAT IF?** In Example 6, what is the height of the body of the dump truck if it is raised 30° above the frame? **7 ft**

**8.** In a 30°-60°-90° triangle, *describe* the location of the shorter side. *Describe* the location of the longer side? ***Sample answer:* The shorter side is adjacent to the 60° angle; the longer side is adjacent to the 30° angle.**

# 7.5 Apply the Tangent Ratio

**Before** You used congruent or similar triangles for indirect measurement.

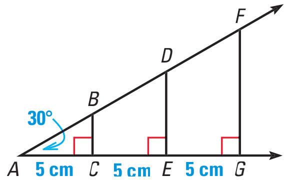
**Now** You will use the tangent ratio for indirect measurement.

**Why?** So you can find the height of a roller coaster, as in Ex. 32.

**Key Vocabulary**
- **trigonometric ratio**
- **tangent**

## ACTIVITY RIGHT TRIANGLE RATIO

**Materials:** metric ruler, protractor, calculator




*STEP 1* **Draw** a 30° angle and mark a point every 5 centimeters on a side as shown. Draw perpendicular segments through the 3 points.

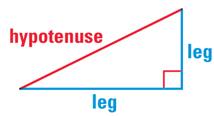
*STEP 2* **Measure** the legs of each right triangle. Copy and complete the table.

| Triangle | Adjacent leg | Opposite leg | $\frac{\text{Opposite leg}}{\text{Adjacent leg}}$ |
|---|---|---|---|
| $\triangle ABC$ | 5 cm | ? | ? |
| $\triangle ADE$ | 10 cm | ? | ? |
| $\triangle AFG$ | 15 cm | ? | ? |

*STEP 3* **Explain** why the proportions $\frac{BC}{DE} = \frac{AC}{AE}$ and $\frac{BC}{AC} = \frac{DE}{AE}$ are true.

*STEP 4* **Make** a conjecture about the ratio of the lengths of the legs in a right triangle. Test your conjecture by using different acute angle measures.

A **trigonometric ratio** is a ratio of the lengths of two sides in a right triangle. You will use trigonometric ratios to find the measure of a side or an acute angle in a right triangle.




The ratio of the lengths of the legs in a right triangle is constant for a given angle measure. This ratio is called the **tangent** of the angle.

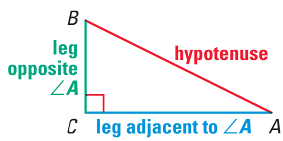
**ABBREVIATE**
Remember these abbreviations:
tangent → tan
opposite → opp.
adjacent → adj.

## KEY CONCEPT *For Your Notebook*

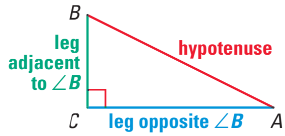
### Tangent Ratio

Let $\triangle ABC$ be a right triangle with acute $\angle A$. The tangent of $\angle A$ (written as $\tan A$) is defined as follows:

$$\tan A = \frac{\text{length of leg opposite } \angle A}{\text{length of leg adjacent to } \angle A} = \frac{BC}{AC}$$

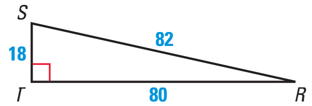


**COMPLEMENTARY ANGLES** In the right triangle, $\angle A$ and $\angle B$ are complementary so you can use the same diagram to find the tangent of $\angle A$ and the tangent of $\angle B$. Notice that the leg adjacent to $\angle A$ is the leg *opposite* $\angle B$ and the leg opposite $\angle A$ is the leg *adjacent* to $\angle B$.




## EXAMPLE 1 Find tangent ratios

**Find tan $S$ and tan $R$. Write each answer as a fraction and as a decimal rounded to four places.**




**Solution**

$$\tan S = \frac{\text{opp. } \angle S}{\text{adj. to } \angle S} = \frac{RT}{ST} = \frac{80}{18} = \frac{40}{9} \approx 4.4444$$

$$\tan R = \frac{\text{opp. } \angle R}{\text{adj. to } \angle R} = \frac{ST}{RT} = \frac{18}{80} = \frac{9}{40} = 0.2250$$

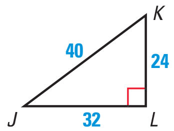
**APPROXIMATE**

Unless told otherwise, you should round the values of trigonometric ratios to the ten-thousandths' place and round lengths to the tenths' place.

## ✓ GUIDED PRACTICE for Example 1

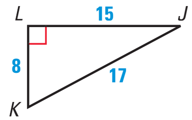
**Find tan $J$ and tan $K$. Round to four decimal places.**

**1.**




0.7500, 1.3333

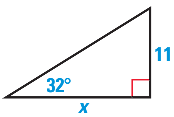
**2.**




0.5333, 1.8750

## EXAMPLE 2 Find a leg length

**ALGEBRA Find the value of $x$.**




**Solution**

Use the tangent of an acute angle to find a leg length.

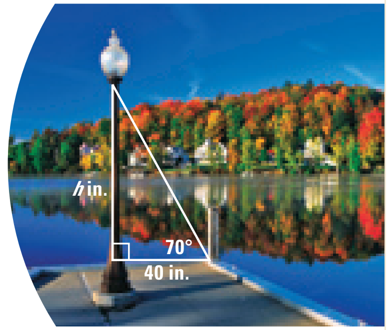
| | |
|---|---|
| $\tan 32° = \frac{\text{opp.}}{\text{adj.}}$ | Write ratio for tangent of 32°. |
| $\tan 32° = \frac{11}{x}$ | Substitute. |
| $x \cdot \tan 32° = 11$ | Multiply each side by $x$. |
| $x = \frac{11}{\tan 32°}$ | Divide each side by tan 32°. |
| $x \approx \frac{11}{0.6249}$ | Use a calculator to find tan 32°. |
| $x \approx 17.6$ | Simplify. |

**ANOTHER WAY**

You can also use the Table of Trigonometric Ratios on p. 925 to find the decimal values of trigonometric ratios.

## EXAMPLE 3 Estimate height using tangent

**LAMPPOST** **Find the height $h$ of the lamppost to the nearest inch.**




$\tan 70° = \frac{\text{opp.}}{\text{adj.}}$ Write ratio for tangent of 70°.

$\tan 70° = \frac{h}{40}$ Substitute.

$40 \cdot \tan 70° = h$ Multiply each side by 40.

$109.9 \approx h$ Use a calculator to simplify.

▶ The lamppost is about 110 inches tall.

**SPECIAL RIGHT TRIANGLES** You can find the tangent of an acute angle measuring 30°, 45°, or 60° by applying what you know about special right triangles.

## EXAMPLE 4 Use a special right triangle to find a tangent

**Use a special right triangle to find the tangent of a 60° angle.**

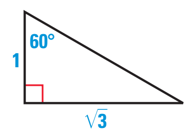
**SIMILAR TRIANGLES**
The tangents of all 60° angles are the same constant ratio. Any right triangle with a 60° angle can be used to determine this value.

**STEP 1** **Because** all 30°-60°-90° triangles are similar, you can simplify your calculations by choosing 1 as the length of the shorter leg. Use the 30°-60°-90° Triangle Theorem to find the length of the longer leg.

longer leg = shorter leg $\cdot \sqrt{3}$ 30°-60°-90° Triangle Theorem

$x = 1 \cdot \sqrt{3}$ Substitute.

$x = \sqrt{3}$ Simplify.




**STEP 2** **Find** tan 60°.

$\tan 60° = \frac{\text{opp.}}{\text{adj.}}$ Write ratio for tangent of 60°.

$\tan 60° = \frac{\sqrt{3}}{1}$ Substitute.

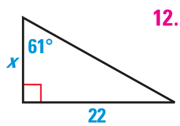
$\tan 60° = \sqrt{3}$ Simplify.

▶ The tangent of any 60° angle is $\sqrt{3} \approx 1.7321$.

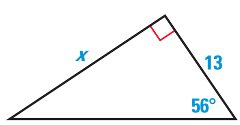
## ✓ GUIDED PRACTICE for Examples 2, 3, and 4

**Find the value of $x$. Round to the nearest tenth.**

**3.** 12.2



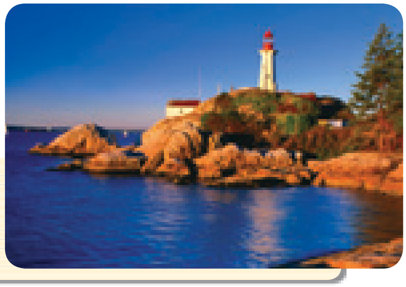

**4.** 19.3




**5.** **WHAT IF?** In Example 4, suppose the side length of the shorter leg is 5 instead of 1. Show that the tangent of 60° is still equal to $\sqrt{3}$.

shorter leg = 5, longer leg = $5\sqrt{3}$, tan 60 = $\frac{5\sqrt{3}}{5} = \sqrt{3}$

# 7.6 Apply the Sine and Cosine Ratios



**Before** You used the tangent ratio.
**Now** You will use the sine and cosine ratios.
**Why** So you can find distances, as in Ex. 39.

**Key Vocabulary**
- **sine**
- **cosine**
- **angle of elevation**
- **angle of depression**

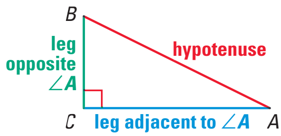
The **sine** and **cosine** ratios are trigonometric ratios for acute angles that involve the lengths of a leg and the hypotenuse of a right triangle.

**ABBREVIATE**
Remember these abbreviations:
sine → sin
cosine → cos
hypotenuse → hyp

## KEY CONCEPT *For Your Notebook*

### Sine and Cosine Ratios

Let $\triangle ABC$ be a right triangle with acute $\angle A$. The sine of $\angle A$ and cosine of $\angle A$ (written $\sin A$ and $\cos A$) are defined as follows:

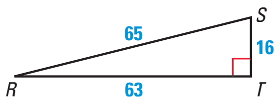



$$\sin A = \frac{\text{length of leg opposite } \angle A}{\text{length of hypotenuse}} = \frac{BC}{AB}$$

$$\cos A = \frac{\text{length of leg adjacent to } \angle A}{\text{length of hypotenuse}} = \frac{AC}{AB}$$

## EXAMPLE 1 Find sine ratios

**Find $\sin S$ and $\sin R$. Write each answer as a fraction and as a decimal rounded to four places.**




**Solution**

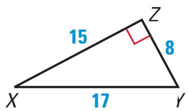
$$\sin S = \frac{\text{opp. } \angle S}{\text{hyp.}} = \frac{RT}{SR} = \frac{63}{65} \approx 0.9692$$

$$\sin R = \frac{\text{opp. } \angle R}{\text{hyp.}} = \frac{ST}{SR} = \frac{16}{65} \approx 0.2462$$

## ✓ GUIDED PRACTICE for Example 1

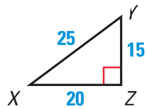
**Find $\sin X$ and $\sin Y$. Write each answer as a fraction and as a decimal. Round to four decimal places, if necessary.**

**1.**




$\frac{8}{17}$ or 0.4706, $\frac{15}{17}$ or 0.8824

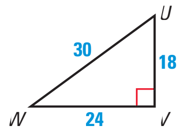
**2.**




$\frac{3}{5}$ or 0.6, $\frac{4}{5}$ or 0.8

## EXAMPLE 2 Find cosine ratios

**Find cos *U* and cos *W*. Write each answer as a fraction and as a decimal.**



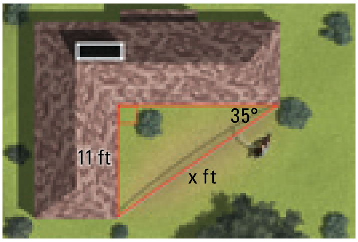

**Solution**

$$\cos U = \frac{\text{adj. to } \angle U}{\text{hyp.}} = \frac{UV}{UW} = \frac{18}{30} = \frac{3}{5} = 0.6000$$

$$\cos W = \frac{\text{adj. to } \angle W}{\text{hyp.}} = \frac{WV}{UW} = \frac{24}{30} = \frac{4}{5} = 0.8000$$

## EXAMPLE 3 Use a trigonometric ratio to find a hypotenuse

**DOG RUN** You want to string cable to make a dog run from two corners of a building, as shown in the diagram. Write and solve a proportion using a trigonometric ratio to approximate the length of cable you will need.




**Solution**

$\sin 35° = \frac{\text{opp.}}{\text{hyp.}}$ — Write ratio for sine of 35°.

$\sin 35° = \frac{11}{x}$ — Substitute.

$x \cdot \sin 35° = 11$ — Multiply each side by $x$.

$x = \frac{11}{\sin 35°}$ — Divide each side by sin 35°.

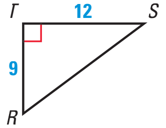
$x \approx \frac{11}{0.5736}$ — Use a calculator to find sin 35°.

$x \approx 19.2$ — Simplify.

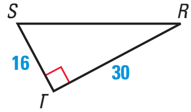
▶ You will need a little more than 19 feet of cable.

## ✓ GUIDED PRACTICE for Examples 2 and 3

**In Exercises 3 and 4, find cos *R* and cos *S*. Write each answer as a decimal. Round to four decimal places, if necessary.**

3. 

0.6, 0.8

4. 

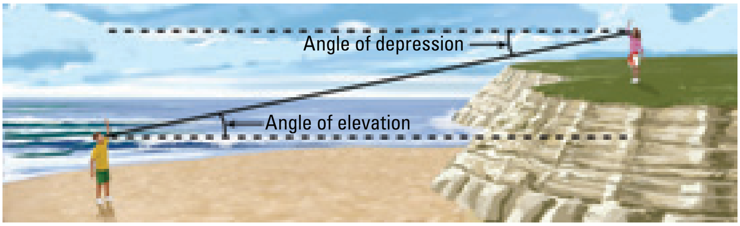
0.8824, 0.4706

5. In Example 3, use the cosine ratio to find the length of the other leg of the triangle formed. about 15.7 ft

**ANGLES** If you look up at an object, the angle your line of sight makes with a horizontal line is called the **angle of elevation**. If you look down at an object, the angle your line of sight makes with a horizontal line is called the **angle of depression**.

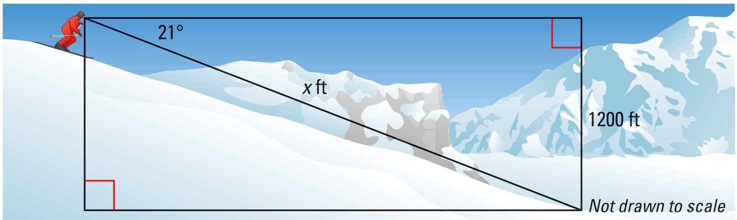
**APPLY THEOREMS**

Notice that the angle of elevation and the angle of depression are congruent by the Alternate Interior Angles Theorem on page 155.




## EXAMPLE 4 Find a hypotenuse using an angle of depression

**SKIING** You are skiing on a mountain with an altitude of 1200 meters. The angle of depression is 21°. About how far do you ski down the mountain?




### Solution

$\sin 21° = \frac{\text{opp.}}{\text{hyp.}}$ **Write ratio for sine of 21°.**

$\sin 21° = \frac{1200}{x}$ **Substitute.**

$x \cdot \sin 21° = 1200$ **Multiply each side by $x$.**

$x = \frac{1200}{\sin 21°}$ **Divide each side by sin 21°.**

$x \approx \frac{1200}{0.3584}$ **Use a calculator to find sin 21°.**

$x \approx 3348.2$ **Simplify.**

▶ You ski about 3348 meters down the mountain.

Animated Geometry at classzone.com

## ✓ GUIDED PRACTICE for Example 4

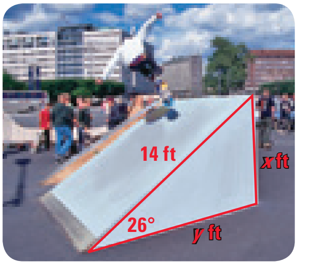
6. **WHAT IF?** Suppose the angle of depression in Example 4 is 28°. About how far would you ski? **about 2556 m**



Differentiated Instruction

## EXAMPLE 5 Find leg lengths using an angle of elevation

**SKATEBOARD RAMP** You want to build a skateboard ramp with a length of 14 feet and an angle of elevation of 26°. You need to find the height and length of the base of the ramp.




**ANOTHER WAY**
For alternative methods for solving the problem in Example 5, turn to page 481 for the **Problem Solving Workshop**.

**Solution**

**STEP 1** **Find** the height.

$\sin 26° = \frac{\text{opp.}}{\text{hyp.}}$ **Write ratio for sine of 26°.**

$\sin 26° = \frac{x}{14}$ **Substitute.**

$14 \cdot \sin 26° = x$ **Multiply each side by 14.**

$6.1 \approx x$ **Use a calculator to simplify.**

▶ The height is about 6.1 feet.

**STEP 2** **Find** the length of the base.

$\cos 26° = \frac{\text{adj.}}{\text{hyp.}}$ **Write ratio for cosine of 26°.**

$\cos 26° = \frac{y}{14}$ **Substitute.**

$14 \cdot \cos 26° = y$ **Multiply each side by 14.**

$12.6 \approx y$ **Use a calculator to simplify.**

▶ The length of the base is about 12.6 feet.

## EXAMPLE 6 Use a special right triangle to find a sine and cosine

**Use a special right triangle to find the sine and cosine of a 60° angle.**

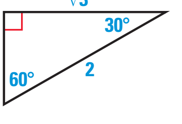
**DRAW DIAGRAMS**
As in Example 4 on page 468, to simplify calculations you can choose 1 as the length of the shorter leg.

**Solution**

Use the 30°-60°-90° Triangle Theorem to draw a right triangle with side lengths of 1, $\sqrt{3}$, and 2. Then set up sine and cosine ratios for the 60° angle.

$$\sin 60° = \frac{\text{opp.}}{\text{hyp.}} = \frac{\sqrt{3}}{2} \approx 0.8660$$

$$\cos 60° = \frac{\text{adj.}}{\text{hyp.}} = \frac{1}{2} = 0.5000$$




### ✓ GUIDED PRACTICE for Examples 5 and 6

7. **WHAT IF?** In Example 5, suppose the angle of elevation is 35°. What is the new height and base length of the ramp? **about 8 ft, about 11.5 ft**

8. Use a special right triangle to find the sine and cosine of a 30° angle. $\frac{1}{2}, \frac{\sqrt{3}}{2}$



Differentiated Instruction

# 7.7 Solve Right Triangles

| | |
|---|---|
| **Before** | You used tangent, sine, and cosine ratios. |
| **Now** | You will use inverse tangent, sine, and cosine ratios. |
| **Why?** | So you can build a saddlerack, as in Ex. 39. |

**Key Vocabulary**
- **solve a right triangle**
- **inverse tangent**
- **inverse sine**
- **inverse cosine**

To **solve a right triangle** means to find the measures of all of its sides and angles. You can solve a right triangle if you know either of the following:

- Two side lengths
- One side length and the measure of one acute angle

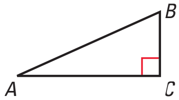
In Lessons 7.5 and 7.6, you learned how to use the side lengths of a right triangle to find trigonometric ratios for the acute angles of the triangle. Once you know the tangent, the sine, or the cosine of an acute angle, you can use a calculator to find the measure of the angle.

**READ VOCABULARY**
The expression "$\tan^{-1} x$" is read as "the inverse tangent of $x$."

**KEY CONCEPT** — *For Your Notebook*

### Inverse Trigonometric Ratios

Let $\angle A$ be an acute angle.



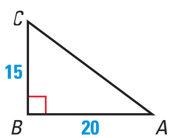

**Inverse Tangent** If $\tan A = x$, then $\tan^{-1} x = m\angle A$. $\quad \tan^{-1}\frac{BC}{AC} = m\angle A$

**Inverse Sine** If $\sin A = y$, then $\sin^{-1} y = m\angle A$. $\quad \sin^{-1}\frac{BC}{AB} = m\angle A$

**Inverse Cosine** If $\cos A = z$, then $\cos^{-1} z = m\angle A$. $\quad \cos^{-1}\frac{AC}{AB} = m\angle A$

## EXAMPLE 1 Use an inverse tangent to find an angle measure

**Use a calculator to approximate the measure of $\angle A$ to the nearest tenth of a degree.**




**Solution**

Because $\tan A = \frac{15}{20} = \frac{3}{4} = 0.75$, $\tan^{-1} 0.75 = m\angle A$. Use a calculator.

$\tan^{-1} 0.75 \approx 36.86989765\ldots$

▶ So, the measure of $\angle A$ is approximately 36.9°.

## EXAMPLE 2 Use an inverse sine and an inverse cosine

**ANOTHER WAY**

You can use the Table of Trigonometric Ratios on p. 925 to approximate $\sin^{-1} 0.87$ to the nearest degree. Find the number closest to 0.87 in the sine column and read the angle measure at the left.

**Let $\angle A$ and $\angle B$ be acute angles in a right triangle. Use a calculator to approximate the measures of $\angle A$ and $\angle B$ to the nearest tenth of a degree.**

**a.** $\sin A = 0.87$ **b.** $\cos B = 0.15$

### Solution

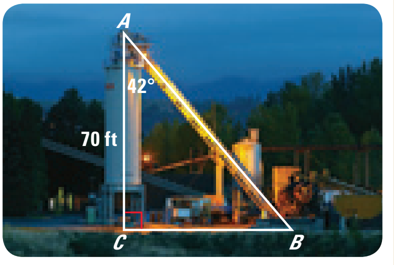
**a.** $m\angle A = \sin^{-1} 0.87 \approx 60.5°$ **b.** $m\angle B = \cos^{-1} 0.15 \approx 81.4°$

## ✓ GUIDED PRACTICE for Examples 1 and 2

1. Look back at Example 1. Use a calculator and an inverse tangent to approximate $m\angle C$ to the nearest tenth of a degree. 53.1°
2. Find $m\angle D$ to the nearest tenth of a degree if $\sin D = 0.54$. 32.7°

## EXAMPLE 3 Solve a right triangle

**Solve the right triangle. Round decimal answers to the nearest tenth.**




### Solution

**STEP 1** **Find** $m\angle B$ by using the Triangle Sum Theorem.

$$180° = 90° + 42° + m\angle B$$
$$48° = m\angle B$$

**STEP 2** **Approximate** $BC$ by using a tangent ratio.

$\tan 42° = \frac{BC}{70}$ Write ratio for tangent of 42°.

$70 \cdot \tan 42° = BC$ Multiply each side by 70.

$70 \cdot 0.9004 \approx BC$ Approximate tan 42°.

$63 \approx BC$ Simplify and round answer.

**ANOTHER WAY**

You could also find *AB* by using the Pythagorean Theorem, or a sine ratio.

**STEP 3** **Approximate** $AB$ using a cosine ratio.

$\cos 42° = \frac{70}{AB}$ Write ratio for cosine of 42°.

$AB \cdot \cos 42° = 70$ Multiply each side by *AB*.

$AB = \frac{70}{\cos 42°}$ Divide each side by cos 42°.

$AB \approx \frac{70}{0.7431}$ Use a calculator to find cos 42°.

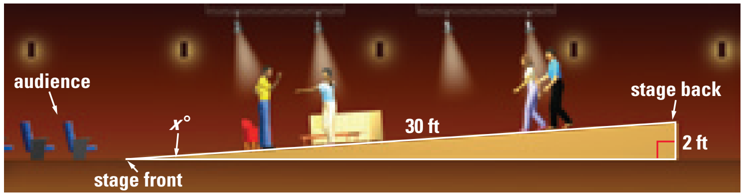
$AB \approx 94.2$ Simplify .

▶ The angle measures are 42°, 48°, and 90°. The side lengths are 70 feet, about 63 feet, and about 94 feet.

## EXAMPLE 4 Solve a real-world problem

**READ VOCABULARY**
A *raked stage* slants upward from front to back to give the audience a better view.

**THEATER DESIGN** Suppose your school is building a *raked stage*. The stage will be 30 feet long from front to back, with a total rise of 2 feet. A rake (angle of elevation) of 5° or less is generally preferred for the safety and comfort of the actors. Is the raked stage you are building within the range suggested?




**Solution**

Use the sine and inverse sine ratios to find the degree measure $x$ of the rake.

$$\sin x° = \frac{\text{opp.}}{\text{hyp.}} = \frac{2}{30} \approx 0.0667$$

$$x \approx \sin^{-1} 0.0667 \approx 3.824$$

▶ The rake is about 3.8°, so it is within the suggested range of 5° or less.

## ✓ GUIDED PRACTICE for Examples 3 and 4

3. Solve a right triangle that has a 40° angle and a 20 inch hypotenuse. **40°, 50°, and 90°, about 12.9 in., about 15.3 in. and 20 in.**

4. **WHAT IF?** In Example 4, suppose another raked stage is 20 feet long from front to back with a total rise of 2 feet. Is this raked stage safe? *Explain*. **No; the rake is 5.7° so it is slightly larger than the suggested range.**

# 7.7 EXERCISES

**HOMEWORK KEY**
○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 5, 13, and 35
★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 9, 29, 30, 35, 40, and 41
◆ = **MULTIPLE REPRESENTATIONS** Ex. 39

## SKILL PRACTICE

A

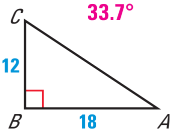
1. **VOCABULARY** Copy and complete: To solve a right triangle means to find the measures of all of its __?__ and __?__. **angles, sides**

2. ★ **WRITING** *Explain* when to use a trigonometric ratio to find a side length of a right triangle and when to use the Pythagorean Theorem. **See margin.**

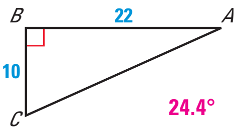
**EXAMPLE 1**
on p. 483 for Exs. 3–5

**USING INVERSE TANGENTS** **Use a calculator to approximate the measure of $\angle A$ to the nearest tenth of a degree.**

3. **33.7°**




4.




**24.4°**

5. **74.1°**